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Theory of linear magnetoelastic effects

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A theory for the elastic properties of magnetostrictive materials is presented. The complete elastic constant tensor is obtained from simple linear magnetoelastic approximations as a function of the magnetoelastic coupling coefficients and the susceptibility. It is also shown that, by using the proper dynamic susceptibility tensor, most of the observed effects can be described. More specifically the field dependence of the attenuation and the difference in the ultrasonic shear-wave velocity between the cases of **H** parallel to the polarization and that of **H** parallel to the propagation direction are explained. A very good agreement is found when the theoretical predictions are compared with some available experimental data.

INTRODUCTION

The elastic properties of magnetically ordered materials depend on the magnetic state of the sample and on the applied field. This dependence can be very large,[1] and has been measured in many different magnetic materials.[2–8] In general, these effects are detected as a dependence of both the ultrasonic wave velocity and attenuation on the intensity and direction of an applied magnetic field **H**. The principal mechanism used to explain these effects is the "Simon effect"[9] predicted directly from the first-order approximation linear-magnetoelastic theory. Higher orders of approximation give rise to other effects (morphic effect, rotational effect) which are often invoked to explain some characteristics of the experimental data unexplained by the Simon effect. This is, for example, the case of the difference in the ultrasonic shear wave velocities between the case of **H** parallel to the polarization and that of **H** parallel to the propagation direction;[5] in addition to this, the dependence of the attenuation on **H** has not yet been accounted for by the available theoretical models.[2]

All the theories for these magnetoelastic effects which have been presented to date are extensions of Simon's theory which include higher-order terms. The elastic properties are derived from the free energy by solving the coupled elastic and magnetic equations of motion.[9–11] This method necessitates the preliminary choice of the wave vector **K** and, as a result, gives the velocity and polarization of the three normal modes.

In the present paper we present a theory for the elastic properties of magnetostrictive materials which, although based on the linear magnetoelastic theory and hence describe only the Simon effect, does not make use of the equations of motion. The complete elastic constant tensor C is derived by using an extension of the formalism commonly used to calculate the magnetostriction. In this way we obtain corrections to the elastic constants which are functions of the magnetoelastic coupling coefficients and of the susceptibility. We show that most of the observed effects can be explained by using a properly calculated susceptibility tensor without the help of higher order effects.

Static linear magnetoelasticity

It is well known that in the first order of approximation the energy of a magnetoelastic system can be written as[12]

$$E=E_A+E_z+E_{\rm el}+E_{\rm me}\ , \tag{1}$$

where the anisotropy energy is

$$E_A=K_1(\alpha_1^2\alpha_2^2+\alpha_2^2\alpha_3^2+\alpha_1^2\alpha_3^2)+\cdots \tag{2}$$

for the cubic system and

$$E_A=K_1(1-\alpha_3^2)+K_2(1-\alpha_3^2)^2+\cdots$$
$$=K_1\sin^2\theta+K_2\sin^4\theta \tag{2'}$$

for the uniaxial system. The α_i are the direction cosines of the magnetization **M** and θ is the angle between **M** and the c axis.

The Zeeman energy is $E_z=-\mathbf{M}\cdot\mathbf{H}$ and the elastic one is

$$E_{\rm el}=\tfrac{1}{2}\sum_{i=1}^{6}\sum_{j=1}^{6}C_{ij}e_ie_j\ , \tag{3}$$

with the normal convention for the indexes:

$1=xx,\ \ 2=yy,\ \ 3=zz\ ,$

$4=zy,\ \ 5=zx,\ \ 6=xy\ .$

The $e_i=e_{\alpha\beta}$ are the strain tensor defined as

$$e_{\alpha\beta}=\left[\frac{\partial u_\alpha}{\partial\beta}+\frac{\partial u_\beta}{\partial\alpha}\right]\ \text{for}\ \alpha\neq\beta\ \text{and}\ e_{\alpha\alpha}=\frac{\partial u_\alpha}{\partial\alpha}\ ,$$

u_α being the α components of the particle displacement.

The number of independent elastic constants C_{ij} is limited by symmetry. One has

$$C_{ij} \Longrightarrow \begin{vmatrix} C_{11} & C_{12} & C_{12} & 0 & 0 & 0 \\ C_{12} & C_{11} & C_{12} & 0 & 0 & 0 \\ C_{12} & C_{12} & C_{11} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{44} \end{vmatrix} \tag{4}$$

for cubic systems, and

$$C_{ij} \Longrightarrow \begin{vmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\ C_{12} & C_{11} & C_{13} & 0 & 0 & 0 \\ C_{13} & C_{13} & C_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & c_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & \dfrac{C_{11}-C_{12}}{2} \end{vmatrix} \tag{4'}$$

for the uniaxial system.

If the system is isotropic, one has the additional condition

$$C_{13}=C_{12},\quad C_{33}=C_{11},\quad C_{44}=C_{55}=C_{66}=(C_{11}-C_{12})/2 .$$

The first-order approximation for the magnetoelastic energy has the general form

$$E_{\text{me}}=b_{ijK}\alpha_i\alpha_j e_K , \tag{5}$$

where i and j vary between 1 and 3 while K varies between 1 and 6.

Due to symmetry this expression reduces to

$$E_{\text{me}}=b_0(e_{xx}+e_{yy}+e_{zz})+b_1(\alpha_x^2 e_{xx}+\alpha_y^2 e_{yy}+\alpha_z^2 e_{zz}) + b_2(\alpha_x\alpha_y e_{xy}+\alpha_y\alpha_z e_{yz}+\alpha_z\alpha_x e_{zx}) , \tag{6}$$

for the cubic system, and reduces to

$$E_{\text{me}}=b_{11}(e_{xx}+e_{yy})+b_{12}e_{zz}+b_{21}(\alpha_z^2-\tfrac{1}{3})(e_{xx}+e_{yy})+b_{22}(\alpha_z^2-\tfrac{1}{3})e_{zz} + b_3[\tfrac{1}{2}(\alpha_x^2-\alpha_y^2)(e_{xx}-e_{yy})+\alpha_x\alpha_y e_{xy}]+b_4(\alpha_z\alpha_x e_{zx}+\alpha_y\alpha_z e_{yz}) \tag{6'}$$

for uniaxial symmetry.

The first step in the analysis of these systems is to find the equilibrium state. This is generally achieved by assuming that the orientation of the magnetization is the independent variable and then determining the equilibrium strain[12] by using the conditions

$$\frac{\partial E}{\partial e_i}=\frac{\partial(E_{\text{el}}+E_{\text{me}})}{\partial e_i}=0 . \tag{7}$$

From the solution of the above six equations the equilibrium strains e_i^0 can be obtained. Since

$$\frac{\partial l}{l}=\sum_{i\geq j} e_{ij}^0\beta_i\beta_j$$

is the elongation of the sample along a given direction, the well-known expressions for the magnetostriction are obtained.[12] For cubic systems,

$$\frac{\Delta l}{l}=\tfrac{3}{2}\lambda_{100}(\alpha_x^2\beta_x^2+\alpha_y^2\beta_y^2+\alpha_z^2\beta_z^2-\tfrac{1}{3}) + 3\lambda_{111}(\alpha_x\alpha_y\beta_x\beta_y+\alpha_y\alpha_z\beta_y\beta_z+\alpha_z\alpha_x\beta_z\beta_x) + \cdots \tag{8}$$

(the ellipsis denotes terms independent of α_i), where

$$\lambda_{100}=-2b_1/[3(C_{11}-C_{12})] ,$$
$$\lambda_{111}=-b_2/(3C_{44}) ,$$

and, for uniaxial systems,

$$\frac{\Delta l}{l}=\lambda_A(\alpha_z^2-\tfrac{1}{3})(\beta_x^2+\beta_y^2)+\lambda_B(\alpha_z^2-\tfrac{1}{3})\beta_z^2 + \lambda_c[\tfrac{1}{2}(\alpha_x^2-\alpha_y^2)(\beta_x^2-\beta_y^2)+2\alpha_x\alpha_y\beta_x\beta_y] + 2\lambda_D(\alpha_y\alpha_z\beta_y\beta_z+\alpha_z\alpha_x\beta_z\beta_x) + \cdots \tag{8'}$$

(the ellipsis denotes terms independent of α_i), where

$$\lambda_A=(-b_{21}C_{33}+b_{22}C_{13})/D ,$$
$$\lambda_B=[2b_{21}C_{13}-b_{22}(C_{11}+C_{12})]/D ,$$
$$\lambda_C=-b_3/(C_{11}-C_{12}),$$
$$\lambda_D=-b_4/2C_{44} ,\quad D=C_{33}(C_{11}+C_{12})-2C_{13}^2 .$$

One can also immediately obtain the equilibrium volume magnetostriction

$$\frac{\Delta V}{V}=e_1^0+e_2^0+e_3^0 ,$$

and for the cubic system it results in being equal to zero. On the contrary, in the case of uniaxial system it is not null and it results in

$$\frac{\Delta V}{V}=(\alpha_3^2-\tfrac{1}{3})\frac{2b_{21}(C_{13}-C_{33})+b_{22}(2C_{13}-C_{11}-C_{12})}{C_{33}(C_{11}+C_{12})-2C_{13}^2} .$$

The energy of the equilibrium state can now be obtained by substituting in E the expressions of the equilibrium strains. The expression for E obtained in this manner depends only on the magnetization **M**. The elastic and magnetoelastic energy terms have now the same form as that of the anisotropy. Their coefficients can be seen as corrections of the second- and fourth-order magnetocrystalline anisotropy.

In the cubic case one obtains

$$\Delta K_1=\frac{b_1^2}{C_{11}-C_{12}}-\frac{b_2^2}{2C_{44}} . \tag{9}$$

Also for the uniaxial systems, the ΔK are not null and they result

$$\begin{aligned}\Delta K_1=&-b_4^2/2C_{44}-\{\tfrac{2}{3}[4b_{21}b_{22}C_{13}-2b_{21}^2C_{33}-b_{22}^2(C_{11}+C_{12})]\\&+[2b_{11}(b_{22}C_{13}-b_{21}C_{33})+2b_{12}b_{21}C_{13}-b_{12}b_{22}(C_{11}+C_{12})]\}/D ,\\ \Delta K_2=&b_4^2/2C_{44}-b_3^2/4(C_{11}-C_{12})+\tfrac{1}{2}[4b_{21}b_{22}C_{13}-2b_{21}^2C_{33}-b_{22}^2(C_{11}+C_{12})]/D .\end{aligned} \tag{9'}$$

At this point the energy depends only on the orientation of the magnetization. Hence the equilibrium direction of $\mathbf{M}$, for a given applied field, is immediately found by minimizing the energy with respect to the M rotations.

One thus obtains both the magnetization curves M versus H, often in implicit forms, and the equilibrium magnetization M for a given field. Knowing the equilibrium state of the system, one can now investigate its response to a magnetic or elastic perturbation.

RESPONSE TO A MAGNETIC PERTURBATION

We can now calculate the response of our system to a perturbing magnetic field $\mathbf{h}$, small compared with the static applied field and/or with the anisotropy.

In insulating materials or at low frequency one can ignore the screening effects; one, moreover, can assume that the elastic distortion rearranges itself immediately to follow any rotation of the magnetization; i.e., the relaxation time for the magnetostriction is short compared with the considered frequency. Consequently the energy of our system does not explicitly contain any elastic or magnetoelastic terms, providing that the effective anisotropy constants $K_i=K_i^0+\Delta K_i$ are used. If, for a given field $\mathbf{H}^0$, θ_0 and φ_0 correspond to the equilibrium magnetization $\mathbf{M}^0$ in a polar-coordinates reference system, after the perturbation the magnetization will rotate to a new θ and φ orientation, where $\theta=\theta_0+\epsilon$ and $\varphi=\varphi^0+\xi$. ϵ and ξ are infinitesimal quantities for infinitesimal perturbations.

Expanding the energy around the θ_0,φ_0 equilibrium position, one can write

$$E=E_0+E_{\theta\theta}\epsilon^2+E_{\varphi\varphi}\xi^2-\mathbf{M}\cdot\mathbf{h} , \tag{10}$$

where

$$E_{\theta\theta}=\frac{1}{2}\left.\frac{\partial^2E(H_0)}{\partial\theta^2}\right|_{\theta=\theta_0\,\varphi=\varphi_0} ,$$

$$E_{\varphi\varphi}=\frac{1}{2}\left.\frac{\partial^2E(H_0)}{\partial\varphi^2}\right|_{\theta=\theta_0\,\varphi=\varphi_0} ,$$

and in which it has been considered that, at equilibrium,

$$E_{\theta\varphi}=\frac{1}{2}\left.\frac{\partial^2E(H_0)}{\partial\theta\,\partial\varphi}\right|_{\theta=\theta_0\,\varphi=\varphi_0}=0 .$$

Minimizing Eq. (10) with respect to ϵ and to ξ, one immediately obtains

$$\epsilon_0=(M_s/2E_{\theta\theta})(h_x\cos\theta_0\cos\varphi_0+h_y\cos\theta_0\sin\varphi_0-h_z\sin\theta_0) ,$$

$$\xi_0=(M_s/2E_{\varphi\varphi}(-h_x\sin\theta_0\sin\varphi_0+h_y\sin\theta_0\cos\varphi_0) .$$

Defining the response to the perturbation as $\mathbf{m}=\mathbf{M}-\mathbf{M}^0$ and writing it in terms of the usual differential susceptibility tensor $\mathbf{m}=\boldsymbol{\chi}\cdot\mathbf{h}$, one finds

$$\begin{aligned}\chi_{xx}&=M_s^2\left[\frac{\cos^2\theta_0\cos^2\varphi_0}{E_{\theta\theta}}+\frac{\sin^2\theta_0\sin^2\varphi_0}{E_{\varphi\varphi}}\right] ,\\ \chi_{yy}&=M_s^2\left[\frac{\cos^2\theta_0\sin^2\varphi_0}{E_{\theta\theta}}+\frac{\sin^2\theta_0\cos^2\varphi_0}{E_{\varphi\varphi}}\right] ,\\ \chi_{zz}&=M_s^2\frac{\sin^2\theta_0}{E_{\theta\theta}} ,\\ \chi_{yz}&=-M_s^2\frac{\sin\theta_0\cos\theta_0\sin\varphi_0}{E_{\theta\theta}} ,\\ \chi_{xz}&=-M_s^2\frac{\sin\theta_0\cos\theta_0\cos\varphi_0}{E_{\theta\theta}} ,\\ \chi_{xy}&=-M_s^2\left[\frac{\cos^2\theta_0\cos\varphi_0\sin\varphi_0}{E_{\theta\theta}}-\frac{\sin^2\theta_0\cos\varphi_0\sin\varphi_0}{E_{\varphi\varphi}}\right] .\end{aligned} \tag{11}$$

Moreover, one obtains the energy difference between the perturbed and unperturbed states:

$$\Delta E=-\mathbf{M}_0\cdot\mathbf{h}-\tfrac{1}{2}\mathbf{m}\cdot\mathbf{h} . \tag{12}$$

RESPONSE TO AN ELASTIC PERTURBATION

If, at constant field, we apply an elastic perturbation as an externally induced distorsion having components e_i, then the magnetization vector, due to the magnetoelastic coupling, will rotate away from its equilibrium position. The new values of the direction cosines will be $\alpha_i=\alpha_i^0+a_i$.

The energy variation of the system, induced by the perturbation, is

$$\Delta E=E(\alpha_i^0+a_i,e_j^0+e_j)-E(\alpha_i^0,e_j^0) .$$

Adding and subtracting from ΔE a term $E(\alpha_i^0+a_i,e_j^0)$, one can see that

$$\Delta E_1 = E(\alpha_i^0 + a_i, e_j^0) - E(\alpha_i^0, e_j^0)$$

represents the energy variation induced by a small rotation of $\mathbf{M}$; hence it is equal to what we have calculated in the case of magnetic perturbation. This term can then be written $\Delta E = E_{\theta\theta}\epsilon^2 + E_{\varphi\varphi}\xi^2$ as in Eq. (10). The energy difference

$$\Delta E_2 = E(\alpha_i^0 + a_i, e_j^0 + e_j) - E(\alpha_i^0 + a_i, e_j^0)$$

has to be calculated by explicitly writing the different energy terms. The magnetocrystalline and Zeeman terms do not contribute to ΔE_2. Moreover, in ΔE_2 there are terms having the form $(C_{ik}e_i^0 + b_{ijK}\alpha_i^0\alpha_j^0)e_K$. These terms are null because the unperturbed system is in equilibrium. In fact, the term in parentheses has to be zero according to Eq. (7). The only terms of ΔE_2 different from zero have the form

$$\Delta E_2 = b_{ijK}\alpha_i^0 a_j e_K + \tfrac{1}{2} C_{ij} e_i e_j \ .$$

The terms of ΔE_2 linear in a_j can be seen as being due to the effect of a perturbing field h^{me} having components

$$h_j^{\mathrm{me}} = \frac{1}{M_s}\frac{\partial E}{\partial a_j} \propto \frac{1}{M_s}(b_{ijK}\alpha_i^0 e_K) \ . \tag{13}$$

Hence, the total perturbation energy takes the form

$$\Delta E = E_{\theta\theta}\epsilon^2 + E_{\varphi\varphi}\xi^2 - \mathbf{m}\cdot\mathbf{h}^{\mathrm{me}} + \tfrac{1}{2} C_{ij} e_i e_j \ ,$$

where $m_i = M_s a_i$ are the components of $\mathbf{m}$.

The magnetic part of ΔE has the same dependence on the rotation of $\mathbf{M}$ calculated in the case of a magnetic perturbation. The expression for the corresponding energy variation has been given in Eq. (12). We can then write

$$\Delta E = -\tfrac{1}{2}\mathbf{m}\cdot\mathbf{h}^{\mathrm{me}} + \tfrac{1}{2} C_{ij} e_i e_j \ . \tag{14}$$

The response of $\mathbf{m}$ to $\mathbf{h}^{\mathrm{me}}$ can be written as $\mathbf{m} = \chi\mathbf{h}^{\mathrm{me}}$, where χ is an appropriate susceptibility tensor. For low frequencies it coincides with the one calculated for a magnetic perturbation and given in Eq. (11). At higher frequency, dynamic screening effects are present. They will be considered in the following paragraph. Substituting $\mathbf{m} = \chi\mathbf{h}^{\mathrm{me}}$ in ΔE, one obtains

$$\Delta E = -\tfrac{1}{2}\chi_{ij} h_i^{\mathrm{me}} h_j^{\mathrm{me}} + \tfrac{1}{2} C_{ij} e_i e_j \ .$$

Being $\mathbf{h}^{\mathrm{me}}$ linear in the strains e_i, according to the definition, one can finally write

$$\Delta E = \tfrac{1}{2}(C_{ij} - \Delta C_{ij}) e_i e_j \ , \tag{15}$$

where

$$\Delta C_{ij} = (1/M_s^2) b_{pqi} b_{rsj} \chi_{qs} \alpha_p^0 \alpha_r^0 \ . \tag{16}$$

Equation (15) indicates that the magnetoelastic interaction can be ignored in the study of the elastic properties of these systems, provided that the elastic constants C_{ij} are corrected by the ΔC_{ij} terms. One must note that the ΔC_{ij} can reduce the symmetry of the system: an isotropic one can be reduced to an uniaxial, a hexagonal to an orthorhombic, etc.

The importance of having an explicit expression for all the effective elastic constants, knowing χ and the direction of $\mathbf{M}$, is that one can then predict all the elastic properties of the systems. The results of the explicit calculation for the magnetoelastic field components and for ΔC_{ij} are reported in Table I for the cubic system and in Table II for the hexagonal one. Finally it must be noted that it is possible to derive the expressions for an isotropic system (random polycrystals) either from the cubic or from the hexagonal case by making the following substitutions. For the elastic constants,

$$C_{11} = C_{22} = C_{33} = \lambda + 2\mu \ ,$$
$$C_{12} = C_{23} = C_{13} = \lambda \ ,$$
$$C_{44} = C_{55} = C_{66} = (C_{11} - C_{12})/2 = \mu \ .$$

TABLE I. Magnetoelastic field components h and elastic constant corrections ΔC_{ij} for a cubic system.

$$h_x^{\mathrm{me}} = -[2b_1 e_1 \alpha_1^0 + b_2(e_6\alpha_2^0 + e_5\alpha_3^0)]/M_s$$
$$h_y^{\mathrm{me}} = -[2b_1 e_2 \alpha_2^0 + b_2(e_6\alpha_1^0 + e_4\alpha_3^0)]/M_s$$
$$h_z^{\mathrm{me}} = -[2b_1 e_3 \alpha_3^0 + b_2(e_5\alpha_1^0 + e_4\alpha_2^0)]/M_s$$

$$\Delta C_{11} = -[4b_1^2\chi_{xx}(\alpha_1^0)^2]/M_s^2$$
$$\Delta C_{22} = -[4b_1^2\chi_{yy}(\alpha_2^0)^2]/M_s^2$$
$$\Delta C_{33} = -[4b_1^2\chi_{zz}(\alpha_3^0)^2]/M_s^2$$
$$\Delta C_{44} = -b_2^2[\chi_{zz}(\alpha_2^0)^2 + 2\chi_{yz}\alpha_2^0\alpha_3^0 + \chi_{yy}(\alpha_3^0)^2]/M_s^2$$
$$\Delta C_{55} = -b_2^2[\chi_{xx}(\alpha_3^0)^2 + \chi_{zz}(\alpha_1^0)^2 + 2\chi_{xz}\alpha_1^0\alpha_3^0]/M_s^2$$
$$\Delta C_{66} = -b_2^2[\chi_{xx}(\alpha_2^0)^2 + \chi_{yy}(\alpha_1^0)^2 + 2\chi_{xy}\alpha_1^0\alpha_2^0]/M_s^2$$
$$\Delta C_{12} = -(4b_1^2\chi_{xy}\alpha_1^0\alpha_2^0)/M_s^2$$
$$\Delta C_{13} = -(4b_1^2\chi_{xz}\alpha_1^0\alpha_3^0)/M_s^2$$
$$\Delta C_{23} = -(4b_1^2\chi_{yz}\alpha_2^0\alpha_3^0)/M_s^2$$
$$\Delta C_{14} = -2b_1b_2(\chi_{xy}\alpha_1^0\alpha_3^0 + \chi_{xz}\alpha_1^0\alpha_2^0)/M_s^2$$
$$\Delta C_{15} = -2b_1b_2[\chi_{xx}\alpha_1^0\alpha_3^0 + \chi_{xz}(\alpha_1^0)^2]/M_s^2$$
$$\Delta C_{16} = -2b_1b_2[\chi_{xx}\alpha_1^0\alpha_2^0 + \chi_{xy}(\alpha_1^0)^2]/M_s^2$$
$$\Delta C_{24} = -2b_1b_2[\chi_{yy}\alpha_2^0\alpha_3^0 + \chi_{yz}(\alpha_2^0)^2]/M_s^2$$
$$\Delta C_{25} = -2b_1b_2(\chi_{xy}\alpha_2^0\alpha_3^0 + \chi_{yz}\alpha_1^0\alpha_2^0)/M_s^2$$
$$\Delta C_{26} = -2b_1b_2[\chi_{xy}(\alpha_2^0)^2 + \chi_{yy}\alpha_1^0\alpha_2^0]/M_s^2$$
$$\Delta C_{34} = -2b_1b_2[\chi_{yz}(\alpha_3^0)^2 + \chi_{zz}\alpha_2^0\alpha_3^0]/M_s^2$$
$$\Delta C_{35} = -2b_1b_2[\chi_{xz}(\alpha_3^0)^2 + \chi_{zz}\alpha_1^0\alpha_3^0]/M_s^2$$
$$\Delta C_{36} = -2b_1b_2(\chi_{xz}\alpha_2^0\alpha_3^0 + \chi_{yz}\alpha_1^0\alpha_3^0)/M_s^2$$
$$\Delta C_{45} = -b_2^2[\chi_{xy}(\alpha_3^0)^2 + \chi_{xz}\alpha_2^0\alpha_3^0 + \chi_{yz}\alpha_1^0\alpha_3^0 + \chi_{zz}\alpha_1^0\alpha_2^0]/M_s^2$$
$$\Delta C_{46} = -b_2^2[\chi_{xy}\alpha_2^0\alpha_3^0 + \chi_{xz}(\alpha_2^0)^2 + \chi_{yy}\alpha_1^0\alpha_3^0 + \chi_{yz}\alpha_1^0\alpha_2^0]/M_s^2$$
$$\Delta C_{56} = -b_2^2[\chi_{xx}\alpha_2^0\alpha_3^0 + \chi_{xy}\alpha_1^0\alpha_3^0 + \chi_{xz}\alpha_1^0\alpha_2^0 + \chi_{yz}(\alpha_1^0)^2]/M_s^2$$

TABLE II. Same as Table I, for a hexagonal system.

$$h_x^{\mathrm{me}} = -[b_3\alpha_1^0(e_1-e_2)+b_3\alpha_2^0 e_6+b_4\alpha_3^0 e_5]/M_s$$
$$h_y^{\mathrm{me}} = -[b_3\alpha_2^0(e_2-e_1)+b_3\alpha_1^0 e_6+b_4\alpha_3^0 e_4]/M_s$$
$$h_z^{\mathrm{me}} = -[2b_{21}\alpha_3^0(e_1+e_2)+2b_{22}\alpha_3^0 e_3+b_4\alpha_1^0 e_5+b_4\alpha_2^0 e_4]/M_s$$

$$\Delta C_{11}=\{-b_3^2[\chi_{xx}(\alpha_1^0)^2+\chi_{yy}(\alpha_2^0)^2-\chi_{xy}\alpha_1^0\alpha_2^0]+2b_{21}b_3\alpha_3^0(\chi_{yz}\alpha_2^0-\chi_{xz}\alpha_1^0)-\chi_{zz}4b_{21}^2(\alpha_3^0)^2\}/M_s^2$$
$$\Delta C_{22}=\{-b_3^2[\chi_{xx}(\alpha_1^0)^2+\chi_{yy}(\alpha_2^0)^2-\chi_{xy}\alpha_1^0\alpha_2^0]+2b_{21}b_3\alpha_3^0(\chi_{xz}\alpha_1^0-\chi_{yz}\alpha_2^0)-\chi_{zz}4b_{21}^2(\alpha_3^0)^2\}/M_s^2$$
$$\Delta C_{33}=-\chi_{zz}4b_{22}^2(\alpha_3^0)^2/M_s^2$$
$$\Delta C_{44}=-b_4^2[\chi_{zz}(\alpha_2^0)^2+\chi_{yy}(\alpha_3^0)^2+\chi_{yz}\alpha_2^0\alpha_3^0]/M_s^2$$
$$\Delta C_{55}=-b_4^2[\chi_{xx}(\alpha_3^0)^2+\chi_{zz}(\alpha_1^0)^2+\chi_{xz}\alpha_1^0\alpha_3^0]/M_s^2$$
$$\Delta C_{66}=-b_3^2[\chi_{xx}(\alpha_2^0)^2+\chi_{yy}(\alpha_1^0)^2+\chi_{xy}\alpha_1^0\alpha_2^0]/M_s^2$$
$$\Delta C_{12}=\{b_3^2[\chi_{xx}(\alpha_1^0)^2+\chi_{yy}(\alpha_2^0)^2-\chi_{xy}\alpha_1^0\alpha_2^0]-4b_{21}^2(\alpha_3^0)^2\chi_{zz}\}/M_s^2$$
$$\Delta C_{13}=[-b_{22}b_3(x_{xz}\alpha_1^0\alpha_3^0-\chi_{yz}\alpha_2^0\alpha_3^0)-4\chi_{zz}b_{21}b_{22}(\alpha_3^0)^2]/M_s^2$$
$$\Delta C_{14}=-\tfrac{1}{2}b_4\{b_3[-2\chi_{yy}\alpha_2^0\alpha_3^0+\chi_{xy}\alpha_1^0\alpha_3^0+\chi_{xz}\alpha_1^0\alpha_2^0-\chi_{yz}(\alpha_2^0)^2]+b_{21}[4\chi_{zz}\alpha_2^0\alpha_3^0+\chi_3^0+2\chi_{yz}(\alpha_3^0)^2]\}/M_s^2$$
$$\Delta C_{15}=-\tfrac{1}{2}b_4\{b_3[2\chi_{xx}\alpha_1^0\alpha_3^0-\chi_{xy}\alpha_2^0\alpha_3^0-\chi_{yz}\alpha_1^0\alpha_2^0+\chi_{xz}(\alpha_1^0)^2]+b_{21}[2(\alpha_3^0)^2\chi_{xz}+4\alpha_1^0\alpha_3^0\chi_{zz}]\}/M_s^2$$
$$\Delta C_{16}=-\tfrac{1}{2}b_3\{2b_3\alpha_1^0\alpha_2^0(\chi_{xx}-\chi_{yy})+b_3\chi_{xy}[(\alpha_1^0)^2-(\alpha_2^0)^2]+2b_{21}\alpha_3^0(\alpha_2^0\chi_{xz}+\alpha_1^0\chi_{yz})\}/M_s^2$$
$$\Delta C_{23}=[b_3b_{22}\alpha_3^0(\alpha_1^0\chi_{xz}-\alpha_2^0\chi_{yz})-4b_{21}b_{22}(\alpha_3^0)^2\chi_{zz}]/M_s^2$$
$$\Delta C_{24}=-\tfrac{1}{2}b_4\{b_3[2\chi_{yy}\alpha_2^0\alpha_3^0-\chi_{xy}\alpha_1^0\alpha_3^0-\chi_{xz}\alpha_1^0\alpha_2^0+\chi_{yz}(\alpha_2^0)^2]+b_{21}[4\chi_{zz}\alpha_2^0\alpha_3^0+2\chi_{yz}(\alpha_3^0)^2]\}/M_s^2$$
$$\Delta C_{25}=-\tfrac{1}{2}b_4\{b_3[-2\chi_{xx}\alpha_1^0\alpha_3^0+\chi_{xy}\alpha_2^0\alpha_3^0-\chi_{xz}(\alpha_1^0)^2+\chi_{yz}\alpha_1^0\alpha_2^0]+b_{21}[4\chi_{zz}\alpha_1^0\alpha_3^0+2\chi_{xz}(\alpha_3^0)^2]\}/M_s^2$$
$$\Delta C_{26}=-\tfrac{1}{2}b_3\{b_3[-2\alpha_1^0\alpha_2^0\chi_{xx}+2\alpha_1^0\alpha_2^0\chi_{yy}+(\alpha_2^0)^2\chi_{xy}-(\alpha_1^0)^2\chi_{xy}]+b_{21}(2\alpha_2^0\alpha_3^0\chi_{xz}+2\alpha_1^0\alpha_3^0\chi_{yz})\}/M_s^2$$
$$\Delta C_{34}=-b_{22}b_4\alpha_3^0(2\alpha_2^0\chi_{zz}+\alpha_3^0\chi_{yz})/M_s^2$$
$$\Delta C_{35}=-b_{22}b_4\alpha_3^0(2\alpha_1^0\chi_{zz}+\alpha_3^0\chi_{xz})/M_s^2$$
$$\Delta C_{36}=-b_{22}b_3\alpha_3^0(\chi_{xz}\alpha_2^0+\chi_{yz}\alpha_1^0)/M_s^2$$
$$\Delta C_{45}=-\tfrac{1}{2}b_4^2[2\alpha_1^0\alpha_2^0\chi_{zz}+(\alpha_3^0)^2\chi_{xy}+\alpha_2^0\alpha_3^0\chi_{xz}+\alpha_1^0\alpha_3^0\chi_{yz}]/M_s^2$$
$$\Delta C_{46}=-\tfrac{1}{2}b_3b_4[2\alpha_1^0\alpha_3^0\chi_{yy}+\alpha_2^0\alpha_3^0\chi_{xy}+(\alpha_2^0)^2\chi_{xz}+\alpha_1^0\alpha_2^0\chi_{yz}]/M_s^2$$
$$\Delta C_{56}=-\tfrac{1}{2}b_3b_4[2\alpha_2^0\alpha_3^0\chi_{xx}+\alpha_1^0\alpha_3^0\chi_{xy}+\alpha_1^0\alpha_2^0\chi_{xz}+(\alpha_1^0)^2\chi_{yz}]/M_s^2$$

For the magnetoelastic constants,

$$b_1=b_2=b$$

starting from cubic, and

$$b_{21}=-b/2,\quad b_{22}=b_3=b,\quad b_4=b/2$$

starting from hexagonal.

SCREENING EFFECTS ON THE SUSCEPTIBILITY TENSOR

To have a complete description of the elastic properties of a magnetostrictive material, within the approximation of the linear magnetoelastic theory, a further analysis of the susceptibility tensor is necessary. χ has already been calculated for the low-frequency limit, where the screening effects can be ignored. For higher frequencies, the dynamic effects have to be taken into consideration. Moreover, one must take into account the fact that the driving field h^{me} is not a true magnetic field but only a description of the internal coupling between **M** and **e**.

These dynamic effects can be calculated by solving the Maxwell equations for the general case of a magnetic conducting media, having conductivity σ. In the cgs system, assuming the dielectric constant to be $\epsilon=1$ and having no free charges, one has

$$\nabla\cdot\mathbf{E}=0\,, \tag{17a}$$

$$\nabla\cdot\mathbf{B}=0\,, \tag{17b}$$

$$\nabla\times\mathbf{E}=\frac{-1}{c}\frac{\partial\mathbf{B}}{\partial t}\,, \tag{17c}$$

$$\nabla\times\mathbf{B}=\frac{1}{c}\left[4\pi\sigma\mathbf{E}+\frac{\partial\mathbf{E}}{\partial t}\right]+4\pi\nabla\times\mathbf{M}\,. \tag{17d}$$

In $\mathbf{B}=\mathbf{H}+4\pi\mathbf{M}$ we have to consider that $\mathbf{H}=\mathbf{H}_0+\mathbf{h}'$ where H_0 is the static field and h' is the screening field to be calculated. h^{me} does not contribute to $\mathbf{H}$ because it is not a true magnetic field. In $\mathbf{M}=\mathbf{M}_0+\mathbf{m}$, we have to introduce the response of M to the perturbation: $\mathbf{m}=\chi^0(h'+h^{\text{me}})$, where the susceptibility tensor χ^0, for frequencies well below the ferromagnetic resonance, can be assumed as equal to the static susceptibility calculated in Eq. (11); all the dynamic effects can be considered as being described by the screening field $\mathbf{h}'$. Simplifying the $4\pi\nabla\times\mathbf{M}$ in Eq. (17d), taking the curl of both the terms of this equation, and then substituting Eq. (17c) in it, one obtains

$$\nabla\times(\nabla\times\mathbf{H})=-\frac{1}{c^2}\frac{\partial^2\mathbf{B}}{\partial t^2}-\frac{4\pi\sigma}{c^2}\frac{\partial\mathbf{B}}{\partial t}\ .$$

Assuming plane-wave solutions for h' and m, $h'=h'_0\exp[i(\mathbf{K}\cdot\mathbf{r}-\omega t)]$ and $\mathbf{m}=\mathbf{m}_0\exp[i(\mathbf{K}\cdot\mathbf{r}-\omega t)]$, propagating at the same frequency and with the same wave vector $|\mathbf{K}|=\omega/v$ as the elastic perturbation, one has

$$(\mathbf{h}'_0\cdot\mathbf{K})\mathbf{K}-|\mathbf{K}|^2\cdot\mathbf{h}'_0=\left[\frac{\omega^2}{c^2}-i\frac{4\pi}{c^2}\sigma\omega\right]\times[(\mathbb{1}+4\pi\chi_0)\mathbf{h}'_0+4\pi\chi_0\mathbf{h}^{\text{me}}]\ .$$

This vector equation can in general be solved for the three components of $\mathbf{h}'_0$. Then from h'_0, using the simple expression

$$\chi\mathbf{h}_0^{\text{me}}=\chi^0(\mathbf{h}'_0+\mathbf{h}_0^{\text{me}})\ ,$$

one obtains the effective dynamic susceptibility χ.

If one chooses a reference system with the z axis parallel to $\mathbf{M}$, then the susceptibility tensor is simply

$$\chi^0=\begin{bmatrix}\chi^0_{xx} & 0 & 0\\ 0 & \chi^0_{yy} & 0\\ 0 & 0 & 0\end{bmatrix}$$

when the orientation of the x and y axes is properly chosen. If, moreover, $\mathbf{K}$ is parallel to one of the principal axes, so as to have $K_iK_j=0$ if $i\neq j$, one then obtains $\chi_{ij}=0$ if $i\neq j$; $\chi_{zz}=0$ and

$$\chi_{ii}=\left[1+\frac{[1-i(\omega_s/\omega)]4\pi\chi^0_{ii}}{(c^2/v^2)(1-\hat{K}_i^2)-[1-i(\omega_s/\omega)](1+4\pi\chi^0_{ii})}\right]\chi^0_{ii}\ , \tag{18}$$

for $i=x,y$, where one has defined $\omega_s=4\pi\sigma$ and $\hat{K}=\mathbf{K}/|K|$.

Separating the real and imaginary part of χ_{ii}, using $\mu^0_{ii}=1+4\pi\chi^0_{ii}$ and $|\mathbf{K}|=\omega/v$, one obtains

$$\begin{aligned}\text{Re}\chi_{ii}&=\left[4\pi\chi^0_{ii}\frac{(c^2/v^2)(1-\hat{K}_i^2)-(1+\omega_s^2/\omega^2)\mu^0_{ii}}{[(c^2/v^2)(1-\hat{K}_i^2)-\mu^0_{ii}]^2+(\omega_s^2/\omega^2)(\mu^0_{ii})^2}+1\right]\chi^0_{ii}\ ,\\ \text{Im}\chi_{ii}&=-\left[4\pi\chi^0_{ii}\frac{(\omega_s/\omega)(c^2/v^2)(1-\hat{K}_i)^2}{[(c^2/v^2)(1-\hat{K}_i^2)-\mu^0_{ii}]^2+(\omega_s^2/\omega^2)(\mu^0_{ii})^2}\right]\chi^0_{ii}\ .\end{aligned} \tag{19}$$

The susceptibility for given directions of the wave vector $\mathbf{K}$ can now be calculated. If $\mathbf{K}||\mathbf{M}||Z$ the transverse components of χ are modified as follows:

$$\begin{aligned}\text{Re}\chi_{ii}&=\left[4\pi\chi^0_{ii}\frac{c^2/v^2-(1+\omega_s^2/\omega^2)\mu^0_{ii}}{(c^2/v^2-\mu^0_{ii})^2+(\omega_s^2/\omega^2)(\mu^0_{ii})^2}+1\right]\chi^0_{ii}\ ,\\ \text{Im}\chi_{ii}&=-\left[4\pi\chi^0_{ii}\frac{(\omega_s/\omega)(c^2/v^2)}{(c^2/v^2-\mu^0_{ii})^2+(\omega_s^2/\omega^2)(\mu^0_{i})^2}\right]\chi^0_{ii}\ .\end{aligned} \tag{20}$$

If $\mathbf{K}$ is perpendicular to $\mathbf{M}$, for the components of χ perpendicular to both $\mathbf{K}$ and $\mathbf{M}$, one obtains the same corrections for χ_{ii} as above. On the other end, for the component parallel to $\mathbf{K}$ one obtains

$$\begin{aligned}\text{Re}\chi_{||K}&=\frac{1}{1+4\pi\chi^0_{ii}}\chi^0_{ii}\ ,\\ \text{Im}\chi_{||K}&=0\ .\end{aligned} \tag{21}$$

The above results indicate a dependence of the susceptibility tensor on the propagation direction. This effect will allow us to give a more complete interpretation of the available experimental data. More specifically the difference in the transverse susceptibility components, parallel and perpendicular to $\mathbf{K}$, will allow us to explain both the presence of an attenuation only in the case of the $\mathbf{K}||\mathbf{M}\perp\mathbf{u}$ configuration and the difference in the ultrasonic shear wave velocities between the $\mathbf{K}||\mathbf{M}\perp\mathbf{U}$ and the $\mathbf{K}\perp\mathbf{M}||\mathbf{U}$

TABLE III. Comparison of the theoretical and experimental data for nickel single crystal. Shear waves at 10 MHz propagating along [110] at $H_{int}=8$ KOe. We have assumed $\lambda_{111}=-19\times10^{-6}$, $\lambda_{100}=-50\times10^{-6}$, $M_s=484$ emu/cm^3, $C_{44}=1.235\times10^{12}$ dyn/cm^2, $(C_{11}-C_{12})/2=0.504\times10^{12}$ dyn/cm^2, and $\sigma=15\times10^{16}$ sec^{-1}. Experimental data are from Ref. 2.

			Theoretical	Experimental
$\mathbf{u}\|\|[1\bar{1}0]$	$\mathbf{M}\|\|[110]$	$\Delta V/V_0$	11.35×10^{-4}	13.5×10^{-4}
		δ (Np/cycle)	10.95×10^{-4}	10.4×10^{-4}
	$(\Delta V/V_0)_{\mathbf{M}\|\|[110]}-(\Delta V/V_0)_{\mathbf{M}\|\|[1\bar{1}0]}$		3.03×10^{-4}	2×10^{-4}
$\mathbf{u}\|\|[001]$	$\mathbf{M}\|\|[110]$	$\Delta V/V_0$	3.07×10^{-4}	5.3×10^{-4}
		δ (Np/cycle)	1.7×10^{-4}	$<3.0\times10^{-4}$
	$(\Delta V/V_0)_{\mathbf{M}\|\|[110]}-(\Delta V/V_0)_{\mathbf{M}\|\|[001]}$		0.12×10^{-4}	$<0.3\times10^{-4}$

cases. We can also note that this difference increases with decreasing conductivity.

DISCUSSION

Our approach to the magnetoelastic effects, within the approximation of the linear magnetoelastic theory, makes it possible to calculate the correction to the elastic constants, without fixing *a priori* the direction of propagation and without solving the relative coupled equations of motion. The resulting corrections to C have the form

$$\Delta C=\frac{b^2}{M_s^2}\alpha_i^0\alpha_j^0\chi \ .$$

Although the susceptibility tensor in this expression differs from the static susceptibility χ^0 for dynamic corrections, these can at first be ignored for a qualitative analysis of the elastic properties of the system. Then, once the applied field and consequently the orientation of $\mathbf{M}$ and χ^0 are known, one can immediately predict the symmetry of the elastic properties of the system. The nature of the normal modes of acoustic wave propagation as well as their degree of magnetoelastic coupling can directly be calculated. Moreover, also peculiar observed effects, such as the acoustic birefringence[13] or the resonance and polarization rotation,[14] can be easily predicted. It must also be noted that the ΔC reported in Tables I and II are consistent with those reported in literature,[4,9] which are calculated, for specific cases, by solving the coupled elastic and magnetic equations of motion.

To obtain correct quantitative values for the variation of the velocity of the elastic waves and of their attenuation, one must calculate the dynamic susceptibility tensor following the procedure described in the above paragraph. For the two main cases of propagation, parallel or perpendicular to $\mathbf{M}$, one can distinguish two dynamic effects on χ. The first is the occurrence of a term $4\pi\mathbf{M}$ to be added to the applied field (and eventually to the anisotropy) in the denominator of the susceptibility. This effect occurs in the component of χ parallel to the propagation direction, and it has been attributed to a dynamic demagnetizing field.[4] The second effect is due to the electron conduction screening; it is frequency and conductivity dependent, and it shows up both as a reduction of the real part χ' and in the appearance of an imaginary part χ'' in the susceptibility components perpendicular to the propagation direction. The presence of χ'' results in an imaginary part $\Delta C''$ in the elastic constants and hence it gives rise to a logarithmic attenuation (in units of Np/cycle) for the elastic waves:

$$\delta=2\pi\frac{\Delta C''}{2C^0} \ .$$

It is this difference between the susceptibility components parallel to the propagation and perpendicular to it that makes it possible to explain the experimental data relative to both the different velocity of the $\mathbf{K}\|\mathbf{M}\perp\mathbf{u}$ and the $\mathbf{K}\perp\mathbf{M}\|\mathbf{u}$ cases and the different attenuation of the various modes.

A comparison between the theoretical predictions and some available experimental data[2,8] is reported in Tables III and IV. The comparison has been made for shear waves in nickel single crystal and in $Pr_2(Co_{0.81}Fe_{0.19})_{17}$ polycrystal for values of the internal field (8 and 10 KOe, respectively) high enough to have complete magnetic saturation.

For nickel crystal we have considered the case of propagation $\mathbf{K}$ along the [110] direction, polarization $\mathbf{u}$ along $[1\bar{1}0]$ or [001], and magnetization either parallel to $\mathbf{K}$ or to $\mathbf{u}$. Considering all the effective elastic constants that, due to the magnetoelastic corrections, can result different from zero in our cases, we have the following expressions for the shear wave velocities:

$$\rho v^2=(C_{11}-C_{12})/2 \quad \text{for } \mathbf{K}\,\|[110],\ \mathbf{u}\|[1\bar{1}0]\ ,$$

$$\rho v^2=(C_{44}+C_{55}+2C_{45})/2 \quad \text{for } \mathbf{K}\,\|[110],\ \mathbf{u}\|[001]\ ,$$

TABLE IV. Comparison of the theoretical and experimental data for $Pr_2(Co_xFe_{1-x})_{17}$ polycrystal; V are the shear wave velocities at 5 MHz. We have assumed $\lambda_{\|}-\lambda_{\perp}=-572\times10^{-6}$, $\mu_0=\rho V_0^2=6.19\times10^{11}$ dyn/cm^2, $b_1=-(\lambda_{\|}-\lambda_{\perp})2\mu_0$, $M_s=1158$ emu/cm^3, $\sigma=1.997\times10^{16}$ sec^{-1}, $H=10$ KOe, and $x=0.81$. Experimental data are from Ref. 8.

	V(expt) (cm/sec)	V(calc) (cm/sec)
$\mathbf{u}\perp\mathbf{M}\perp\mathbf{K}$	2.73×10^5	
$\mathbf{u}\|\mathbf{M}\perp\mathbf{K}$	2.704×10^5	2.704×10^5
$\mathbf{u}\perp\mathbf{M}\|\mathbf{K}$	2.69×10^5	2.695×10^5

in the cases we considered one has to take the susceptibility from Eq. (20) when $\mathbf{M}||\mathbf{K}$ and from Eq. (21) when $\mathbf{M}$ is parallel to $\mathbf{u}$. As shown in Table III, the prediction made using a reasonable set of parameters are in very good agreement with the experimental data.

For the case of $Pr_2(Co_{0.81}Fe_{0.19})_{17}$, in which the magnetoelastic coupling reduces the symmetry from isotropic to uniaxial, we have considered the three cases of $\mathbf{M}$ being parallel to the propagation, to the polarization or perpendicular to both. The last case, in the simple approximation of considering a polycrystal as an isotropic single crystal, resulted unaffected by the magnetoelastic coupling, the velocity being given by $\rho v^2=(C_{11}-C_{12})/2=\mu_0$. When $\mathbf{M}$ is parallel to $\mathbf{K}$ or to $\mathbf{u}$, the shear wave velocity is given by $\rho v^2=\mu_0-b_4^2\chi_{xx}/M_s^2$. When $\mathbf{M}||\mathbf{k}$, the susceptibility is given by Eq. (20), while for $\mathbf{M}||\mathbf{u}$, one must use Eq. (21). Again the agreement between theory and experiment is very good (see Table IV). For this sample we have also made the comparison between theory and experiment at different values of field, in the range 4 to 12 kOe, obtaining always a good agreement.

We can finally also note that the peak in the softening observed, at low field, in nickel single crystals[15,16] and films[3] can probably be explained by the present model if one takes into account that the transverse susceptibility has a strong peak at $H=H_a$.

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