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Magnetic-field-enhanced electron-electron scattering in the resistivity of copper

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Our measurements of the electrical resistivity $\rho_d(T)$ on high-quality copper whiskers reveal a T^2 variation at very low temperatures. With the taking into account of appreciable corrections for surface scattering, the coefficient for electron-electron scattering in bulk copper could be determined, $A=\Delta\rho/T^2=27$ fΩ cm K^{-2}, in good agreement with calculations assuming an isotropic relaxation time $\tau(\mathbf{k})$. For the first time, a strong enhancement of A by a longitudinal magnetic field is observed, which is related to an anisotropy of $\tau(\mathbf{k})$. At 4 T we find $A\simeq 110$ fΩ cm K^{-2} independent of an induced dislocation density of about 10^9 cm^{-2}.

I. INTRODUCTION

At very low temperatures, the existence of an electron-electron scattering term in the bulk resistivity

$$\Delta\rho(T)=\rho(T)-\rho(0)=AT^2 \tag{1}$$

is now well established for the noble metals. However, the magnitude of the coefficient A appears to be sample dependent. This may be related to an anisotropy in the electron relaxation time[1-3] $\tau(\mathbf{k})$ arising from dislocations.[4] Recently, Kaveh and Wiser (KW) used the expression, originally derived for the alkali metals,[2,3] to also describe for copper[5] the enhancement of A by dislocations,

$$A\simeq A_l[1+(A_a/A_l-1)/(1+\rho_{\text{imp}}/\rho_{\text{dis}})^2] \quad . \tag{2}$$

ρ_{imp} and ρ_{dis} denote the contributions of impurity and dislocation scattering to the residual resistivity $\rho(0)$. For the isotropic and anisotropic limits of A, KW[5] inferred from earlier calculations[6-8] $A_l=27$ fΩ cm K^{-2} and $A_a\simeq 3A_l=81$ fΩ cm K^{-2}, respectively. At the present stage of the theory, the numerical uncertainties for these values amount to at least 50%.

In this Rapid Communication we report on an investigation of $\Delta\rho_d(T)$ on pure single-crystalline copper whiskers, with diameters d small compared with the mean free path of the electrons $l_e=v_F\tau_0$. The evaluation of the temperature-dependent part of the bulk resistivity for these almost dislocation-free samples ($\rho_{\text{dis}} << \rho_{\text{imp}}$) is based on enhancement factors due to surface scattering,

$$\Delta\rho_d(T)=G\Delta\rho(T) \quad , \tag{3}$$

which have been determined in a previous study at $T>2$ K.[9] According to a recent theory of Sambles and Preist,[10] G is given by

$$G=n_s+(1-n_s)\rho_d(0)/\rho(0) \quad , \tag{4}$$

where for $d/l_e << 1$ the parameter n_s depends only on the surface roughness. Using this appreciable size correction we obtain as one central result from our samples with $d=6.9$ and 22 μm in the region $0.4\lesssim T\lesssim 2$ K the same value $A=27$ fΩ cm K^{-2}, in surprisingly good agreement with the estimate of KW[5] for the isotropic limit.

By straining the samples, slightly larger values of A are obtained as qualitatively expected from the effect of anisotropy on $\tau(\mathbf{k})$. As an outstanding feature, we find a strong enhancement of A by a longitudinal magnetic field, which we tentatively ascribe to a field-induced anisotropy. A field of 4 T drives A to 110 fΩ cm K^{-2}, and no further increase by straining, i.e., by generating a dislocation density of about 10^9 cm^{-2}, is observed. Thus it appears that this field brings the electron-electron scattering contribution to the resistivity close to its anisotropic limit and the second prediction of KW,[5] $A_a\simeq 3A_l$, is only approximately valid for copper.

II. MEASUREMENTS AND RESULTS

The filamentary grown single crystals of copper ("whiskers") with $\langle 111\rangle$ growth axes were annealed under low pressure of oxygen as previously described.[11] Resistances were measured by a four-wire technique using inductive voltage compensation and lock-in detection at 84 Hz reaching a relative accuracy of 1 ppm for the thinnest sample. The very low temperatures were produced by a ^{3}He-^{4}He dilution refrigerator, and the longitudinal magnetic field was provided by a superconducting magnet in the persistent mode.

From Figs. 1 and 2 it is evident that the measured resistivities obey a T^2 law at low temperatures for both samples in the annealed and strained states and in zero and finite longitudinal magnetic field as well. The only exception occurs in the thinner sample 2 in the unstrained state, where an anomalous drop of $\Delta\rho_d(T)$ is observed below 0.4 K [Fig. 1(b)]. This effect is not yet understood but will be pursued further on still finer whiskers. The ranges of validity for the T^2 behavior are compiled in Table I.

Obviously, by applying a magnetic field or by straining, the upper limit of this range is extended from about 1.5 to 2.3 K, so that the contribution of electron-phonon scattering to $\Delta\rho_d$ appears to be suppressed.[12] The slopes of the $\Delta\rho_d$ vs T^2 data define the parameter A_d which depends on sample diameter d, i.e., on the ratio $\rho_d(0)/\rho(0)$, magnetic field B, and strain-induced dislocation content (see Table I). For the unstrained samples $\rho(0)$ was obtained from a size-effect analysis.[9,13,14]

To illustrate the influence of the magnetic field on ρ_d at a fixed temperature, Fig. 3 shows the longitudinal magnetoresistances (LMR) recorded at 2 K up to 4 T. The maxi-

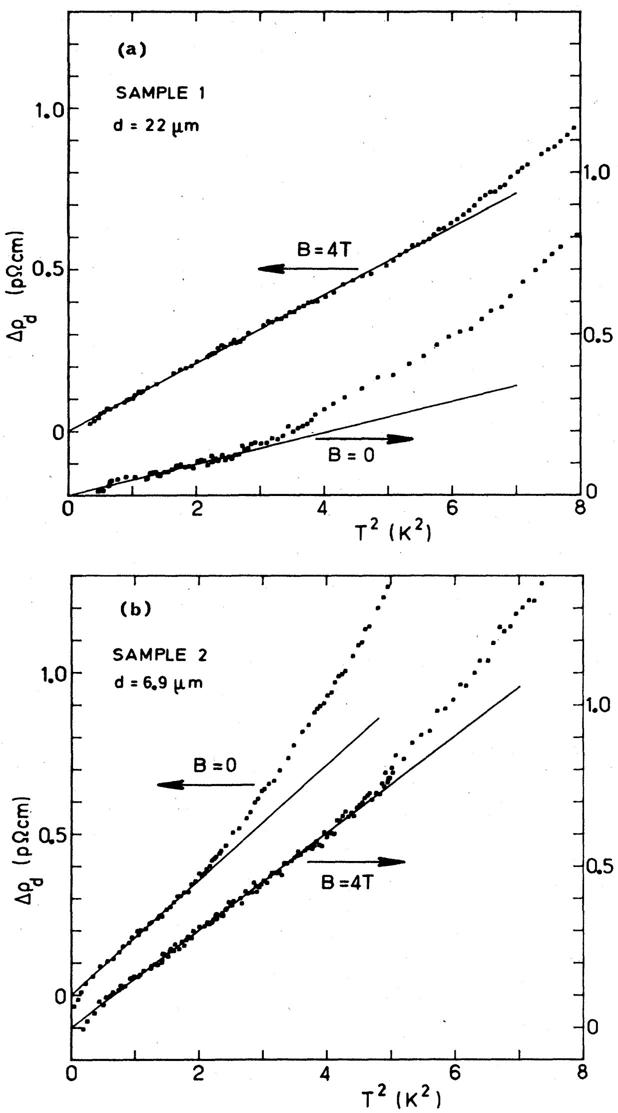

FIG. 1. Temperature-dependent part of the resistivity of the unstrained samples in zero magnetic field and in a longitudinal field of $B=4$ T. The straight lines represent fits in the temperature ranges listed in Table I.

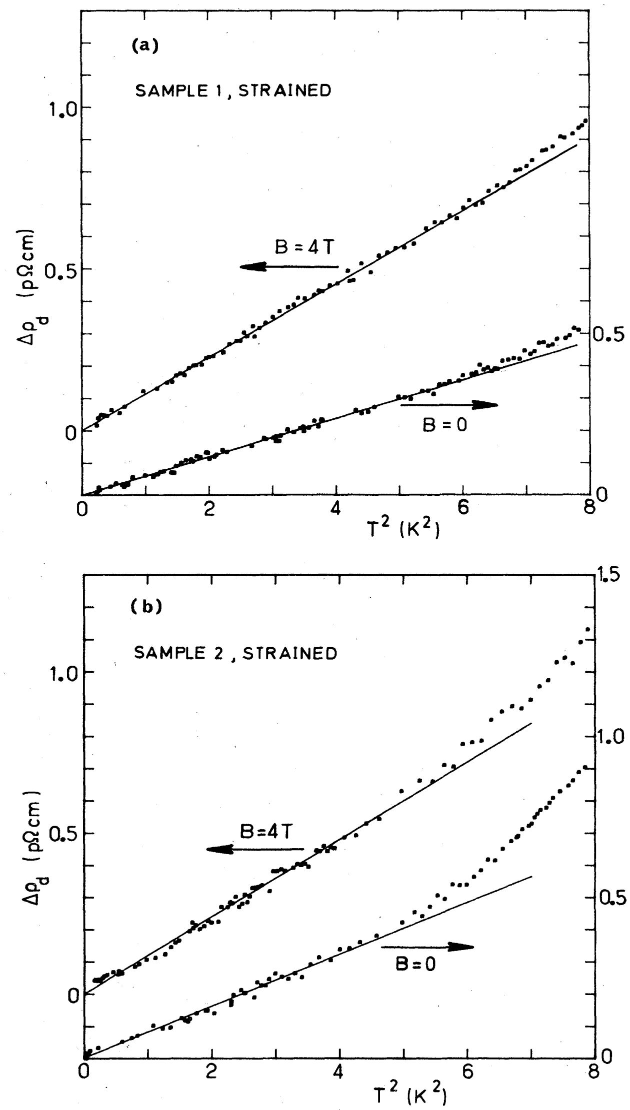

FIG. 2. Effect of straining on the temperature-dependent part of the resistivity of the same samples as in Fig. 1.

TABLE I. Sample parameters and data analysis.

Sample	d (μm)	B (T)	Temperature range (K)	A_d (fΩ cm K^{-2})	$\rho_d(0)$ (nΩ cm)	$\rho(0)$ (nΩ cm)	G	A_d/G (fΩ cm K^{-2})
1	22	0	0.65–1.6	49 ± 3	3.60	0.27	1.94 ± 0.06	26 ± 3
		4	0.58–2.0	107 ± 2			1.02 ± 0.01	105 ± 3
2	6.9	0	0.48–1.43	179 ± 2	7.84	0.27	6.7 ± 0.2	26.7 ± 0.7
		4	0.5–2.2	152 ± 1			1.3 ± 0.1	117 ± 9
1 strained	22	0	0.47–2.3	59 ± 2	4.75	1.42	1.2 ± 0.1	49 ± 4
		4	0.4–2.3	114 ± 2			1.02 ± 0.01	112 ± 3
2 strained	6.9	0	0.15–2.2	82 ± 2	8.45	0.88	2.7 ± 0.8	31 ± 10
		4	0.4–2.2	120 ± 2			1.2 ± 0.1	100 ± 10
a	1500	0				0.40	$\simeq1$	26.8 ± 0.5
b		0	0.4–1			7.29	1	121 ± 4

[a]Reference 12, sample *Cu* 6 from Khoshnevisan, Pratt, Jr., Schroeder, and Steenwyk (Ref. 17).
[b]Reference 18, sample *Cu*Ag-*A*4 heavily strained.

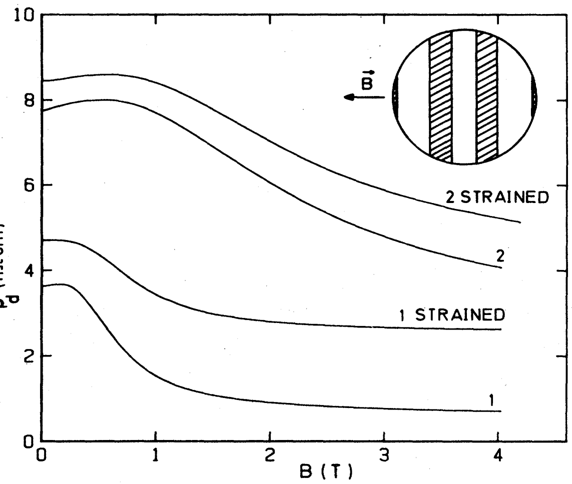

FIG. 3. The longitudinal magnetoresistivity $\rho_d(B)$ for both samples at $T = 2$ K. Inset: simplified drawing of the Fermi surface of copper for $\mathbf{B} \parallel \langle 111 \rangle$ (see text).

ma in finite fields arise from the superposition of a positive LMR of the bulk and of a negative size effect (e.g., Ref. 15). The latter is related to the fact that with increasing B a growing number of electrons orbiting around the longitudinal field is prevented from surface scattering.

In order to correct the experimental A_d values ranging from 49 to 179 $\mathrm{f\Omega\,cm\,K^{-2}}$ for surface scattering, we evaluated the enhancement factors G from Eq. (4), using $n_s = 0.923$ for the thicker sample 1 (etched surface) and $n_s = 0.795$ for sample 2 (as-annealed surface), as displayed in Fig. 2 of Ref. 9. The ratio $\rho_d(0)/\rho(0)$ was determined for $B = 0$ from the experimental values (Table I) and for $B = 4$ T from the theory of Chambers[16] yielding 1.28 and 2.43 for the samples 1 and 2, respectively. Within their error margins, the corrected coefficients $A = A_d/G$ listed in Table I can be divided into three groups as follows: (i) $A \simeq 27$ $\mathrm{f\Omega\,cm\,K^{-2}}$ for the unstrained samples in zero field, (ii) $A > 27$ $\mathrm{f\Omega\,cm\,K^{-2}}$ depending on the dislocation density in the strained samples at $B = 0$, and (iii) $A \simeq 110$ $\mathrm{f\Omega\,cm\,K^{-2}}$ for unstrained and strained samples at $B = 4$ T.

III. DISCUSSION

A. Zero magnetic field

Since the size-corrected coefficients A for the unstrained samples are identical, we may consider them as values for a bulk material of low dislocation density ($< 10^6$ $\mathrm{cm^{-2}}$). This conclusion is strongly supported by the excellent agreement of this number with that of Steenwyk, Rowlands, and Schroeder,[12] $A = 26.8$ $\mathrm{f\Omega\,cm\,K^{-2}}$, obtained on a bulk sample ($d \simeq 1500$ μm).[17] Moreover, this value is also identical with the theoretical estimate by KW,[5] $A_i = 27$ $\mathrm{f\Omega\,cm\,K^{-2}}$, based on an isotropic relaxation time $\tau(\mathbf{k})$. Although we may assume that the low dislocation density in our unstrained whiskers does not give rise to any significant anisotropy of $\tau(\mathbf{k})$, we believe that the identity between experimental and calculated values is more or less accidental in view of the approximations in the theory.

After application of strain, the coefficients A increased along with the residual resistivities $\rho(0)$. However, they remained much smaller than $A = 121$ $\mathrm{f\Omega\,cm\,K^{-2}}$ measured by Zwart, Pratt, Jr., Schroeder, and Caplin[18] on a heavily strained *Cu*Ag sample ($\rho_{\mathrm{dis}}/\rho_{\mathrm{imp}} \simeq 10$). To test the KW prediction for A [Eq. (2)] we determined from the $\rho(0)$ values the ratios $\rho_{\mathrm{dis}}/\rho_{\mathrm{imp}} = 4$ and 2 for samples 1 and 2, respectively, and obtain then $A = 62$ $\mathrm{f\Omega\,cm\,K^{-2}}$ and 51 $\mathrm{f\Omega\,cm\,K^{-2}}$. Both numbers follow the trend of the experimental results, but are about 15 $\mathrm{f\Omega\,cm\,K^{-2}}$ larger than these. A more detailed test of Eq. (2) is under investigation.

Qualitatively, this increase of A is related to the anisotropy of $\tau(\mathbf{k})$ on the Fermi surface of copper. For dislocations, $\tau(\mathbf{k})$ is much smaller in the nonspherical neck regions of the Fermi surface than in the belly regions (e.g., Ref. 4). Then normal electron-electron scattering events, which otherwise do not contribute to the resistivity, will redistribute the electrons by scattering them into regions of small $\tau(\mathbf{k})$. Consequently, the coefficient A increases with increasing anisotropy towards a limiting value A_a.

B. Longitudinal magnetic field

From Table I it is evident that straining has no significant effect on the coefficient A at $B = 4$ T. We obtain an average value of $A_d/G \simeq 110$ $\mathrm{f\Omega\,cm\,K^{-2}}$. This suggests that the effects of dislocations and of the longitudinal magnetic field on the electron-electron scattering resistivity are very similar.

Therefore, we also apply the anisotropy model to the situation in a longitudinal magnetic field and obtain a rather similar picture. Under the condition $\omega_c\tau_0 = 80$ at $B = 4$ T, electrons on cyclotron orbits passing through a neck region give a negligible contribution to the current, because of Bragg reflection at the necks. According to Pippard,[19] who suggested this model in his explanation of the bulk LMR $\rho(B)$ in copper, the rotation of the electrons around the magnetic field axis can be replaced by a rotation of the whole Fermi surface in the opposite direction. From this point of view the fast rotating necks seem to be forming belts on the Fermi surface perpendicular to $\mathbf{B}$ as indicated by the inset of Fig. 3. Within these belts $\tau(\mathbf{k})$ tends to zero with increasing field, while in the belly regions $\tau(\mathbf{k})$ remains unchanged. So this effect seems to account for the high anisotropy required for the explanation of the strongly enhanced electron-electron scattering. Owing to $\omega_c\tau_0 = 80$ we expect this field-induced anisotropy to be saturated. This is also supported by the fact that straining leaves the coefficient A essentially unchanged. Identifying $A = 110$ $\mathrm{f\Omega\,cm\,K^{-2}}$ with the anisotropic limit, we obtain for the ratio $A_a/A_i = 4$, a slightly larger value than 3 calculated by KW.[5] A systematic study of this strain- and field-induced anisotropy has been started.

Finally we should remark that, in view of the preceding discussion, the magnetic field effect upon A is not confined to thin samples, but should also be observable in bulk, polycrystalline samples of sufficient purity.

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[1]A. Bergmann, M. Kaveh, and N. Wiser, J. Phys. F **10**, L71 (1980).
[2]M. Kaveh and N. Wiser, J. Phys. F **10**, L37 (1980).
[3]M. Kaveh and N. Wiser, J. Phys. F **12**, 935 (1982).
[4]A. Bergmann, M. Kaveh, and N. Wiser, Phys. Rev. B **24**, 6807 (1981).
[5]M. Kaveh and N. Wiser, J. Phys. F **13**, 1207 (1983).
[6]W. E. Lawrence and J. W. Wilkins, Phys. Rev. B **7**, 2317 (1973).
[7]J. E. Black, Can. J. Phys. **56**, 708 (1978).
[8]A. H. MacDonald and M. J. Laubitz, Phys. Rev. B **21**, 2638 (1980).
[9]G. Thummes, V. Kuckhermann, and H. H. Mende, J. Phys. F (to be published).
[10]J. R. Sambles and T. W. Preist, J. Phys. F **12**, 1971 (1982).
[11]G. Thummes and H. H. Mende, Phys. Status Solidi A **26**, 243 (1974).
[12]S. D. Steenwyk, J. A. Rowlands, and P. A. Schroeder, J. Phys. F **11**, 1623 (1981).
[13]V. Kuckhermann, H. H. Mende, G. Thummes, and J. Kötzler, in *Proceedings of the Seventeenth International Conference on Low Temperature Physics, Karlsruhe, Germany, 1984,* edited by U. Eckern *et al.* (North-Holland, Amsterdam, 1984), p. 1093.
[14]V. Kuckhermann, G. Thummes, and H. H. Mende (unpublished).
[15]V. Kuckhermann, G. Thummes, and H. H. Mende, Phys. Status Solidi A **73**, 439 (1982).
[16]R. G. Chambers, Proc. R. Soc. London, Ser. A **202**, 378 (1950).
[17]M. Koshnevisan, W. P. Pratt, Jr., P. A. Schroeder, and S. D. Steenwyk, Phys. Rev. B **19**, 3873 (1979).
[18]J. W. Zwart, W. P. Pratt, Jr., P. A. Schroeder, and A. D. Caplin, J. Phys. F **13**, 2595 (1983).
[19]A. B. Pippard, Proc. R. Soc. London **282**, 464 (1964).