Critical relaxation of the one-dimensional Blume-Emery-Griffiths model

Ya'akov Achiam*
Department of Physics, University of Pennsylvania, Philadelphia, Pennsylvania 19104-3859
(Received 5 June 1984)

A model for the critical relaxation in the one-dimensional Ising-type $S=1$ spin system is presented. This model is equivalent to the Blume-Emery-Griffiths model and exhibits two simple critical points and one tricritical point. The kinetic behavior is studied using the real-space renormalization-group approach. In the two critical points we find that the critical slowing down is described by the dynamic exponent z, $z=2$. In each point this exponent belongs to the critical order parameter, while the second order parameter relaxes faster, with $z=1$ or 0. At the critical point the two order parameters relax with the same z, $z=1$.

I. INTRODUCTION

The Blume-Emery-Griffiths model[1] (BEG) describes a system of spins. Each spin has a magnitude of one, and interacts with its neighbors via an Ising-like interaction. The model allows for interactions which depend on s, the z component of the spin, S, or on the s^2. This enables us to describe a system having two degrees of freedom. Blume *et al.*[1] proposed their model to describe a ^{3}He-^{4}He mixture. Using the two degrees of freedom they were able to present (as an Ising-like system) a phase separation and superfluid ordering. The model was reinterpreted to describe phase transitions in simple[2] and multicomponent[2,3] fluids, and magnetic systems.[4]

The model exhibits a variety of properties such as critical, tricritical, and first-order transitions[5] due to the existence of two degrees of freedom. This behavior causes the model to be very attractive. It was studied using the mean-field approximation,[1–4] series,[6] and Monte Carlo[7,8] methods, as well as renormalization-group (RG) techniques.[5,9,10]

While the statics of the $S=1$ model have been studied extensively, we have not had a profound study of its dynamics. Obokata[11] suggested a multidimensional kinetic model based on the Glauber[12] model. He used the mean-field technique, and was not able to discern the peculiarity of this system. A similar model was given by Batten and Lemberg.[13] They also used the mean-field technique, but in a more sophisticated way. They found an Ising-like behavior, with a one-time scale for the two order parameters of the system. This is consistent with the conventional theory[14] which is expected for the mean field. In both studies, only the behavior near the two simple Ising points was examined.

In this paper we analyze the relaxation of a spin-one model in one dimension. This one-dimensional model has two critical points. One describes 1 to -1 transition of the spin-Z component. The second describes the 0 to 1 transition of the absolute value of the spin-Z component. The model has a third second-order transition at a tricritical point when these two transitions compete with each other. It is natural to describe the model using two order parameters.[5] This description enables us to construct a kinetic model in which each order parameter has Glauber[12]-like dynamics.

To study the dynamics, we use the real space[15] RG technique,[16] as was proposed by us,[17,18] to analyze the time-dependent properties of systems near their critical point. This technique, which fits the low-dimensional system, was used to calculate z, the dynamic exponent,[14] of a few one-[19] and two-dimensional[20] Ising-like models. In the following model we obtained $z=2$ at the critical points, and $z=1$ at the tricritical point. These values agree with the exact result known for the one-dimensional kinetic Ising model,[12] and do not agree with the expected value for the tricritical point, as was studied using the ϵ expansion.[21]

This paper is organized as follows. In Sec. II the model is presented. In Sec. III the main ideas of the time-dependent real-space RG (TRG) approach are reviewed and the calculation of the TRG is performed. A discussion of the results is given in Sec. IV.

II. THE KINETIC $S=1$ MODEL

The model under consideration is defined by the one-dimensional Hamiltonian

$$\bar{H}=\sum_n (Js_ns_{n+1}+Ks_n^2s_{n+1}^2-\Delta s_n^2)\ , \tag{2.1}$$

where $s=1,0,-1$. This is the BEG Hamiltonian with a zero magnetic field. Different versions of this Hamiltonian which contain odd-spin parameters appear in the literature.[5] The most general form in one dimension was studied by Krinsky and Furman,[9] and we shall refer to it below.

The spin-one Hamiltonian is used to describe a system having two types of species. One of them, which constitutes a pure system, exhibits simple Ising-like behavior (of $S=\frac{1}{2}$). The second species enters as an annealed impurity (or vacancy). References to physical systems which fit this description were given in the preceding section. Each lattice site is occupied by one and only one species. The details of the species are given by the value of s. The impurity is characterized by $s=0$. The critical species, the one which exhibits the $S=\frac{1}{2}$ property, is characterized by

the value $s=\pm 1$. The sign in the last case is the local order parameter of the critical species.

Instead of the s variable, the Hamiltonian can be rewritten using two two-value variables, σ and μ, which are related to s via[22]

$$s_i^2=(\mu_i+1)/2 , \tag{2.2}$$

$$s_i=\sigma_i s_i^2 . \tag{2.3}$$

The μ variable is well defined by (2.2), and can have the values ± 1. On the other hand, only when $\mu_i=1$ is the value of $\sigma_i=\pm 1$ determined by (2.3). When $\mu_i=-1$ the value of σ_i is irrelevant. Treating σ as an independent variable causes the partition function to be multiplied by $2^{[\sum_i(1-\mu_i)/2]}$. Dividing the partition function by this factor[22] allows us to express the Hamiltonian (2.1) using the $\{\sigma,\mu\}$ variables as

$$\begin{aligned}\bar{H}&\equiv\left[\sum_n V(\sigma_n,\mu_n;\sigma_{n+1},\mu_{n+1})\right]-N(K-\Delta)/2\\&=\sum_n[(J/4)\sigma_n\sigma_{n+1}+(K/4)\mu_n\mu_{n+1}+(J/4)\sigma_n\sigma_{n+1}(\mu_n+\mu_{n+1})\\&\quad+(J/4)\sigma_n\sigma_{n+1}\mu_n\mu_{n+1}+(H/2)(\mu_n+\mu_{n+1})-N(K-\Delta)/2] ,\end{aligned} \tag{2.4}$$

where $N=\sum_i 1$ and $H=\frac{1}{2}(K-\Delta+\ln 2)$. The two Hamiltonians (2.1) and (2.4) are equivalent. The reasons for using one of them instead of the other are the details of the calculations which have to be carried out, and the particular test of the investigator. We studied a kinetic generalization of the statics described by (2.1) and found the form (2.4) more suitable for this purpose. The statics of the model is studied extensively in Ref. 9 [using the form (2.1)]. The variety of phase transitions found in two dimensions (Ref. 5) degenerates in one dimension to three high-order transitions.

(1) $S=\frac{1}{2}$ Ising-like transition which is characterized by a long-range order of the σ degree of freedom. (The "magneticlike" transition.)

(2) $S=\frac{1}{2}$ Ising-like transition which is characterized by a long-range order of the μ degree of freedom. (The "lattice-gas" transition.)

(3) A tricritical point.

All these transitions take place at zero temperature and should be regarded as pseudocritical. The static behavior which we found using (2.4) is identical, of course, to that stated above.

The kinetic model is a straightforward generalization of the Glauber model[12] for the $S=\frac{1}{2}$ system. The model describes the time-dependent behavior of a large interacting spin system whose equilibrium is determined by the Hamiltonian (2.4). The system is brought into a state of constrained equilibrium. Then, at time $t=0$ the constraint is removed, and the system relaxes towards the final equilibrium via an interaction with a heat bath. The heat bath is not treated explicitly in the model, and is characterized by a bare time scale, τ. During the relaxation neither the average value of the spins nor the energy of the system is conserved. Only one kind of spin, σ or μ, is allowed to flip each time, and only at one lattice site, with a transition probability rate $W_i^{\sigma}(\{\sigma,\mu\})$ or $W_i^{\mu}(\{\sigma,\mu\})$, respectively. This procedure can be described by an empirical master equation for the spin probability distribution, $P(\{\sigma,\mu\};t)$,

$$\begin{aligned}\tau\left[\frac{d}{dt}\right]P(\{\sigma,\mu\};t)&=\sum_{i=1}^{N}-W_i^{\mu}(\sigma_1,\mu_1,\ldots,\sigma_i,\mu_i,\ldots,\sigma_N,\mu_N)P(\sigma_1,\mu_1,\ldots,\sigma_i,\mu_i,\ldots,\sigma_N,\mu_N;t)\\&\quad-\tfrac{1}{2}(1+\mu_i)W_i^{\sigma}(\sigma_1,\mu_1,\ldots,\sigma_i,\mu_i,\ldots)P(\sigma_1,\mu_1,\ldots,\sigma_i,\mu_i,\ldots;t)\\&\quad+W_i^{\mu}(\sigma_1,\mu_1,\ldots,\sigma_i,-\mu_i,\ldots)P(\sigma_1,\mu_1,\ldots,\sigma_i,-\mu_i,\ldots;t)\\&\quad+\tfrac{1}{2}(1+\mu_i)W_i^{\sigma}(\sigma_1,\mu_1,\ldots,-\sigma_i,\mu_i,\ldots)P(\sigma_1,\mu_1,\ldots,-\sigma_i,\mu_i,\ldots;t)\\&\equiv L(\alpha)P(\alpha,t)\equiv-\sum_i\sum_{\alpha=\sigma,\mu}(1-p_i^{\alpha})W_i^{\alpha}(\alpha)P(\alpha;t) ,\end{aligned} \tag{2.5}$$

where α denotes the spin type (i.e., $\sum_{\alpha}f^{\alpha}=f^{\sigma}+f^{\mu}$), and p_i^{α} is a spin-flip operator:

$$\begin{aligned}p_i^{\alpha}f(\sigma_1,\mu_1,\ldots,\alpha_i,\ldots,\sigma_N,\mu_N)\\=f(\sigma_1,\mu_1,\ldots,-\alpha_i,\ldots,\sigma_N,\mu_N) .\end{aligned} \tag{2.6}$$

The transition probability satisfies the detailed balance which ensures the ergodicity of the system:

$$(1-p_i^{\alpha})W_i^{\alpha}(\alpha)P_e(\alpha)=0 , \tag{2.7}$$

where e denotes equilibrium, $P_e(\alpha)=P(\alpha,t=\infty)$. Equa-

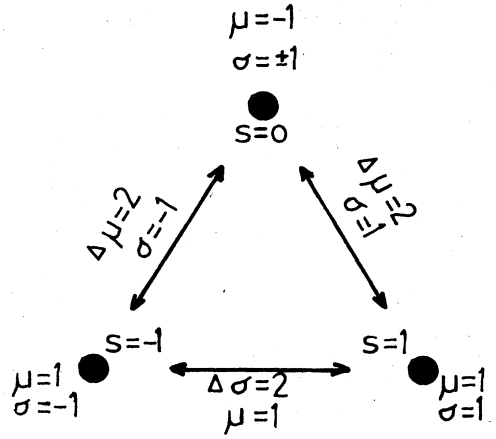

FIG. 1. Schematic representation of the spin states in s and in σ-μ space. The transitions between them (in σ-μ space) are indicated.

tion (2.5) describes two independent kinetic processes. The transition between the states $s=\pm 1$ is accomplished by flipping σ. The transition to the $s=0$ state is obtained by flipping μ. It is easy to see that there are six possible transitions in the s space. These are the transitions between $s=1,0,-1$ in cyclic and anticyclic orders, as plotted in Fig. 1. In the σ-μ space there are eight transitions. Six of them are equivalent to those in the s space, and appear in Fig. 1. The two extra transitions are the flipping of σ_i in the $\mu_i=-1$ state. These transitions are between two states having the same energy. Inclusion of these transitions does not affect the long-time dynamics of the model. The relation (2.7) does not determine W_i^α uniquely. We shall use

$$W_i^\alpha(\{\sigma,\mu\})=[P_e(\{\sigma,\mu\})/p_i^\alpha P_e(\{\sigma,\mu\})]^{-1/2} . \quad (2.8)$$

The master equation (2.5) can be written in a slightly different form, namely,

$$\tau dP(\alpha,t)/dt=-\mathcal{L}\phi(\alpha,t) , \quad (2.9)$$

where $\phi(\alpha,t)$ measures the deviation from equilibrium,

$$\phi(\alpha,t)=P(\alpha,t)/P_e(\alpha) . \quad (2.10)$$

The operator $\mathcal{L}$ is obtained from (2.5)—(2.7),

$$\mathcal{L}=\sum_{i,\alpha}\mathcal{L}_i^\alpha,\quad \mathcal{L}_i^\alpha=P_e W_i^\alpha(1-p_i^\alpha) . \quad (2.11)$$

For further properties of the Liouville operator L (or $\mathcal{L}$), the reader is referred to Ref. 23 and references therein. We shall only note that since W does not depend on the history of the system, the model is a Markoffian one.

We limit ourselves to the relaxation from infinitely small perturbation from equilibrium. This limit is physically accessible in the critical slowing-down problem. We study the magneticlike perturbation which consists of odd spin operators. Hence, the perturbation ϕ is

$$\phi(\alpha)=1+h_\sigma\sum_i\sigma_i+h_\mu\sum_i\mu_i . \quad (2.12)$$

III. THE RG TRANSFORMATION

The TRG transformation is composed of two stages.

A. The renormalization of the space

The RG transformation is applied to the two sides of the master equation (2.9):

$$\tau T(\alpha';\alpha)\frac{d}{dt}P(\alpha;t)=-T(\alpha';\alpha)L(\alpha)P(\alpha;t) . \quad (3.1)$$

We choose the RG transformation to be independent of time. Thus the transformation on the left-hand side of (3.1) is nothing other than the standard RG static transformation.[15,16] This transformation maps the set of the spin variables $\{\sigma,\mu\}$ to a new set of spin variables, $\{\sigma',\mu'\}$, defined on a lattice whose dimensions are scaled by a factor b, while preserving the free energy of the system. The transformation is performed using the operator $T(\sigma',\mu';\sigma,\mu)$ operating on $P(\sigma,\mu;t)$. The T fulfills some conditions so as to conserve the probability distribution after the transformation.[15,16]

The simplest choice for T is the decimation transformation,

$$T(\sigma',\mu';\sigma,\mu)=\prod_i\delta(\sigma_i'-\sigma_{2i})\delta(\mu_i'-\mu_{2i}) , \quad (3.2)$$

by which every b spin becomes a member of the renormalized lattice, and all the spins between the new ones are summed out of the probability distribution. We are not interested, for the present, in "antiferromagnetic" ordering; hence, we limit ourselves to $b=2$.

The RG transformation is easily carried out using the transfer matrix method. The transfer matrix $\underline{M}$ is

$$\underline{M}_{n,n+1}(\vec{\mathrm{K}})=\exp V(\alpha_n,\alpha_{n+1}) , \quad (3.3)$$

where $\vec{\mathrm{K}}[=(J,K,H)]$, is the parameter space of the Hamiltonian. Each term $\underline{M}_{n,n+1}$ corresponds to a particular state of α_n and α_{n+1}, and the terms in $\underline{M}$ are organized according to the following key:

σ		1	−1	1	−1	$n+1$
	μ	1	1	−1	−1	
1	1					
−1	1					
1	−1		$\underline{M}$			
−1	−1					
	n					

The outer multiplication of these matrices (divided by the partition function) is a matrix representation of the probability distribution. The decimation transformation is performed by the inner multiplication of pairs of $\underline{M}$. The renormalized transfer matrix is

$$A(\vec{\mathrm{K}})\underline{M}_{n,n+1}(\vec{\mathrm{K}}')\equiv\underline{M}_{2n,2n+1}(\vec{\mathrm{K}})\cdot\underline{M}_{2n+1,2(n+1)}(\vec{\mathrm{K}}) , \quad (3.4)$$

where $A(\vec{\mathrm{K}})$ is the contribution to the free energy from the decimation. Relation (3.4) determines the RG recursion relations,

$$\vec{K}'=\underline{R}\vec{K} . \tag{3.5}$$

The fixed points of this transformation,

$$\vec{K}^*=\underline{R}\vec{K}^* , \tag{3.6}$$

are associated with critical points (or zero correlation).

Using the following functions in the parameter space,

$$x=\exp(J),\quad y=\exp(K/4),\quad w=\exp(H) , \tag{3.7}$$

The recursion relations derived from (3.4) are

$$Ax'y'w'=y^2w^2(x^2+x^{-2})+2/y^2 , \tag{3.8a}$$

$$Ay'w'/x'=2y^2w^2+2/y^2 , \tag{3.8b}$$

$$A/y'=w(x+x^{-1})+2/w , \tag{3.8c}$$

$$Ay'/w'=2y^{-2}+2y^2/w^2 . \tag{3.8d}$$

The fixed points $\vec{K}^*=(x^*,y^*,w^*)$ of (3.8) are

$$\begin{aligned}&(\infty,\infty,\infty) \text{ at point } A\text{: Ising (ferromagnet)} ,\\ &(1,\infty,1) \text{ at point } B\text{: Ising(lattice gas)} ,\\ &(\infty,\infty,0) \text{ at point } C\text{: tricritical} .\end{aligned} \tag{3.9}$$

In a similar way the RG transformation of ϕ is calculated. There are two different contributions to it. When the spin site i of α_i is $i=2n$ (a spin which is not summed out in the decimation), the first contribution is just α'_n. The second contribution is when $i=2n+1$. In this case a sum of the form $\underline{M}_{n,n+1}\cdot\underline{\sigma}_{n+1}\cdot\underline{M}_{n+1,n+2}$, where

$$\underline{\mu}=\begin{bmatrix}\underline{I} & \underline{0}\\ \underline{0} & \underline{I}\end{bmatrix},\quad \underline{\sigma}=\begin{bmatrix}\underline{\sigma}_z & \underline{0}\\ \underline{0} & \sigma_z\end{bmatrix},\quad \underline{I}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix},$$

$$\underline{0}=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix},\quad \text{and } \underline{\sigma}_z=\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix},$$

has to be carried out.

To the leading terms, the calculation gives

$$\underline{M}\cdot\underline{\sigma}_{2n+1}\cdot\underline{M}\sim\begin{cases}A\underline{M}(\vec{K}')(\underline{\sigma}'_n+\underline{\sigma}'_{n+1})/2 & \text{at fixed point } A ,\\ 0 & \text{at fixed points } B,C ,\end{cases} \tag{3.10}$$

$$\underline{M}\cdot\underline{\mu}_{2n+1}\cdot\underline{M}\sim A\underline{M}(\vec{K}')(\underline{\mu}'_n+\underline{\mu}'_{n+1})/2 \quad \text{at fixed points } A,B, \text{ and } C .$$

The two contributions are collected together and give the linearized RG transformation of the perturbation ϕ at the fixed points $Q=A,B,C$:

$$h'_\sigma=\lambda_\sigma^Q h_\sigma,\quad h'_\mu=\lambda_\mu^Q h_\mu , \tag{3.11}$$

where,

$$\lambda_\sigma^A=2,\quad \lambda_\sigma^B=1,\quad \lambda_\sigma^C=2 ,$$

$$\lambda_\mu^A=2,\quad \lambda_\mu^B=2,\quad \lambda_\mu^c=2 .$$

The calculation of the transformation of the right-hand side of (3.1) is more tedious. Ideally, we would expect the result

$$TL(\alpha)P(\alpha;t)=b^{-z}L'(\alpha')P'(\alpha';t) \tag{3.12}$$

which is of the same form as (2.5).

In this expression the transformation of $L(\alpha)$ into $L'(\alpha')$ is determined by that of $P_e(\alpha)$ since $L(\alpha)$ is composed of terms which appear in $P_e(\alpha)$ [Eq. (2.8)]. W_i^α enters into $\mathscr{L}$ [Eq. (2.11)] via the expression

$$P^{(\alpha_i)}\equiv P_e W_i^\alpha(\{\alpha\}) . \tag{3.13}$$

The symbol $P^{(\alpha_i)}$ means that all the interactions around the spin mentioned in the brackets are excluded from P_e. The fact that $P^{(\alpha_i)}$, as given by (3.13), is independent of α_i, is a direct consequence of the detailed balance (2.7). Later, we shall use this lack of dependence on α_i to identify expressions as being $P^{(\alpha_i)}$, or $P'^{(\alpha'_i)}$. $P'^{(\alpha'_i)}$ denotes a renormalized probability distribution with the Hamiltonian (2.4), from which the interactions with α'_i are excluded. Thus, $P'^{(\alpha'_i)}$ is equivalent to

$$P'^{(\alpha'_1)}=P'_e(\{\alpha'\})W'^{\alpha}_i(\{\alpha\}) . \tag{3.14}$$

The term $(1-p_i^\alpha)\phi$ [Eq. (2.11)], which multiplies $P^{(\alpha_i)}$ to form the i term on the right-hand side of the kinetic equation (2.9), simply multiplies $P^{(\alpha_i)}$ by $2h_\alpha\alpha_i$. As already noted, $P^{(\alpha_i)}$ is independent of α_i, so that α_i appears only as $\alpha_i T(\alpha';\alpha)$. The $T(\alpha';\alpha)$ depends on α_i only if α_i is a spin which is renormalized to a α'_i variable. Thus, if $i=2n+1$ (a spin which is summed out) α_i appears linearly in $TP^{(\alpha_i)}\alpha_i$. The contribution of the trace over its values is therefore zero. If, on the other hand, $i=2n$ (the α_i is transformed into a α'_n variable) we shall have to perform the trace on it. The only effect of the RG on α_i is to transform it into α'_{2n}. There is no difference between terms connected to σ or μ, and they are treated equally in the discussion below under the general symbol α. The new variables α'_{n-1} and α'_{n+1} decouple the cells in which α'_n is found from the rest of the lattice. Thus, the trace over the rest of the lattice, which can be performed independently, gives

$$\sum_n\sum_\alpha P'^{(2n-2,2n+2)}(\underline{M}_{2n-2,2n-1}\cdot\underline{M}^{(\alpha_{2n})}_{2n-1,2n})\otimes(\underline{M}^{(\alpha_{2n})}_{2n,2n+i}\cdot\underline{M}_{2n+1,2n+2}) , \tag{3.15}$$

where $P'^{(2n-2,2n+2)}$ is the contribution to the renormalized partition function from the spins which range between 0 to $2n-2$ and $2n+2$ to N. $\underline{M}^{(\alpha_i)}$ is the transfer matrix which is obtained from the relation

$$\underline{M}^{(\alpha_i)}_{i-1,i}\otimes\underline{M}^{(\alpha_i)}_{i,i+1}=\exp(V_{i-1,i})W_i^\alpha\exp(V_{i,i+1}) . \tag{3.16}$$

The detailed expressions for the $\underline{M}^{(\alpha)}$ are

$$\underline{M}^\sigma=yw\begin{bmatrix}\underline{U} & 1/(y^2w)\underline{U}\\ 1/(y^2w)\underline{U} & w^{-2}\underline{U}\end{bmatrix},$$

$$\underline{M}^\mu=w^{1/2}\begin{bmatrix}\underline{X}^{1/2} & \underline{X}^{1/2}\\ w^{-1}\underline{U} & w^{-1}\underline{U}\end{bmatrix}, \tag{3.17}$$

where

$$\underline{X}^{1/2}=\begin{bmatrix} x^{1/2} & x^{-1/2} \\ x^{-1/2} & x^{1/2} \end{bmatrix} \text{ and } \underline{U}=\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} .$$

The multiplication $\underline{M}\cdot\underline{M}^{(\alpha)}$ at the three fixed points gives

$$\underline{M}\cdot\underline{M}^{(\sigma)}=\begin{cases} 2^{-1/2}A\underline{M}'^{(\sigma')} & \text{point } A \ , \\ A\underline{M}'^{(\sigma')} & \text{point } B \ , \\ 2^{-1/2}A\underline{M}'^{(\sigma')} & \text{point } C \ , \end{cases} \tag{3.18}$$

$$\underline{M}\cdot\underline{M}^{(\mu)}=\begin{cases} A\underline{M}'^{(\mu')} & \text{point } A, \\ 2^{-1/2}A\underline{M}'^{(\mu')} & \text{point } B, \\ A\underline{M}'^{(\mu')} & \text{point } C \ . \end{cases}$$

Substituting (3.18) and (3.17) gives

$$\sum_{\alpha} P'^{(\alpha'_n)}\omega_n^{Q}\alpha'_n \ , \tag{3.19}$$

where

$$\omega_\sigma^A=\tfrac{1}{2},\ \omega_\sigma^B=1,\ \omega_\sigma^C=1 \ ,$$
$$\omega_\mu^A=1,\ \omega_\mu^B=\tfrac{1}{2},\ \omega_\mu^C=1 \ .$$

In the derivative of (3.19) we used the fact that the factor A^2, which is the contribution to the free energy from the decimation between the spins $2(n-1)-2(n+1)$ has to be absorbed into $P'^{(\alpha')}$ in order to keep the normalization of the probability distribution. The reader is referred to Ref. 18 in which a detailed graphic representation of this kind of calculation is given.

Using (3.19), the left-hand side of the renormalized master equation (3.1) is

$$2P'_e(\{\alpha'\})\omega^{\sigma}\sum W^{\sigma'_i}\sigma'_i+2P'_e(\{\alpha'\})\omega^{\mu}\sum W_i^{\mu'_i}\mu'_i = \mathscr{L}'\left[1+h_\sigma\omega_\sigma\sum\sigma'_i+h_\mu\omega_\mu\sum\mu'_i\right] . \tag{3.20}$$

To express the perturbation in terms of h', we use the recursion relations (3.11). The equation (3.1) becomes

$$\frac{d}{dt}P'=\mathscr{L}'\left[1+(\omega_\sigma/\lambda_\sigma)h'_\sigma\sum\sigma'_i+(\omega_\mu/\lambda_\mu)h'_\mu\sum\mu'_i\right] . \tag{3.21}$$

B. The renormalization of the time

The renormalization of the space transformation transformed the master equation into (3.21). This equation has two time scales. In order to have a self-consistent description of the kinetic procedure, the two bare time scales should be included in the master equation *a priori*. With the two time scales τ_σ and τ_μ, which characterize the interaction with the heat bath of the fields h_σ and h_μ, respectively, the master equation,

$$\frac{d}{dt}P(\alpha;t)=\mathscr{L}\left[(h_\sigma/\tau_\sigma)\sum\sigma_i+(h_\mu/\tau_\mu)\sum\mu_i\right] , \tag{3.22}$$

becomes

$$\frac{d}{dt}P'(\alpha';t)=\mathscr{L}'\left[(1/\tau_\sigma)(\omega_\sigma/\lambda_\sigma)h'_\sigma\sum\sigma'_i + (1/\tau_\mu)(\omega_\mu/\lambda_\mu)h'_\mu\sum\mu'_i\right] . \tag{3.23}$$

The use of the renormalized bare time scales,

$$\tau'_\sigma=(\lambda_\sigma/\omega_\sigma)\tau_\sigma=b^{-z_\sigma}\tau_\sigma \ ,$$
$$\tau'_\mu=(\lambda_\mu/\omega_\mu)\tau_\mu=b^{-z_\mu}\tau_\mu \ , \tag{3.24}$$

brings the renormalized master equation into the invariant form,

$$\frac{d}{dt}P'(\alpha';t)=\mathscr{L}'\left[\sum_{\alpha'}(1/\tau'_\alpha)h'_{\alpha'}\sum_i\alpha'_i\right] . \tag{3.25}$$

The standard ideas of the RG transformation identify z in the time-dependent problems[17,18,21] as the dynamic exponent.

IV. CONCLUSION

Six exponents characterize the relaxation of the perturbation at the three critical points,

$$z_\sigma^A=2,\ z_\sigma^B=0,\ z_\sigma^C=1 \ ,$$
$$z_\mu^A=1,\ z_\mu^B=2,\ z_\mu^C=1 \ . \tag{4.1}$$

The slowest relaxation mechanism governs the critical slowing down of the system. Hence the dynamic exponents are

$$z=2 \ , \tag{4.2}$$

$$z=1 \ , \tag{4.3}$$

at the two simple critical points and at the tricritical point, respectively. The kinetic behavior near the two simple critical points is not surprising. The static behavior of the system near these two points is an Ising-like behavior. The kinetic process is a one-spin-flip mechanism, as in the Glauber model.[12] Hence, we would expect the $z=2$ exponent known for the one-dimensional (1D) Glauber model. However, examination of the relaxation of the faster dynamic mode shows a difference between the two Ising points. At the magneticlike critical point (point A) the two order parameters suffer from critical slowing down. The critical order parameter is the slowest mode ($z=2$). The nonordering parameter relaxes faster with $z=1$. From Eq. (3.11) it is clear that the static behavior of the nonordering parameter (characterized with λ_α) is the one responsible for its slowing down, and it is the kinetics (3.19) which distinguish between the two parameters. The coupling between the two order parameters resembles the coupling in model A of Halperin *et al.*,[21] which presents the kinetic of a purely dissipative system near four dimensions. The model A is assumed to present the tricritical behavior of a metamagnet (the Gaussian fixed point). However, the nonordering parameter in model A behaves as the energy of the system. In one dimension the energy relaxes with the same z as the critical order parameter,[18] thereby ending the resemblance.

At the second critical point B, the lattice-gas-like point, a different behavior is found. In this case the nonordering parameter is totally irrelevant, in both its static and dynamic behavior, and has $z=0$.

The different behavior is noticed at the tricritical point (C). Now the two order parameters have similar critical behavior, both in the statics and in the dynamics. The dynamic exponent is the same for the two order parameters, $z_\sigma = z_\mu = 1$, and its main contribution is from the λ_μ.

We are not aware of similar low-dimensional systems having tricritical behavior which have been analyzed using the TRG. Hence we can only make a comparison with studies that used the ϵ expansion, on systems near four dimensions. Siggia and Nelson[24] studied the dynamics of ^{3}He-^{4}He near the tricritical point. They also extended their study to the dynamic of the metamagnet, which corresponds to model A of Halperin *et al.*[21] Although the original BEG model was assumed to give a representation of the statics of the ^{3}He-^{4}He mixture, it undoubtedly represents a system which has an order parameter that belongs to different symmetry class. The dynamics of the ^{3}He-^{4}He mixture is affected by conservation laws which do not appear in our purely dissipative model. Hence, comparison can be made only with respect to model A.[21,24]

The statics of model A is that of a system having an order parameter and of nonordering parameter.[24,25] One can include in the model extra terms which would allow the nonordering parameter to be critical itself, and not only at the tricritical point. However, that has not been done, and their information is restricted to the two critical points corresponding to points A and C. We already mentioned in the discussion of the relaxation at point A, the difference between the relaxation of the nonordering parameter in model A (such as the energy) and in our model (which is faster than the energy). At the tricritical point the two models also exhibit different behavior: our model is characterized by $z = 1$, while model A is characterized by $z = 2$ (for all orders in ϵ). Thus, the two models, which describe similar systems but in different dimensions, exhibit different critical slowing down. One should remember that the expansion around four dimensions is not valid below two dimensions. Hence, although the result $z = 2$ was obtained for all order in ϵ, it would be mere coincidence if it would be valid in one dimension.

The impurity dynamics in a 1D chain was studied in Ref. 26. The model has similar dynamics to ours but it is restricted to only one impurity. Hence we can compare only the behavior near the simple Ising-like point, A. Not all the cases given in this reference correspond to our model. The one case which does correspond (case "a" with constant α and a varied J, in the notation of Ref. 26) reveals the same $z = 2$ as the one that we obtained.

ACKNOWLEDGMENT

The work was supported by the National Science Foundation, under Grant No. DMR-82-16178.

*Permanent address: Nuclear Research Centre—Negev, P. O. Box 9001 Beer-Sheva, Israel.

[1]M. Blume, V. J. Emery, and R. B. Griffiths, Phys. Rev. A **4**, 1071 (1971).

[2]J. Lajzerowicz and J. Sivardiere, Phys. Rev. A **11**, 2079 (1975); J. Sivardiere and J. Lajzerowicz, *ibid.* **11**, 2090 (1975); **11**, 2101 (1975).

[3]D. Mukamel and M. Blume, Phys. Rev. A **10**, 610 (1974).

[4]M. Blume, Phys. Rev. **141**, 517 (1966); H. W. Capel, Physica (Utrecht) **32**, 966 (1966); M. Blume and R. E. Watson, J. Appl. Phys. **38**, 991 (1967).

[5]A. N. Berker and M. Wortis, Phys. Rev. B **14**, 4946 (1976).

[6]D. M. Saul, M. Wortis, and D. Stauffer, Phys. Rev. B **9**, 4964 (1974).

[7]A. K. Jain and D. P. Landau, Bull. Am. Phys. Soc. **21**, 231 (1976).

[8]B. L. Arora and D. P. Landa, *Magnetism and Magnetic Materials—1972 (Denver)*, proceedings of the 18th Annual Conference on Magnetism and Magnetic Materials, edited by C. D. Graham and J. J. Rhyne (AIP, New York, 1973), p. 870.

[9]S. Krinsky and D. Furman, Phys. Rev. B **11**, 2602 (1975).

[10]Th. W. Burkhardt, Phys. Rev. B **14**, 1196 (1976).

[11]T. Obokata, J. Phys. Soc. Jpn. **26**, 895 (1969).

[12]R. G. Glauber, J. Math. Phys. **4**, 297 (1963).

[13]G. L. Batten, Jr. and H. L. Lemberg, J. Chem. Phys. **70**, 2934 (1978).

[14]P. C. Hohenberg and B. I. Halperin, Rev. Mod. Phys. **49**, 435 (1977).

[15]T. Niemeijer and M. J. van Leeuwen, in *Phase Transitions and Critical Phenomena,* edited by C. Domb and M. S. Green (Academic, New York, 1976), Vol. 6.

[16]K. G. Wilson and J. Kogut, Phys. Rep. C **12**, 75 (1974).

[17]Y. Achiam and J. M. Kosterlitz, Phys. Rev. Lett. **41**, 128 (1978).

[18]Y. Achiam, Physica **120A**, 279 (1983).

[19]Y. Achiam, J. Phys. A **11**, 975 (1978); **13**, 1825 (1980); **13**, L93 (1980); Phys. Lett. **74A**, 247 (1979).

[20]Y. Achiam, J. Phys. A **11**, L129 (1978); **13**, 1355 (1980); Phys. Rev. B **19**, 376 (1979); Phys. Lett. **72A**, 35 (1979).

[21]B. I. Halperin, P. C. Hohenberg, and S. -K. Ma, Phys. Rev. B **10**, 139 (1974); **13**, 4119 (1976).

[22]R. B. Griffiths, Physica (Utrecht) **33**, 689 (1967).

[23]H. Yahata and M. Suzuki, J. Phys. Soc. Jpn. **27**, 1421 (1969); H. Yahata, *ibid.* **30**, 657 (1971).

[24]E. D. Siggia and D. R. Nelson, Phys. Rev. B **15**, 1427 (1977).

[25]Y. Achiam and Y. Imry, Phys. Rev. B **12**, 2768 (1975).

[26]D. F. Calef, J. Stat. Phys. **32**, 81 (1983).