Effects of energy dependence in the electronic density of states on the far-infrared absorption in superconductors

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Allen's theory of far-infrared absorption in superconductors is generalized to the case when the electronic density of states, $N(\epsilon)$, cannot be taken as constant in the vicinity of the Fermi level. Such a situation can arise in several of the $A15$ compounds. Numerical calculations show that $N(\epsilon)\neq$const can yield an additional frequency dependence in the absorption spectrum.

I. INTRODUCTION

One of the features of several $A15$ superconductors, such as V_3Si, V_3Ga, and Nb_3Sn, is a rapidly varying electronic density of states, $N(\epsilon)$, in the vicinity of the chemical potential $\epsilon=0$. With $[1/N(\epsilon)]dN(\epsilon)/d\epsilon=O(1/\Omega_{\max})$ near $\epsilon=0$, where $\Omega_{\max}$ is the maximum phonon frequency, it is expected that the energy dependence in $N(\epsilon)$ would influence experiments which measure the electron-phonon (EP) coupling function $\alpha^2(\Omega)F(\Omega)$. Indeed, detailed calculations[1,2] have shown that an ϵ dependence of $N(\epsilon)$, which is found in the band-structure calculations, affects the results of tunneling experiments. Unfortunately, the tunneling experiments on $A15$ superconductors are often influenced by the proximity effect due to degraded superconducting properties within a first few atomic layers of the $A15$ electrode near the tunneling barrier.[3–6] At the present time it is not possible to separate the effects of nonconstant $N(\epsilon)$ from the influence of the proximity effect on tunneling characteristics.

Far-infrared absorption measurements can also be utilized to determine the EP coupling function of a metal.[7–9] These experiments probe the sample through a skin depth (which is about 100 nm for $A15$ materials) and, therefore, are less sensitive to the state of the surface than the tunneling experiments, which probe the sample through a coherence length (which is about 5 nm for the high superconducting T_c $A15$ compounds[10]). In a recent paper[11] it has been shown that the far-infrared transmission measurements on a thin film of *normal* V_3Si (Ref. 9) can be explained with a transport EP coupling function $\alpha^2_{\rm tr}F$ which is consistent with the neutron scattering results,[12] only by invoking an energy dependent $N(\epsilon)$. Farnworth and Timusk[8] have shown that more direct information about $\alpha^2_{\rm tr}F$ can be obtained by combining the far-infrared absorption measurements in both the normal and the superconducting states, within Allen's golden-rule theory.[7] In the present work, we extend a study of the effect of nonconstant $N(\epsilon)$ on infrared behavior to the superconducting state.

In Sec. II we generalize Allen's theory to include $N(\epsilon)\neq$const. In Sec. III we present the results of our numerical calculations with a discussion and Sec. IV contains conclusions.

II. THEORY

The effect of EP and electron-impurity (EI) collisions on the ac conductivity $\sigma(\mathbf{q},\omega)$, at the wave vector $\mathbf{q}$ and frequency ω, can be obtained from the equation (throughout this paper we take $\hbar=1$)

$$|\mathbf{E}|^2 \mathrm{Re}\sigma(\mathbf{q},\omega)=\omega\Gamma(\mathbf{q},\omega)\ ,$$

which relates the rate of energy dissipation by the electric field $\mathbf{E}$ to the transition probability per unit time $\Gamma(\mathbf{q},\omega)$ calculated within the golden rule. In this way, Allen[7] has obtained the expressions for EP and EI scattering contributions to the conductivity in the superconducting state:

$$\mathrm{Re}\sigma_{s,\mathrm{ph}}(\omega)=\frac{\pi e^2}{3\omega^3}\sum_{\mathbf{k},\mathbf{k}'}|g_{\mathbf{k}\mathbf{k}'}|^2(\mathbf{v}_{\mathbf{k}}-\mathbf{v}_{\mathbf{k}'})^2\frac{1}{2}\left[1-\frac{\epsilon_{\mathbf{k}}\epsilon_{\mathbf{k}'}-\Delta_{\mathbf{k}}\Delta_{\mathbf{k}'}}{E_{\mathbf{k}}E_{\mathbf{k}'}}\right]\delta(E_{\mathbf{k}}+E_{\mathbf{k}'}+\Omega_{\mathbf{k}-\mathbf{k}'}-\omega)\ , \tag{1}$$

$$\mathrm{Re}\sigma_{s,i}(\omega)=\frac{\pi e^2}{3\omega^3}\sum_{\mathbf{k},\mathbf{k}'}n_i|V_{\mathbf{k}\mathbf{k}'}|^2(\mathbf{v}_{\mathbf{k}}-\mathbf{v}_{\mathbf{k}'})^2\frac{1}{2}\left[1-\frac{\epsilon_{\mathbf{k}}\epsilon_{\mathbf{k}'}-\Delta_{\mathbf{k}}\Delta_{\mathbf{k}'}}{E_{\mathbf{k}}E_{\mathbf{k}'}}\right]\delta(E_{\mathbf{k}}+E_{\mathbf{k}'}-\omega)\ . \tag{2}$$

In Eqs. (1) and (2) e is the electron charge, $\mathbf{k}$ labels the Bloch states, $g_{\mathbf{k}\mathbf{k}'}(V_{\mathbf{k}\mathbf{k}'})$ is the EP (EI) scattering matrix element, $\mathbf{v}_{\mathbf{k}}$ ($\epsilon_{\mathbf{k}}$) is the electron velocity (band energy), n_i is the impurity concentration, $\Omega_{\mathbf{Q}}$ is the phonon energy, and

$$E_{\mathbf{k}}=\left[\epsilon^2_{\mathbf{k}}+\Delta^2_{\mathbf{k}}\right]^{1/2}\ , \tag{3}$$

where $\Delta_{\mathbf{k}}$ is the superconducting energy gap.

By using a set of coordinates in which electron energy ϵ is one of the coordinates and by ignoring the anisotropy effects, Eq. (1) can be rewritten in the form

$$\mathrm{Re}\sigma_{s,\mathrm{ph}}(\omega)=\frac{1}{8\omega^3}\int_{-\infty}^{+\infty}d\epsilon\int_{-\infty}^{+\infty}d\epsilon'\left[\frac{N(\epsilon)}{N(0)}\Omega_p^2(\epsilon')+\frac{N(\epsilon')}{N(0)}\Omega_p^2(\epsilon)\right]\frac{1}{2}\left[1-\frac{\epsilon\epsilon'-\Delta^2}{E(\epsilon)E(\epsilon')}\right]\alpha_{\mathrm{tr}}^2F(\omega-E(\epsilon)-E(\epsilon'))\ , \tag{4}$$

where

$$\Omega_p^2(\epsilon)=\frac{8\pi N(\epsilon)v^2(\epsilon)e^2}{3} \tag{5}$$

is the square of the Drude plasma frequency,

$$N(\epsilon)=\sum_{\mathbf{k}}\delta(\epsilon-\epsilon_{\mathbf{k}}) \tag{6}$$

is the single-spin-band density of states, and

$$v^2(\epsilon)=\sum_{\mathbf{k}}v_{\mathbf{k}}^2\delta(\epsilon-\epsilon_{\mathbf{k}})\Big/N(\epsilon)\ . \tag{7}$$

Also, the transport electron-phonon coupling function is defined by

$$\alpha_{\mathrm{tr}}^2(\Omega)F(\Omega)=N(0)\sum_{\mathbf{k},\mathbf{k}'}|g_{\mathbf{k}\mathbf{k}'}|^2(v_{\mathbf{k}}-v_{\mathbf{k}'})^2\delta(\Omega-\Omega_{\mathbf{k}-\mathbf{k}'})\delta(\epsilon-\epsilon_{\mathbf{k}})\delta(\epsilon'-\epsilon_{\mathbf{k}'})\left[\sum_{\mathbf{k},\mathbf{k}'}(v_{\mathbf{k}}-v_{\mathbf{k}'})^2\delta(\epsilon-\epsilon_{\mathbf{k}})\delta(\epsilon'-\epsilon_{\mathbf{k}'})\right]^{-1}\ , \tag{8}$$

where the (ϵ,ϵ') dependence of the right-hand side is assumed to be negligible.

Now, we will use the fact, established in band-structure calculations for $A15$ compounds,[13] that the ϵ dependence of $\Omega_p^2(\epsilon)$ can be ignored compared to the ϵ dependence of $N(\epsilon)$ in the vicinity of $\epsilon=0$. Then Eq. (4) reduces to

$$\mathrm{Re}\sigma_{s,\mathrm{ph}}(\omega)=\frac{\Omega_p^2}{4\pi\omega}\frac{1}{\omega}\frac{1}{\tau_{s,\mathrm{ph}}(\omega)}\ , \tag{9}$$

where

$$\frac{1}{\tau_{s,\mathrm{ph}}(\omega)}=\frac{2\pi}{\omega}\int_{-\infty}^{+\infty}d\epsilon\int_{-\infty}^{+\infty}d\epsilon'\frac{1}{2}\left[\frac{N(\epsilon)}{N(0)}+\frac{N(\epsilon')}{N(0)}\right]\times\frac{1}{4}\left[1-\frac{\epsilon\epsilon'-\Delta^2}{(\epsilon^2+\Delta^2)^{1/2}(\epsilon'^2+\Delta^2)^{1/2}}\right]\alpha_{\mathrm{tr}}^2F(\omega-(\epsilon^2+\Delta^2)^{1/2}-(\epsilon'+\Delta^2)^{1/2})\ . \tag{10}$$

After some algebra, Eq. (10) can be rewritten in the form

$$\frac{1}{\tau_{s,\mathrm{ph}}(\omega)}=\frac{2\pi}{\omega}\int_0^{\omega-2\Delta}d\Omega(\omega-\Omega)\alpha_{\mathrm{tr}}^2(\Omega)F(\Omega)D\left[\frac{2\Delta}{\omega-\Omega}\right]\ , \tag{11}$$

where

$$D(a)\equiv\int_0^1dx\left\{(1+a)\frac{\left[1-\left[\frac{1-a}{1+a}\right]^2x^2\right]^{1/2}}{(1-x^2)^{1/2}}-2\frac{a}{1+a}\frac{1}{(1-x^2)^{1/2}\left[1-\left[\frac{1-a}{1+a}\right]^2x^2\right]^{1/2}}\right\}$$
$$\times\frac{1}{2}\left[\frac{1}{2}\left[\frac{N\left[\frac{\Delta}{a}(1-a^2)^{1/2}\left[(1-x)\left[1-\frac{1-a}{1+a}x\right]\right]^{1/2}\right]}{N(0)}+\frac{N\left[-\frac{\Delta}{a}(1-a^2)^{1/2}\left[(1-x)\left[1-\frac{1-a}{1+a}x\right]\right]^{1/2}\right]}{N(0)}\right]\right.$$
$$\left.+\frac{1}{2}\left[\frac{N\left[\frac{\Delta}{a}(1-a^2)^{1/2}\left[(1+x)\left[1+\frac{1-a}{1+a}x\right]\right]^{1/2}\right]}{N(0)}+\frac{N\left[-\frac{\Delta}{a}(1-a^2)^{1/2}\left[(1+x)\left[1+\frac{1-a}{1+a}x\right]\right]^{1/2}\right]}{N(0)}\right]\right] \tag{12}$$

with $|a| \leq 1$. In the case when $N(\epsilon)=$const,

$$D(a)=E(1-a^2) ,$$

where

$$E(m)=\int_0^1 dx \frac{(1-mx^2)^{1/2}}{(1-x^2)^{1/2}}$$

is the complete elliptic integral of the second kind, and Eq. (11) reduces to Allen's result.[7]

In an analogous manner, one gets for the impurity conductivity

$$\mathrm{Re}\sigma_{s,i}(\omega)=\frac{\Omega_p^2}{4\pi\omega}\frac{1}{\omega}\frac{1}{\tau_{s,i}(\omega)} , \tag{13}$$

where

$$\frac{1}{\tau_{s,i}(\omega)}=\frac{1}{\tau_i}D\left[\frac{2\Delta}{\omega}\right] \tag{14}$$

and

$$\frac{1}{\tau_i}=\pi n_i N(0)\sum_{\mathbf{k},\mathbf{k}'}|V_{\mathbf{k},\mathbf{k}'}|^2(v_{\mathbf{k}}-v_{\mathbf{k}'})^2\delta(\epsilon_{\mathbf{k}})\delta(\epsilon_{\mathbf{k}'}) \times\left[\sum_{\mathbf{k},\mathbf{k}'}(v_{\mathbf{k}}-v_{\mathbf{k}'})^2\delta(\epsilon_{\mathbf{k}})\delta(\epsilon_{\mathbf{k}'})\right]^{-1} . \tag{15}$$

Equation (14) reduces to Allen's result[7] when $N(\epsilon)=$const. In the normal state one gets

$$\mathrm{Re}\sigma_{N,\mathrm{ph}}(\omega)=\frac{\Omega_p^2}{4\pi\omega}\frac{1}{\omega}\frac{1}{\tau_{N,\mathrm{ph}}(\omega)} , \tag{16}$$

$$\frac{1}{\tau_{N,\mathrm{ph}}(\omega)}=\frac{2\pi}{\omega}\int_0^{\omega}d\Omega\,\alpha_{\mathrm{tr}}^2(\Omega)F(\Omega) \times\int_0^{\omega-\Omega}d\epsilon\left[\frac{1}{2}\left[\frac{N(\epsilon)}{N(0)}+\frac{N(-\epsilon)}{N(0)}\right]\right] , \tag{17}$$

$$\mathrm{Re}\sigma_{N,i}(\omega)=\frac{\Omega_p^2}{4\pi\omega}\frac{1}{\omega}\frac{1}{\tau_{N,i}(\omega)} , \tag{18}$$

$$\frac{1}{\tau_{N,i}(\omega)}=\frac{1}{\tau_i}\frac{1}{\omega}\int_0^{\omega}d\epsilon\left[\frac{1}{2}\left[\frac{N(\epsilon)}{N(0)}+\frac{N(-\epsilon)}{N(0)}\right]\right] . \tag{19}$$

Again, expressions (17) and (19) reduce to Allen's results for the normal state when $N(\epsilon)=$const.

It is not difficult to show that

$$\int_0^{+\infty}d\omega\,\mathrm{Re}\sigma_{N,\mathrm{ph}}(\omega)=\frac{\Omega_p^2}{8}\lambda_{\mathrm{tr}} , \tag{20}$$

with

$$\lambda_{\mathrm{tr}}=\bar{\lambda}_{\mathrm{EP}}(\omega=0) \tag{21}$$

and

$$\bar{\lambda}_{\mathrm{EP}}(\omega)=2\int_0^{\Omega_{\max}}\frac{d\Omega}{\Omega}\alpha_{\mathrm{tr}}^2(\Omega)F(\Omega) \times\int_0^{+\infty}d\epsilon\,\frac{1}{2}\left[\frac{N(\epsilon)}{N(0)}+\frac{N(-\epsilon)}{N(0)}\right] \times\frac{\Omega}{\omega^2}\ln\left|\frac{(\epsilon+\Omega)^2}{(\epsilon+\Omega)^2-\omega^2}\right| . \tag{22}$$

We point out that $\bar{\lambda}_{\mathrm{EP}}$ is related to $1/\tau_{N,\mathrm{ph}}(\omega)$ by the Hilbert transform

$$\omega\bar{\lambda}_{\mathrm{EP}}(\omega)=\frac{1}{\pi}\int_{-\infty}^{+\infty}d\omega'\frac{1/\tau_{N,\mathrm{ph}}(\omega')}{\omega'-\omega} . \tag{23}$$

As noted by Allen,[7] result (20) implies that the total conductivity $\sigma_{\mathrm{tot}}(\mathbf{q},\omega)$ (collisionless plus EP) can be made to satisfy the f sum rule,

$$\int_0^{+\infty}d\omega\,\mathrm{Re}\sigma_{\mathrm{tot}}(\mathbf{q},\omega)=\frac{\Omega_p^2}{8} , \tag{24}$$

if Ω_p^2 in the expressions for both the collisionless and the electron-phonon conductivity is replaced by

$$(\Omega_p^*)^2=\Omega_p^2/(1+\lambda_{\mathrm{tr}}) . \tag{25}$$

This *ad hoc* procedure makes the results of the golden-rule theory closer to the results of a more rigorous treatment[7,14] which is based on Holstein's theory.[15] The latter gives for the local ($\mathbf{q}=0$) conductivity in the normal state[14]

$$\sigma(\omega)=\frac{\Omega_p^2/[1+\bar{\lambda}(\omega)]}{4\pi}\frac{\tau^*(\omega)}{1-i\omega\tau^*(\omega)} , \tag{26}$$

where

$$\frac{1}{\tau^*(\omega)}=\frac{1/\tau_{N,\mathrm{ph}}(\omega)+1/\tau_{N,i}(\omega)}{1+\bar{\lambda}(\omega)} , \tag{27}$$

with $1/\tau_{N,\mathrm{ph}}$ and $1/\tau_{N,i}$ given by Eqs. (17) and (19) and

$$\bar{\lambda}(\omega)=\bar{\lambda}_{\mathrm{EP}}(\omega)+\bar{\lambda}_i(\omega) , \tag{28}$$

where $\bar{\lambda}_{\mathrm{EP}}(\omega)$ is given by Eq. (22) and

$$\bar{\lambda}_i(\omega)=\frac{1}{\pi\tau_i\omega}\int_0^{\infty}d\epsilon\left[\frac{1}{2}\left[\frac{N(\epsilon)}{N(0)}+\frac{N(-\epsilon)}{N(0)}\right]\ln\left|\frac{\epsilon^2}{\epsilon^2-\omega^2}\right|\right] . \tag{29}$$

The golden-rule result for the normal-state conductivity due to collisions (EP plus EI)

$$\mathrm{Re}\sigma_N(\omega)=\frac{\Omega_p^2}{4\pi\omega}\frac{1}{\omega}\frac{1}{\tau_N(\omega)} , \tag{30}$$

$$1/\tau_N(\omega)=1/\tau_{N,\mathrm{ph}}(\omega)+1/\tau_{N,i}(\omega) , \tag{31}$$

can be cast into a form resembling the lowest-order term in the expansion of (26) into the powers of $(\omega\tau^*)^{-1}$. This can be done by replacing Ω_p^2 with $(\Omega_p^*)^2$, Eq. (25) with $\bar{\lambda}(\omega)$ instead of λ_{tr}, and by renormalizing $1/\tau_N(\omega)$ by $[1+\bar{\lambda}(\omega)]$, i.e.,

$$1/\tau_N(\omega)\rightarrow[1/\tau_N(\omega)]/[1+\bar{\lambda}(\omega)] . \quad (32)$$

An analysis[14] of Holstein's theory also indicates that $N(\epsilon)$ in the equations for $1/\tau_{N,\mathrm{ph}}$ and $1/\tau_{N,i}$ should be interpreted as the quasiparticle density of states

$$N(\epsilon)=-\frac{1}{\pi}\mathrm{Im}\sum_{\mathbf{k}} G(\mathbf{k},\epsilon+i0^+) , \quad (33)$$

where G is the electron Green's function. This density of states includes the self-energy effects, i.e., the lifetime broadening and the energy renormalization, and can be quite different from its bare band-electron counterpart $N_0(\epsilon)$ when $N_0(\epsilon)\neq$const near $\epsilon=0$ (Ref. 16).

Following Allen,[7] the volume absorption $A_{N,v}$ in the normal state can be written as

$$A_{N,v}=A_{\mathrm{tot}}-A_s=\Phi_N(\omega)\frac{2}{\Omega_p}\frac{1}{\tau_N(\omega)} , \quad (34a)$$

or as

$$A_{N,v}=A_{\mathrm{tot}}-A_s =\Phi_N(\omega)\frac{2}{\Omega_p}\frac{1}{\tau_N(\omega)}\frac{1}{[1+\bar{\lambda}(\omega)]^{1/2}} , \quad (34b)$$

where A_{tot} and A_s are the total and the surface absorption, respectively, and $\Phi_N(\omega)$ is a weakly varying function of frequency. Equation (34a) is the golden-rule-theory result, while Eq. (34b) is a modification of (34a) which includes some strong-coupling corrections.

In the case of the superconducting state, the golden-rule theory gives for the volume absorption

$$A_{s,v}=\Phi_s(\omega)\frac{2}{\Omega_p}\frac{1}{\tau_s(\omega)} , \quad (35a)$$

while the result which is analogous to (34b) would be

$$A_{s,v}=\Phi_s(\omega)\frac{2}{\Omega_p}\frac{1}{\tau_s(\omega)}\frac{1}{[1+\bar{\lambda}(\omega)]^{1/2}} , \quad (35b)$$

where

$$1/\tau_s(\omega)=1/\tau_{s,\mathrm{ph}}(\omega)+1/\tau_{s,i}(\omega) ,$$

with $1/\tau_{s,\mathrm{ph}}$ and $1/\tau_{s,i}$ given by Eqs. (11) and (14), and where $\Phi_s(\omega)$ is a weakly varying function of ω. However, in contrast to the normal-state case, Eq. (35b) could not be justified from the strong-coupling theory and is really an ansatz based on the fact that $A_{s,v}$ should become equal to $A_{N,v}$ when $\Delta\rightarrow 0$. In fact, one can expect that instead of using $\bar{\lambda}(\omega)$ in Eq. (35b) one should use a different renormalization parameter $\bar{\lambda}_s(\omega)$. On the basis of the difference between the mass renormalization parameters in the normal and the superconducting states, it is expected that $\bar{\lambda}_s-\bar{\lambda}=O((\Delta/\Omega_{\max})^2\ln(\Delta/\Omega_{\max}))$ (Ref. 17).

Finally we point out that $N(\epsilon)$ in the equations for $1/\tau_{s,\mathrm{ph}}$ and $1/\tau_{s,i}$ will also be interpreted as the *normal-state* quasiparticle density of states. There is no basis for this assumption within the golden-rule theory. It is only the requirement $A_{s,v}\rightarrow A_{N,v}$ as $\Delta\rightarrow 0$ that perhaps justifies such an interpretation of $N(\epsilon)$. Also, one intuitively expects that the effects of the lifetime broadening, due to impurity scattering, etc., should be contained in the theory.

III. NUMERICAL RESULTS AND DISCUSSION

Farnworth and Timusk[8] have shown that detailed information about the electron-phonon coupling function $\alpha^2_{\mathrm{tr}}F$ is contained in the derivative $S'(\omega)$ of the difference between the absorption in the superconducting state and the normal-state absorption:

$$S'(\omega)=dS(\omega)/d\omega , \quad (36)$$

$$S(\omega)=A_{s,v}-A_{N,v} . \quad (37)$$

Within Allen's golden-rule theory for $N(\epsilon)=$const one has

$$S'(\omega)=\frac{4\pi^2\Delta}{\Omega_p\omega}\alpha^2_{\mathrm{tr}}F(\omega-2\Delta)+\frac{4\pi}{\omega^2\Omega_p}\int_0^{\omega-2\Delta}d\Omega\,\alpha^2_{\mathrm{tr}}(\Omega)F(\Omega)\left[\Omega E(k^2)+\omega\frac{1-k^2}{k^2}[E(k^2)-K(k^2)]\right]$$
$$-\frac{4\pi}{\omega^2\Omega_p}\int_0^{\omega}d\Omega\,\Omega\alpha^2_{\mathrm{tr}}(\Omega)F(\Omega)+\frac{1}{\tau_i}\frac{2}{\omega\Omega_p}\frac{1-k'^2}{k'^2}[E(k'^2)-K(k'^2)] , \quad (38)$$

where

$$k'^2=1-\left[\frac{2\Delta}{\omega}\right]^2 , \quad (39a)$$

$$k^2=1-\left[\frac{2\Delta}{\omega-\Omega}\right]^2 , \quad (39b)$$

and

$$K(m)=\int_0^1 dx\frac{1}{(1-x^2)^{1/2}(1-mx^2)^{1/2}} . \quad (40)$$

Also, two weakly varying functions, $\Phi_s(\omega)$ and $\Phi_N(\omega)$, are assumed to be constant and equal to their high-frequency value of 1.

Although Eq. (38) is expected to work better for the weak-coupling superconductors than for the strong couplers like Pb, Farnworth and Timusk have demonstrated that Eq. (38) describes their high-resolutions measurements on Pb quite well. Moreover they have solved the integral equation (38) for $\alpha^2_{\mathrm{tr}}F$ with the experimental $S'(\omega)$. The inverted $\alpha^2_{\mathrm{tr}}F$ spectrum was in good agreement with the α^2F function for Pb, measured in the tunneling experiments.

Our program for studying the effects of nonconstant $N(\epsilon)$ on the far-infrared absorption is as follows. First we calculate $S'(\omega)$ for an $N(\epsilon)\neq$const by using the theory described in Sec. II and compare that $S'(\omega)$ with the cor-

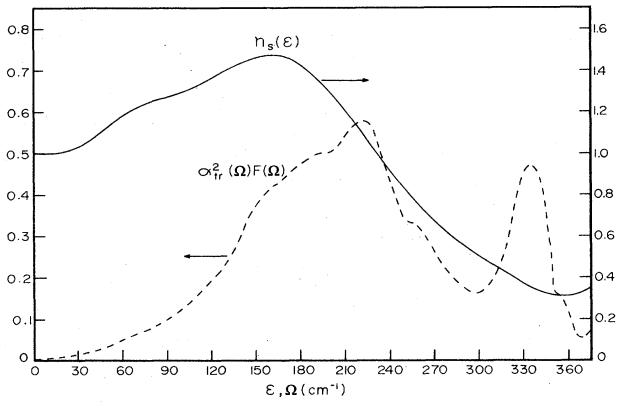

FIG. 1. Symmetrized density of states $n_s(\epsilon)$ and $\alpha^2_{\rm tr}(\Omega)F(\Omega)$ which are used in the numerical work.

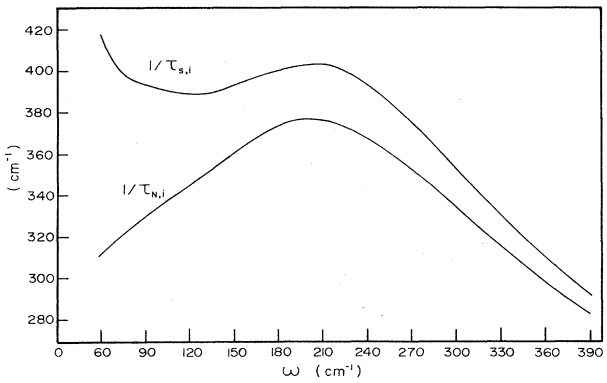

FIG. 3. Impurity scattering times for the superconducting and the normal states.

responding quantity calculated for $N(\epsilon)=$const. Second, we invert $S'(\omega)$ obtained for nonconstant $N(\epsilon)$, assuming that the usual theory [$N(\epsilon)=$const] is applicable, i.e., we solve Eq. (38) for $\alpha^2_{\rm tr}F$. The resulting effective coupling function $(\alpha^2_{\rm tr}F)_{\rm eff}$ may differ from the true $\alpha^2_{\rm tr}F$ if the effects of nonconstant $N(\epsilon)$ are important. In our numerical calculations we use the EP coupling spectrum $\alpha^2_{\rm tr}F$ and the symmetrized density of states

$$n_s(\epsilon)=\frac{1}{2}\left[\frac{N(\epsilon)}{N(0)}+\frac{N(-\epsilon)}{N(0)}\right] \tag{41}$$

(Fig. 1) which were used to fit the normal-state far-infrared transmission data on thin film of V_3Si (Ref. 11).

In Fig. 2 we show $S'(\omega)$ calculated for $1/\tau_i=296$ cm^{-1} (Ref. 11), $\Omega_p=3$ eV (Ref. 18), and $2\Delta=45$ cm^{-1} (Ref. 19). Note that a different choice for Ω_p would only change the vertical scale in Fig. 2. In generating $S'(\omega)$ we have used the unrenormalized expressions (34a) and (35a) for $A_{N,v}$ and $A_{s,v}$, respectively. Also, the weakly varying functions $\Phi_s(\omega)$ and $\Phi_N(\omega)$ are assumed to be constant and equal to their high-frequency value of 1. The work of Farnworth and Timusk[8] indicates that both of these approximations are reasonable. Later on we will examine the effect of the renormalization factor $[1+\bar{\lambda}(\omega)]^{-1/2}$ on $S'(\omega)$.

The difference between $S'(\omega)$ calculated for nonconstant $N(\epsilon)$ (solid line in Fig. 2) and the one calculated for $N(\epsilon)=$const (dashed line in Fig. 2) is mainly due to impurity contribution via $1/\tau_{s,i}$ and $1/\tau_{N,i}$, Fig. 3. $S'(\omega)$ calculated by leaving out the impurity scattering did not depend much on the structure in $N(\epsilon)$.

The $S'(\omega)$ data for nonconstant $N(\epsilon)$ were inverted by solving Eq. (38) for $\alpha^2_{\rm tr}F$. The resulting effective spectrum $(\alpha^2_{\rm tr}F)_{\rm eff}$ is shown in Fig. 4 (solid line) together with the true input spectrum (dashed line). There is a transfer of weight in $(\alpha^2_{\rm tr}F)_{\rm eff}$ to lower frequencies as compared to $\alpha^2_{\rm tr}F$. A similar behavior was found in the study of the influence of a peak in $N(\epsilon)$ around the Fermi level on superconductive tunneling.[1,2] We note, however, that in the case of infrared absorption this effect is dominated by the impurity scattering, while in the case of tunneling the transfer of weight in $(\alpha^2F)_{\rm eff}$ was obtained without explicitly including the impurity scattering. Another difference is that there is a new structure in $(\alpha^2_{\rm tr}F)_{\rm eff}$ at $\Omega=34$ cm^{-1} ($\cong$4 meV) as compared to the true $\alpha^2_{\rm tr}F$ spectrum. In Refs. 1 and 2 no new structure was introduced in the effective α^2F due to the variation in $N(\epsilon)$ [except for the

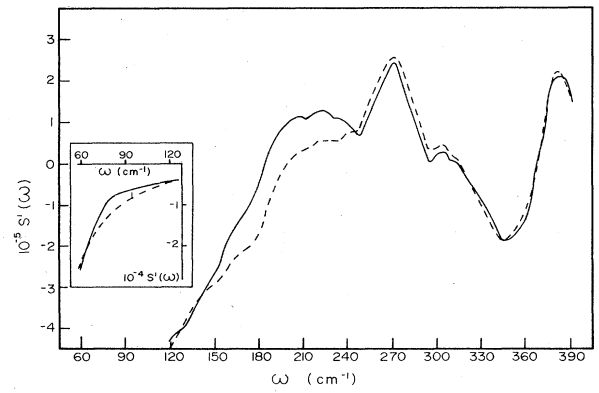

FIG 2. Derivative of the absorption spectrum. Solid line is calculated using $n_s(\epsilon)$ from Fig. 1 and the dashed line is calculated using $n_s(\epsilon)=1$. The inset shows the extension of the two curves to lower frequencies.

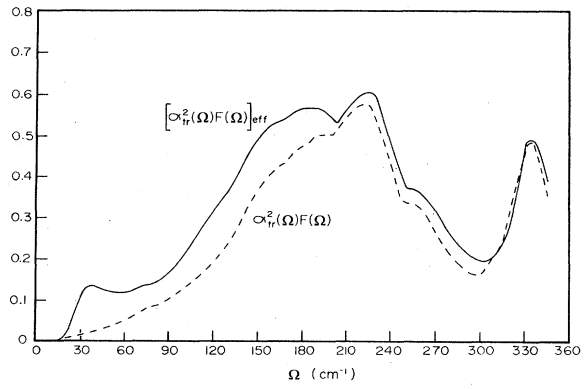

FIG. 4. The effective $\alpha^2_{\rm tr}F$ spectrum (solid line) and the true point $\alpha^2_{\rm tr}F$ (dashed line).

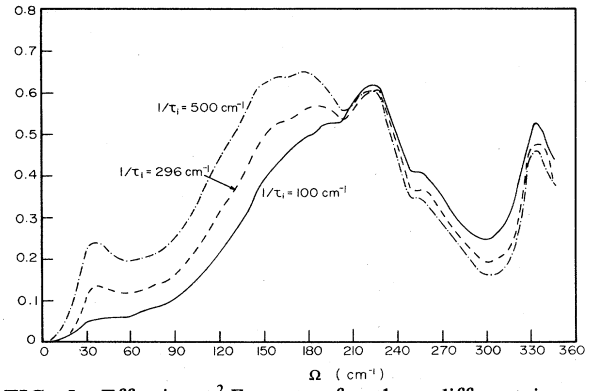

FIG. 5. Effective $\alpha^2_{tr}F$ spectra for three different impurity scattering parameters.

negative tail at $\Omega \geq \Omega_{max}$ (Ref. 1)]. To illustrate that this new feature in $(\alpha^2_{tr}F)_{eff}$ is introduced by the impurity contribution to the scattering times with our choice for $N(\epsilon)$, we have calculated and inverted two additional $S'(\omega)$ spectra for $1/\tau_i = 100$ cm^{-1} and $1/\tau_i = 500$ cm^{-1}, while keeping the remaining parameters the same as before. The results are shown in Fig. 5. We point out that a larger $1/\tau_i$ would introduce more smearing in the electronic density of states, while we used a fixed $n_s(\epsilon)$ in obtaining all three curves in Fig. 5.

It should be noted that all these effects of a nonconstant $N(\epsilon)$ depend on the actual form of the density of states. For instance, a triangular model for $n_s(\epsilon)$,

$$n_s(\epsilon) = \left[1 - \frac{1}{2\epsilon_c}\epsilon\right]\Theta(\epsilon_c - \epsilon) + 0.5\Theta(\epsilon - \epsilon_c)\,,$$

$$\epsilon_c = 190 \text{ cm}^{-1}$$

where Θ is a step function, gave an $(\alpha^2_{tr}F)_{eff}$ which was shifted down with respect to $\alpha^2_{tr}F$ by about 0.05 and, thus, had a negative tail.

Finally we comment that all results obtained for $n_s(\epsilon)$ from Fig. 1 remained qualitatively unchanged when we performed the calculations with Eqs. (34b) and (35b) which include the renormalization factor $[1+\bar{\lambda}(\omega)]^{-1/2}$.

IV. CONCLUSIONS

We have generalized Allen's theory of far-infrared absorption in superconductors to the case of nonconstant electronic density of states $N(\epsilon)$. Our numerical calculations using the parameters which fit the far-infrared transmission data on normal thin film of V_3Si (Ref. 11) show an additional frequency dependence in the derivative $S'(\omega)$, due to nonconstant $N(\epsilon)$. As a result of this, if $S'(\omega)$ data were inverted conventionally [i.e., assuming $N(\epsilon)=$const] the resulting effective $\alpha^2_{tr}F$ spectrum will contain an extra frequency dependence as compared to the true microscopic EP coupling function. This situation is analogous to the case of superconductive tunneling,[1,2] but in the case of infrared absorption the effect is dominated by the impurity scattering. We stress that the energy renormalization due to electron-phonon interaction makes the structure in the quasiparticle density of states, which appears in the theory of infrared absorption and, in effect, in the theory of tunneling,[1,2] much sharper[16] than the structure in the bare band-structure density of states broadened by the disorder scattering.

High-resolution far-infrared absorption measurements on good quality homogeneous $A15$ samples of varying residual resistivities can provide more information about the importance of energy dependence in the band density of states in these materials.

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