# Particle-hole symmetry violation in normal liquid $^3$He

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We study the effects of the particle-hole asymmetry terms contained in the Landau-Silin equations on the collective modes of normal $^3$He. We find that there are no frequency shifts, first order in particle-hole asymmetry terms, of either the zero sound mode or the spin wave modes of normal $^3$He. Propagating spin waves cannot be excited by zero sound; but, because of particle-hole asymmetry, zero sound can drive small magnetic oscillations with frequencies ($\omega_{\text{zero sound}} \pm \Omega_{\text{Larmor}}$) in the combined pulsed-NMR and zero sound experiment proposed by Ketterson.

In this paper we present a systematic study of the small particle-hole asymmetry (PHA) terms that are contained in the Landau-Silin equations[1,2] for the normal phase of liquid $^3$He. Our main result is that zero sound can induce small oscillations of the magnetization with frequencies ($\omega \pm \Omega_0$), where $\omega$ is the zero sound frequency and $\Omega_0$ is the Larmor frequency, and wavelength equal to that of zero sound. Our investigation was largely motivated by the paper of Ketterson[3] (hereafter referred to as JK), which suggests that because of PHA zero sound can be used as a probe of the transverse spin wave modes of $^3$He. However, we come to qualitatively different conclusions regarding the effects of PHA on the spectra of collective modes, and on the coupling between spin waves and zero sound. Thus, part of this paper is a commentary on the paper of JK. To begin, we briefly review PHA where it is most clearly exhibited, in the superfluid phases of $^3$He.

The particle-hole transformation is defined by a unitary operator $C$ which takes a quasiparticle with energy $\xi_{\vec{p}} = \epsilon_{\vec{p}} - \epsilon_F > 0$ above the Fermi surface (FS) into a quasiparticle with energy $-\xi_{\vec{p}}$ and rotates the spin by $\pi$. In terms of the quasiparticle field operators, $a_{\vec{p}\alpha} \to C a_{\vec{p}\alpha} C^\dagger = (i\sigma^y)_{\alpha\beta} a^\dagger_{\tilde{\vec{p}}\beta}$, where $\tilde{\vec{p}}$ is defined by $\xi_{\tilde{\vec{p}}} = -\xi_{\vec{p}}$ and $\hat{\tilde{p}} = \hat{p}$. In the energy band, $|\xi_{\vec{p}}| < \epsilon_0 \ll \epsilon_F$, the particle-hole transformation is an approximate symmetry of the low-energy (quasiparticle) Hamiltonian.[4] The particle-hole transformation is not an exact symmetry; the order of magnitude of the particle-hole symmetry violating terms is determined by the finite slope of the quasiparticle density of states at the FS, $N(\xi_{\vec{p}}) = N(0) + N'(0)\xi_{\vec{p}} + \cdots$. For one spin population $N(0) = m^* k_F (2\pi^2\hbar^2)^{-1}$ is determined by the effective mass $m^*$ and the Fermi wave vector $k_F = (3\pi^2 n)^{1/3}$, where $n$ is the equilibrium density. The slope $N'(0)$ is not related to parameters in the Fermi-liquid theory; however, from the Fermi-gas theory $N'(0) = N(0)/(2\epsilon_F)$, so it is to be expected that PHA is a small effect at low temperature $k_B T \ll \epsilon_F$; low excitation energy $\hbar\omega \ll \epsilon_F$, and magnetic fields $\gamma\hbar H \ll \epsilon_F$—the usual conditions for the validity of Landau's Fermi-liquid theory.

Particle-hole asymmetry is observable when it is associated with a broken symmetry or the singular response of a collective variable. In the superfluid phases PHA is clearly exhibited and leads to several striking effects. The $A_1$ phase of $^3$He is a condensate of quasiparticle pairs with their spins aligned parallel to the magnetic field lines. The range of temperatures in which the $A_1$ phase is thermodynamically stable is proportional to the magnetic field and determined by PHA.[5] Particle-hole asymmetry has also been observed in the dynamic response of superfluid $^3$He-B. The sharp resonance observed in the ultrasound attenuation by Giannetta *et al.*[6] and Mast *et al.*[7] has been identified as the $J = 2+$ mode (the real squashing mode).[8,9] The observed coupling of this mode to zero sound is nonzero because of PHA.[9] The recently reported gyromagnetic effect in rotating $^3$He-B has been interpreted as evidence that the vortex cores in $^3$He-B are ferromagnetic.[10] Theoretical models for the vortex magnetic moment in $^3$He-B have been investigated recently by Ohmi *et al.* and Salomaa and Volovik.[11] Ferromagnetic vortices were first proposed theoretically for the $^3P_2$ superfluid inside neutron stars.[12] In that case, but more generally for any ferromagnetic vortex in a $p$-wave superfluid, the magnetic moment is nonzero only because of PHA.[12] Given the attention that has been paid to these small, but impressive PHA effects in superfluid $^3$He, it seems worthwhile to study PHA in the normal phase of $^3$He, which is the main subject of the rest of this paper.

Recently, Ketterson[3] suggested that a combination of pulsed NMR and acoustics may be used to study the transverse spin wave modes of normal $^3$He. Ketterson's proposal is to use pulsed NMR to prepare the $^3$He in a uniform, but nonequilibrium, state in which the magnetization is tipped at an angle $\theta_H$ relative to the original static field $\vec{H}_0$. The magnetization then rotates about the static field at the Larmor frequency. The tipping angle $\theta_H$ is constant for times short compared with the spin-relaxation time $T_1$ (due to the combination of spin-flip scattering at the walls and the nuclear dipole interaction). Since $T_1$ can be made to be several hours for temperatures below 10 mK (Ref. 13) by coating the walls with $^4$He, there is more than sufficient time to probe the nonequilibrium spin system with zero sound. Ketterson's idea is that PHA provides the necessary coupling between zero sound and transverse spin waves. With $^3$He initially prepared as a bath of homogeneous spin waves, zero sound can be used to excite transverse spin waves with finite wave vector in a process where the $\vec{q}=0$ spin waves stimulate the decay of zero sound phonons into transverse spin waves with frequency determined by energy and momentum conservation, $\omega(\vec{q})-\Omega_0=\omega_{SW}(\vec{q})$, where $\omega(\vec{q})=c_L q$ is the longitudinal zero sound frequency, $\Omega_0$ is the $\vec{q}=0$ spin wave frequency, and $\omega_{SW}(\vec{q})$ is the final-state transverse spin wave frequency. Thus, by tuning the magnetic field, the zero sound frequency, and observing the decay of zero sound it should be possible to determine the transverse spin wave spectrum $\omega_{SW}(\vec{q})$. While this picture has its appeal, it is not possible to excite the transverse spin waves [with frequencies $\omega_{SW}(\vec{q})$] using zero sound because there is *no* coupling between zero sound and any propagating mode of the spin system. However, there is an interesting magnetic response to zero sound at frequencies $(\omega\pm\Omega_0)$ due to PHA, in the experimental arrangement proposed by Ketterson. But these magnetic oscillations are more accurately described as the driven response of the longitudinal magnetization in the rotating frame, modulated by the uniform rotation of the original homogeneous magnetization.[14] This is a different physical picture than that proposed by Ketterson; the spin variables that couple to zero sound—with both PHA and a finite $\theta_H$—have no natural oscillations. They are not propagating spin wave modes, and consequently, there is no resonant transfer of energy between the spin wave system and zero sound as is implied by the quantum decay process suggested in JK.

For Ketterson's proposed experiment the PHA terms imply that there is a small magnetization wave, induced by zero sound $\delta n(\vec{r},t)\propto e^{i(\vec{q}\cdot\vec{r}-\omega t)}$, that is modulated by the uniformly rotating magnetization. The components of this magnetization wave, in a coordinate system $(x,y,z)$ with $\hat{z}$ being the direction of the static magnetic field $\vec{H}_0$, are

$$\delta M_x \pm i\delta M_y=\eta_M\left[\frac{\gamma\hbar}{2}\right]\delta n(\vec{r},t)\sin(\theta_H)e^{\pm i\Omega_0 t}, \quad (1a)$$

$$\delta M_z=\eta_M\left[\frac{\gamma\hbar}{2}\right]\delta n(\vec{r},t)\cos(\theta_H). \quad (1b)$$

where $\eta_M\sim(\hbar\Omega_0/\epsilon_F)$ is a measure of the PHA to be defined below. Thus, the $\delta M_x$ and $\delta M_y$ components are nonzero only for finite tipping angles $(\theta_H\neq 0)$ and oscillate at frequencies $(\omega\pm\Omega_0)$, which should facilitate their detection since the induced magnetic oscillations, which are small in magnitude, are separated in frequency from both the unperturbed rotating magnetization and the sound wave.

This PHA effect may still be difficult to detect. The magnitude of the magnetization is small,[15]

$$|\delta\vec{M}|\sim\left[\frac{\hbar\Omega_0}{\epsilon_F}\right]\left[\frac{\gamma\hbar}{2}\right]|\delta n|\cong 10^{-8}\text{ G }[H_0/(10\text{ kG})],$$

which for an area $A_{\text{cell}}=2\text{ cm}^2$ and $H_0=10$ kG gives a magnetic flux $|\Phi|=|\delta\vec{M}|A_{\text{cell}}$ equal to approximately one-tenth of a flux quantum. The simplest geometry for detecting these oscillations is a $^3$He cell with the static magnetic field along the $\hat{z}$ axis, with a conducting loop around the cell in the $x$-$z$ plane, and—after the magnetization is tipped away from the $\hat{z}$ axis—with sound waves transmitted along the $\hat{y}$ axis through the conducting loop. The magnetic oscillations induce an ac voltage of magnitude

$$|V|\cong\frac{1}{c}|\Phi|\Delta\omega$$
$$\cong 2\times 10^{-6}\text{ V }[H_0/(10\text{ kG})][\Delta\omega/(10^6\text{ sec}^{-1})],$$

where $\Delta\omega=\omega_0\pm\Omega_0$. This value will be smaller if the wave front is not uniform over the full area $A_{\text{cell}}$. These voltage oscillations might be observable, but their high frequency, their short wavelength ($\lambda=3.5\times 10^{-2}$ cm $[1\text{ MHz}/(\omega_0/2\pi)]$), and the finite (but long) spin-relaxation time $T_1$ may make their detection difficult.

It is shown in Ref. 14 (see also the Appendix) that the form of Eqs. (1)—including the dependence on $\theta_H$, $\omega\pm\Omega_0$, and $\eta_M$—can be obtained from a symmetry analysis of the retarded Green's functions that determine the density and spin-density response to a scalar field. However, not all questions regarding the effects of the small PHA terms on the collective mode spectrum and the coupling between these modes can be resolved by symmetry arguments. For this purpose we analyze the PHA terms contained in the collisionless Landau-Silin equation.

A general description of the nonequilibrium properties of normal $^3$He would start from a quantum transport equation for the Wigner distribution function that describes the quasiparticle excitations. However, for the cases we consider the wave vector and frequency of the disturbances are sufficiently small, $\hbar\omega$ and $\hbar q v_F \ll k_B T=\beta^{-1}$, that the quantum transport equation reduces to the Landau-Silin equation for the semiclassical spin-matrix distribution function $\hat{n}_{\vec{p}}(\vec{r},t)$,[16]

$$\partial_t\hat{n}_{\vec{p}}-i[\hat{n}_{\vec{p}},\hat{\epsilon}_{\vec{p}}]+\tfrac{1}{2}\{\partial^i_{\vec{p}}\hat{\epsilon}_{\vec{p}},\partial^i_{\vec{r}}\hat{n}_{\vec{p}}\}$$
$$-\tfrac{1}{2}\{\partial^i_{\vec{r}}\hat{\epsilon}_{\vec{p}},\partial^i_{\vec{p}}\hat{n}_{\vec{p}}\}=\hat{I}_{\vec{p}}, \quad (2)$$

where $[\hat{A},\hat{B}]$ ($\{\hat{A},\hat{B}\}$) is a commutator (anticommutator) in spin space, $\hat{I}_{\vec{p}}$ is a collision integral, and the quasiparticle energy $\hat{\epsilon}_{\vec{p}}$ is a functional of the distribution function. Specifically, for a given perturbation $\delta\hat{n}_{\vec{p}}$ of the distribution function, the corresponding change in quasiparticle energy is

$$[\delta\epsilon_{\vec{p}}]_{\alpha\gamma}=\int\frac{d^3p'}{(2\pi)^3}f(\vec{p},\vec{p}\,')_{\alpha\beta:\gamma\rho}[\delta\hat{n}_{\vec{p}\,'}]_{\rho\beta}, \quad (3)$$

where

$$f(\vec{p},\vec{p}\,')_{\alpha\beta:\gamma\rho}=f^s(\vec{p},\vec{p}\,')\delta_{\alpha\gamma}\delta_{\beta\rho}+f^a(\vec{p},\vec{p}\,')\vec{\sigma}_{\alpha\gamma}\cdot\vec{\sigma}_{\beta\rho}$$

is the quasiparticle interaction function for magnetic fields $\gamma\hbar H\ll\epsilon_F$. External fields contribute to the quasiparticle energy implicitly, as well as explicitly, by modifying the quasiparticle distribution function. In the presence of a magnetic field $\vec{H}=-(\gamma\hbar/2)^{-1}\vec{h}(\vec{r},t)$ and a scalar field $u(\vec{r},t)$, the quasiparticle energy becomes

$$\hat{\epsilon}_{\vec{p}}=\epsilon_{\vec{p}}\hat{1}+\vec{h}_{\vec{p}}\cdot\hat{\vec{\sigma}}\ , \tag{4a}$$

$$\epsilon_{\vec{p}}=\xi_{\vec{p}}+u+2\int\frac{d^3p'}{(2\pi)^3}f^s(\vec{p},\vec{p}\,')\delta n_{\vec{p}\,'}\ , \tag{4b}$$

$$\vec{h}_{\vec{p}}=\vec{h}+2\int\frac{d^3p'}{(2\pi)^3}f^a(\vec{p},\vec{p}\,')\delta\vec{m}_{\vec{p}\,'}\ , \tag{4c}$$

where $\delta n_{\vec{p}}$ and $\delta\vec{m}_{\vec{p}}$ are scalar and vector components of the response of the distribution function, $\delta\hat{n}_{\vec{p}}=\delta n_{\vec{p}}\hat{1}+\delta\vec{m}_{\vec{p}}\cdot\hat{\vec{\sigma}}$, to the external fields. These response functions are determined by a coupled set of integral-differential equations for the scalar and vector components of Eq. (2) and by the initial distribution function. For $\hat{n}_{\vec{p}}=n_{\vec{p}}\hat{1}+\vec{m}_{\vec{p}}\cdot\hat{\vec{\sigma}}$, Eq. (2) becomes

$$\partial_t n_{\vec{p}}+(\partial^i_{\vec{p}}\epsilon_{\vec{p}})(\partial^i_{\vec{r}}n_{\vec{p}})+(\partial^i_{\vec{p}}h^\mu_{\vec{p}})(\partial^i_{\vec{r}}m^\mu_{\vec{p}}) -(\partial^i_{\vec{r}}\epsilon_{\vec{p}})(\partial^i_{\vec{p}}n_{\vec{p}})-(\partial^i_{\vec{r}}h^\mu_{\vec{p}})(\partial^i_{\vec{p}}m^\mu_{\vec{p}})=I_{\vec{p}}\ , \tag{5a}$$

$$\partial_t\vec{m}_{\vec{p}}-2(\vec{h}_{\vec{p}}\times\vec{m}_{\vec{p}})+(\partial^i_{\vec{p}}\epsilon_{\vec{p}})(\partial^i_{\vec{r}}\vec{m}_{\vec{p}})+(\partial^i_{\vec{p}}\vec{h}_{\vec{p}})(\partial^i_{\vec{r}}n_{\vec{p}}) -(\partial^i_{\vec{r}}\epsilon_{\vec{p}})(\partial^i_{\vec{p}}\vec{m}_{\vec{p}})-(\partial^i_{\vec{r}}\vec{h}_{\vec{p}})(\partial^i_{\vec{p}}n_{\vec{p}})=\vec{I}_{\vec{p}}\ . \tag{5b}$$

Here we study only the collisionless excitations, and so we set $\hat{I}_{\vec{p}}=0$. At sufficiently low temperatures such that $\omega\tau\gg1$, where $\omega$ is the characteristic frequency of an excitation and $\tau\propto T^{-2}$ is the quasiparticle relaxation time, damping of the collective excitations by quasiparticle scattering can be neglected, or included perturbatively.

The well-known collisionless collective excitations of $^3$He (longitudinal and transverse zero sound, and the spin wave modes) are reviewed in detail by Baym and Pethick.[16] Here we briefly review the derivation of the equations of motion for these modes since the notation and results are useful for what follows. The collisionless modes are the solutions to Eqs. (5) linearized about the equilibrium distribution,

$$\hat{n}^{\rm eq}_{\vec{p}}=n_0(\xi_{\vec{p}})\hat{1}+\vec{m}_0(\xi_{\vec{p}})\cdot\hat{\vec{\sigma}}\ , \tag{6a}$$

$$n_0(\xi_{\vec{p}})=(1+e^{\beta\xi_{\vec{p}}})^{-1}\ , \tag{6b}$$

$$\vec{m}_0(\xi_{\vec{p}})=n_0'\vec{h}_{\rm eq}(\xi_{\vec{p}})\ , \tag{6c}$$

where $n_0'=\partial n_0/\partial\xi_{\vec{p}}$, $\vec{h}_{\rm eq}(\xi_{\vec{p}})=\vec{h}_0[1-F^a_0(\xi_{\vec{p}})/(1+F^a_0)]$ includes the molecular-field correction to the static field $\vec{h}_0=-(\gamma\hbar/2)\vec{H}_0$, $F^a_0(\xi_{\vec{p}})$ is the energy-dependent $l=0$ Landau parameter defined in Eqs. (17), $F^a_0=F^a_0(0)$ is the usual Landau parameter, and $\xi_{\vec{p}}=\epsilon^{(0)}_{\vec{p}}-\epsilon_F$ is the unperturbed quasiparticle energy measured relative to the Fermi energy. The linearized equations, written in terms of the deviations, $\delta n_{\vec{p}}=n_{\vec{p}}-n_0$, $\delta\vec{m}_{\vec{p}}=\vec{m}_{\vec{p}}-\vec{m}_0$, $\delta\epsilon_{\vec{p}}=\epsilon_{\vec{p}}-\xi_{\vec{p}}$, and $\delta\vec{h}_{\vec{p}}=\vec{h}_{\vec{p}}-\vec{h}_{\rm eq}$ become

$$\partial_t\delta n_{\vec{p}}+\vec{v}_{\vec{p}}\cdot\vec{\partial}_{\vec{r}}\left[\delta n_{\vec{p}}-n_0'\delta\epsilon_{\vec{p}}-n_0''\vec{h}_{\rm eq}(0)\cdot\delta\vec{h}_{\vec{p}}+\left[\frac{\partial}{\partial\xi_{\vec{p}}}\vec{h}_{\rm eq}\right]\cdot\delta\vec{m}_{\vec{p}}\right]=0\ , \tag{7a}$$

$$\partial_t\delta\vec{m}_{\vec{p}}-2(\vec{h}_{\rm eq}\times\delta\vec{m}_{\vec{p}})-2(\delta\vec{h}_{\vec{p}}\times\vec{m}_0)+\vec{v}_{\vec{p}}\cdot\vec{\partial}_{\vec{r}}\left[\delta\vec{m}_{\vec{p}}-n_0'\delta\vec{h}_{\vec{p}}-n_0''\vec{h}_{\rm eq}(0)\delta\epsilon_{\vec{p}}+\left[\frac{\partial}{\partial\xi_{\vec{p}}}\vec{h}_{\rm eq}\right]\delta n_{\vec{p}}\right]=0\ . \tag{7b}$$

At low temperatures $n_0'\cong-\delta(\xi_{\vec{p}})$, thus only those excitations very near the FS are relevant for the linear dynamics. The next step, which is the usual procedure in simplifying the low-temperature kinetic equation, is to integrate Eqs. (7) over the band of quasiparticle energies $|\xi_{\vec{p}}|<\epsilon_0$, and so derive equations for the much simpler "$\xi_{\vec{p}}$-integrated" scalar and spin-density distribution functions

$$\nu_{\hat{p}}=\int_{-\epsilon_0}^{\epsilon_0}d\xi_{\vec{p}}\delta n_{\vec{p}}\ , \tag{8a}$$

$$\vec{\sigma}_{\hat{p}}=\int_{-\epsilon_0}^{\epsilon_0}d\xi_{\vec{p}}\delta\vec{m}_{\vec{p}}\ , \tag{8b}$$

which are functions only of the direction $\hat{p}$ of the quasiparticle momentum $\vec{p}$ near the FS. This method works because the quasiparticle velocity, $\vec{v}_{\vec{p}}=\vec{\partial}_{\vec{p}}\xi_{\vec{p}}$, and molecular fields are slowly varying functions of $\xi_{\vec{p}}$ near the FS, so to a good approximation they can be replaced by their values on the FS. (What is left out of this procedure are the PHA effects.) When this step is carried out the $\xi_{\vec{p}}$-integrated equations for $\nu_{\hat{p}}$ and $\vec{\sigma}_{\hat{p}}$ decouple, and when written in terms of Fourier components

$$\nu_{\hat{p}}(\vec{q},\omega)=\int d^3r\int dt\, e^{-i(\vec{q}\cdot\vec{r}-\omega t)}\nu_{\hat{p}}(\vec{r},t)\ ,$$

etc., these equations become

$$(\omega-\vec{q}\cdot\vec{v}_{\hat{p}})\nu_{\hat{p}}-(\vec{q}\cdot\vec{v}_{\hat{p}})\delta\epsilon_{\hat{p}}=(\vec{q}\cdot\vec{v}_{\hat{p}})u\ , \tag{9a}$$

$$\omega\vec{\sigma}_{\hat{p}}-2i\vec{h}_{\rm eq}\times(\vec{\sigma}_{\hat{p}}+\delta\vec{h}_{\hat{p}})-(\vec{q}\cdot\vec{v}_{\hat{p}})[\vec{\sigma}_{\hat{p}}+\delta\vec{h}_{\hat{p}}] =2i\vec{h}_{\rm eq}\times\vec{h}_{\rm xt}+(\vec{q}\cdot\vec{v}_{\hat{p}})\vec{h}_{\rm xt}\ , \tag{9b}$$

where $\vec{v}_{\hat{p}}=v_F\hat{p}$ is the Fermi velocity and the external fields $(u,\vec{h}_{\rm xt})$ are shown explicitly. The molecular fields are given by

$$\delta\epsilon_{\hat{p}}=\int\frac{d\Omega_{\hat{p}'}}{4\pi}F^s(\hat{p}\cdot\hat{p}\,')\nu_{\hat{p}'}\ , \tag{10a}$$

$$\delta\vec{h}_{\hat{p}}=\int\frac{d\Omega_{\hat{p}'}}{4\pi}F^a(\hat{p}\cdot\hat{p}\,')\vec{\sigma}_{\hat{p}'}\ , \tag{10b}$$

where

$$2N(0)f^{s,a}(p_F\hat{p},p_F\hat{p}\,')=F^{s,a}(\hat{p}\cdot\hat{p}\,') =\sum_{l\geq0}F^{s,a}_lP_l(\hat{p}\cdot\hat{p}\,')$$

are the Landau interaction functions evaluated on the FS.

Here we briefly discuss the solutions to these equations.[16] Both longitudinal and transverse (with respect to $\vec{q}$) zero sound are eigenmodes of Eq. (9a), with linear dispersion relations $\omega_{\rm L,T}=c_{\rm L,T}q$. These modes propagate in the absence of collisions because the quasiparticle interaction $F^s$ provides a restoring force for density and current fluctuations. For longitudinal zero sound the restoring force (due primarily to $F_0^s$ and $F_1^s$) is large and leads to a high sound velocity, $c_L \gg v_F$; consequently, $(qv_F/\omega)$ is a useful expansion parameter for this mode.

The spin wave modes of $^3$He are obtained by solving Eq. (9b) for the components of $\vec{\sigma}_{\hat{p}}$ in a coordinate system $(\hat{x},\hat{y},\hat{z})$ with $\hat{z}\,||\,\vec{h}_{eq}$. The equations of motion become

$$\omega\sigma_{\hat{p}}^z - \vec{q}\cdot\vec{v}_{\hat{p}}(\sigma_{\hat{p}}^z + \delta h_{\hat{p}}^z) = \vec{q}\cdot\vec{v}_{\hat{p}} h_{xt}^z \;, \tag{11a}$$

$$\omega\sigma_{\hat{p}}^{\pm} \pm 2h_{eq}(\sigma_{\hat{p}}^{\pm}+\delta h_{\hat{p}}^{\pm}) - \vec{q}\cdot\vec{v}_{\hat{p}}(\sigma_{\hat{p}}^{\pm}+\delta h_{\hat{p}}^{\pm}) = (\mp 2h_{eq} + \vec{q}\cdot\vec{v}_{\hat{p}}) h_{xt}^{\pm} \;, \tag{11b}$$

where $\sigma_{\hat{p}}^{\pm} = (\sigma_{\hat{p}}^x \pm i\sigma_{\hat{p}}^y)/\sqrt{2}$. Equation (11a) for $\sigma_{\hat{p}}^z$ is formally the same as that for the scalar distribution function $\nu_{\hat{p}}$, except that the interaction $F^s$ is replaced by $F^a$. In consequence, there are no spin wave modes associated with the variable $\sigma_{\hat{p}}^z$, because in $^3$He the interaction $F^a$ does not provide a restoring force for longitudinal fluctuations of $\vec{\sigma}_{\hat{p}}$ (with respect to $\vec{h}_{eq}$); the antisymmetric Landau parameters $(F_0^a, F_1^a, F_2^a)$ are all below the threshold values for propagating longitudinal spin wave modes. The spin wave excitations that exist in $^3$He are eigenmodes of the transverse components $\sigma_{\hat{p}}^{\pm}$, and have eigenfrequencies that depend on the angular momentum quantum numbers of the particle-hole pairs. The spherically symmetric components $(l=0)$, $\sigma_{00}^{\pm} = \int (d\Omega_{\hat{p}}/4\pi)\sigma_{\hat{p}}^{\pm}$, corresponding to the transverse oscillations of the magnetization, have eigenfrequencies (at $\vec{q}=0$) $\omega_{00}^{\pm} = \mp\Omega_0$, where $\Omega_0 = (2/\hbar)h_0 = -\gamma H_0$ is the Larmor frequency. The higher angular momentum components $\sigma_{lm}^{\pm}$ are also eigenmodes, but with frequencies that are separated from the Larmor frequency because of Fermi-liquid interactions,

$$\omega_{lm}^{\pm}(\vec{q}=0) = \mp\Omega_0[1+F_l^a/(2l+1)]/(1+F_0^a) \;.$$

In the standard NMR experiment a spatially uniform transverse magnetic field excites only the $l=0$ transverse modes. However, Doniach[17] has noted that the $l\geq 1$ transverse modes can be inferred from the experimental observation[18] of the Leggett-Rice effect.[19] Recent NMR measurements on the $^3$He-$^4$He system at temperatures below 25 mK show the first evidence of $l \geq 1$ transverse spin waves in that system.[20]

Now we consider the effects of PHA on the sound and spin dynamics of $^3$He, in the presence of a nonequilibrium tipped magnetization. Immediately after an NMR tipping pulse is applied to $^3$He, the distribution function that describes the rotating magnetic state is

$$\hat{n}_{\hat{p}} = n_0(\xi_{\vec{p}})\hat{1} + \vec{m}_t(\xi_{\vec{p}})\cdot\hat{\vec{\sigma}} \;, \tag{12}$$

where $\vec{m}_t = n_0' h_{eq}\hat{\mu}(t)$ describes an instantaneous rotation of the equilibrium spin-density distribution $\vec{m}_0 = n_0' h_{eq}\hat{z}$. The magnetization

$$\vec{M}(t) = -\left[\frac{\gamma\hbar}{2}\right] 2N(0) h_{eq}\hat{\mu}(t)$$

precesses about the static field $\vec{h}_0 = h_0\hat{z}$ according to

$$\partial_t\hat{\mu} - 2\vec{h}_0\times\hat{\mu} = 0 \;. \tag{13}$$

By convention we choose $\hat{\mu}(t=0) = \vec{R}(\hat{y},\theta_H)\cdot\hat{z}$, a rotation of the equilibrium magnetization about the $\hat{y}$ axis by an angle $\theta_H$, so that

$$\hat{\mu}(t) = \sin(\theta_H)[\cos(\Omega_0 t)\hat{x} + \sin(\Omega_0 t)\hat{y}] + \cos(\theta_H)\hat{z} \;. \tag{14}$$

Next we linearize the collisionless Landau-Silin kinetic equations [Eqs. (9) and (10)] in the deviation of the distribution function (and molecular field) from the time-dependent distribution describing the rotating state [Eqs. (12)—(14)]. The linear equations of motion for the perturbations, $\delta n_{\vec{p}} = n_{\vec{p}} - n_0(\xi_{\vec{p}})$ and $\delta\vec{m}_{\vec{p}} = \vec{m}_{\vec{p}} - \vec{m}_t(\xi_{\vec{p}})$ become

$$\partial_t\delta n_{\vec{p}} + \vec{v}_{\vec{p}}\cdot\vec{\partial}_{\vec{r}}\left[\delta n_{\vec{p}} - n_0'\delta\epsilon_{\vec{p}} - n_0'' h_{eq}\hat{\mu}(t)\cdot\delta\vec{h}_{\vec{p}} + \left[\frac{\partial}{\partial\xi_{\vec{p}}}\delta\vec{h}_{rot}(\xi_{\vec{p}})\right]\cdot\delta\vec{m}_{\vec{p}}\right] = \vec{v}_{\vec{p}}\cdot\vec{\partial}_{\vec{r}}[n_0'\epsilon_{xt} + n_0'' h_{eq}\hat{\mu}(t)\cdot\vec{h}_{xt}] \;, \tag{15a}$$

$$\begin{aligned}&\partial_t\delta\vec{m}_{\vec{p}} - 2\{(\vec{h}_0+\delta\vec{h}_{rot})\times\delta\vec{m}_{\vec{p}} - n_0' h_{eq}[\hat{\mu}(t)\times\delta\vec{h}_{\vec{p}}]\} \\ &\quad + \vec{v}_{\vec{p}}\cdot\vec{\partial}_{\vec{r}}\left[\delta\vec{m}_{\vec{p}} - n_0'\delta\vec{h}_{\vec{p}} + \left[\frac{\partial}{\partial\xi_{\vec{p}}}\delta\vec{h}_{rot}\right]\delta n_{\vec{p}} - n_0'' h_{eq}\hat{\mu}(t)\delta\epsilon_{\vec{p}}\right] \\ &= n_0' h_{eq}(\vec{h}_{xt}\times\hat{\mu}) + \vec{v}_{\vec{p}}\cdot\vec{\partial}_{\vec{r}}[n_0'\vec{h}_{xt} + n_0'' h_{eq}\hat{\mu}(t)\epsilon_{xt}] \;.\end{aligned} \tag{15b}$$

In addition to the molecular fields generated by the perturbation of the distribution function, there is a time-dependent molecular field $\delta\vec{h}_{rot}$ generated by the magnetization of the unperturbed rotating state,

$$\begin{aligned}\delta\vec{h}_{rot} &= 2\int\frac{d^3p'}{(2\pi)^3} f^a(\vec{p},\vec{p}\,')\vec{m}_t(\xi_{\vec{p}'}) \\ &= -2N(0)h_{eq}\hat{\mu}(t)\int\frac{d\Omega_{\hat{p}'}}{4\pi} f^a(|\vec{p}|\hat{p}, p_F\hat{p}') \;,\end{aligned} \tag{16}$$

which depends only on the magnitude of the quasiparticle momentum, or equivalently the quasiparticle energy $\xi_{\vec{p}}$ near the FS. It is convenient to define energy-dependent Landau parameters,

$$F^{s,a}(\hat{p}\cdot\hat{p}'\,|\,\xi_{\vec{p}}) = 2N(0)f^{s,a}(|\vec{p}|\hat{p}, p_F\hat{p}') \;, \tag{17a}$$

$$F_l^{s,a}(\xi) = (2l+1)\int_{-1}^{1}\frac{dx}{2}F^{s,a}(x\,|\,\xi)P_l(x) \;. \tag{17b}$$

These quantities, as well as the density of states $N(\xi)$ and the quasiparticle velocity $\vec{v}_{\vec{p}} = v(\xi_{\vec{p}})\hat{p}$, enter kinetic equations when PHA is included. The molecular field generated by $\vec{m}_t$ can then be written

$$\delta\vec{h}_{\rm rot} = -h_{\rm eq} F_0^a(\xi_{\vec{p}})\hat{\mu}(t) \,. \tag{18}$$

Inspection of Eqs. (15) shows that the $\xi_{\vec{p}}$-integrated distribution functions ($\nu_{\hat{p}}$ and $\vec{\sigma}_{\hat{p}}$) become weakly coupled when PHA corrections are included. In addition to a direct coupling between these variables, there is also an indirect coupling via the energy moments of the distribution functions,

$$\beta_{\hat{p}} = -h_{\rm eq}^{-1} \int d\xi_{\vec{p}} \xi_{\vec{p}} \delta n_{\vec{p}} \,, \tag{19a}$$

$$\vec{\zeta}_{\hat{p}} = -h_{\rm eq}^{-1} \int d\xi_{\vec{p}} \xi_{\vec{p}} \delta\vec{m}_{\vec{p}} \,, \tag{19b}$$

where the prefactor is a convenient normalization. The coupled equations for the $\xi_{\vec{p}}$-integrated functions follow from Eqs. (15)—(19).

The scalar equations are

$$\partial_t \nu_{\hat{p}} + \vec{v}_{\hat{p}} \cdot \vec{\partial}_{\vec{r}} (\nu_{\hat{p}} + \delta\epsilon_{\hat{p}} + \eta_v \beta_{\hat{p}} + \hat{\mu}\cdot\vec{\Sigma}_{\hat{p}}) = -\vec{v}_{\hat{p}} \cdot \vec{\partial}_{\vec{r}} (\epsilon_{\rm xt} + \eta_v \hat{\mu}\cdot\vec{h}_{\rm xt}) \,, \tag{20a}$$

$$\partial_t \beta_{\hat{p}} + \vec{v}_{\hat{p}} \cdot \vec{\partial}_{\vec{r}} (\beta_{\hat{p}} + \hat{\mu}\cdot\delta\vec{h}_{\hat{p}} + \eta_a \hat{\mu}\cdot\vec{\zeta}_{\hat{p}}) = -\vec{v}_{\hat{p}} \cdot \vec{\partial}_{\vec{r}} [\hat{\mu}\cdot\vec{h}_{\rm xt}] \,. \tag{20b}$$

The vector (spin) equations are

$$\partial_t \vec{\sigma}_{\hat{p}} - 2[\vec{h}_0 \times \vec{\sigma}_{\hat{p}} - h_{\rm eq}\hat{\mu}\times(F_0^a \vec{\sigma}_{\hat{p}} - \delta\vec{h}_{\hat{p}} + \eta_a \vec{\zeta}_{\hat{p}})] + \vec{v}_{\hat{p}} \cdot \vec{\partial}_{\vec{r}} (\vec{\sigma}_{\hat{p}} + \delta\vec{h}_{\hat{p}} + \eta_v \vec{\zeta}_{\hat{p}} + \hat{\mu} A_{\hat{p}}) = 2h_{\rm eq}(\hat{\mu}\times\vec{h}_{\rm xt}) - \vec{v}_{\hat{p}} \cdot \vec{\partial}_{\vec{r}} (\vec{h}_{\rm xt} + \eta_v \epsilon_{\rm xt}\hat{\mu}) \,, \tag{20c}$$

$$\partial_t \vec{\zeta}_{\hat{p}} - 2[\vec{h}_0 \times \vec{\zeta}_{\hat{p}} - h_{\rm eq} F_0^a (\hat{\mu}\times\vec{\zeta}_{\hat{p}})] + \vec{v}_{\hat{p}} \cdot \vec{\partial}_{\vec{r}} (\vec{\zeta}_{\hat{p}} + \hat{\mu}\delta\epsilon_{\hat{p}} + \eta_a \hat{\mu}\beta_{\hat{p}}) = -\vec{v}_{\hat{p}} \cdot \vec{\partial}_{\vec{r}} (\hat{\mu}\epsilon_{\rm xt}) \,. \tag{20d}$$

The PHA parameters $\eta_v$ and $\eta_a$ [Eqs. (23b) and (23c)] couple the scalar and vector functions explicitly. There are additional PHA couplings from the energy dependence of the molecular fields and quasiparticle velocity,

$$\vec{\Sigma}_{\hat{p}} = \int \frac{d\Omega_{\hat{p}'}}{4\pi} H^a(\hat{p}\cdot\hat{p}')\vec{\sigma}_{\hat{p}'} + \eta_a \vec{\sigma}_{\hat{p}} \,, \tag{21a}$$

$$A_{\hat{p}} = \int \frac{d\Omega_{p'}}{4\pi} H^s(\hat{p}\cdot\hat{p}')\nu_{\hat{p}'} + \eta_a \nu_{\hat{p}} \,, \tag{21b}$$

as well as PHA corrections to the molecular field that come from the energy dependence of the density of states and the quasiparticle interactions. Thus, $\delta\epsilon_{\hat{p}}$ and $\delta\vec{h}_{\hat{p}}$ are given by

$$\delta\epsilon_{\hat{p}} = \int \frac{d\Omega_{p'}}{4\pi} [F^s(\hat{p}\cdot\hat{p}')\nu_{\hat{p}'} + G^s(\hat{p}\cdot\hat{p}')\beta_{\hat{p}'}] \,, \tag{22a}$$

$$\delta\vec{h}_{\hat{p}} = \int \frac{d\Omega_{p'}}{4\pi} [F^a(\hat{p}\cdot\hat{p}')\vec{\sigma}_{\hat{p}'} + G^a(\hat{p}\cdot\hat{p}')\vec{\zeta}_{\hat{p}'}] \,. \tag{22b}$$

There is one additional PHA parameter $\eta_N$ which enters the calculation of a one-body observable from the distribution function; thus the PHA parameters are

$$\eta_N = -h_{\rm eq} \left[ \frac{\partial}{\partial\xi} \frac{N(\xi)}{N(0)} \right]_{\xi=0} \,, \tag{23a}$$

$$\eta_v = -h_{\rm eq} \left[ \frac{1}{v} \frac{\partial}{\partial\xi} v(\xi) \right]_{\xi=0} \,, \tag{23b}$$

$$\eta_a = -h_{\rm eq} \left[ \frac{\partial}{\partial\xi} F_0^a(\xi) \right]_{\xi=0} \,, \tag{23c}$$

$$H^{s,a}(x) = -h_{\rm eq} \frac{\partial}{\partial\xi} \left[ \frac{v(\xi)}{v(0)} F^{s,a}(x\mid\xi) \right]_{\xi=0} \,, \tag{23d}$$

$$G^{s,a}(x) = -h_{\rm eq} \frac{\partial}{\partial\xi} \left[ \frac{N(\xi)}{N(0)} F^{s,a}(x\mid\xi) \right]_{\xi=0} \,. \tag{23e}$$

Equations (20)—(23) are the complete set of Landau-Silin equations, linearized about the rotating state, which include all first order PHA terms.

The PHA parameters are typically of order $\hbar\Omega_0/\epsilon_0$, where $\epsilon_0$ is the relevant energy scale of the quasiparticle density of states, velocity, or interactions. If we assume $\epsilon_0 = \epsilon_F \cong 1$ K, then $\hbar\Omega_0/\epsilon_F = 1.5\times10^{-3}$ for a 10-kG magnetic field. There are some indications that the energy scale is considerably smaller than the Fermi energy[21] (but still large compared with millikelvin temperatures), $\epsilon_0 \lesssim 0.1\epsilon_F$, which would imply correspondingly larger PHA parameters. In any event these corrections are small for any conceivable laboratory magnetic field. Finally, we note that Eqs. (8) and (9) of JK include only part of the PHA terms; those arising indirectly from $\beta_{\hat{p}}$ and $\vec{\zeta}_{\hat{p}}$ are omitted.

Equations (20) are complicated by the time dependence of $\hat{\mu}(t)$. However, this time dependence can be formally removed by transforming to a reference frame that rotates at an angular speed equal to the Larmor frequency. For any spin vector $\vec{B}$ we write

$$\vec{B} = \sum_{i=1}^{3} B^i \hat{u}_i(t) \,, \tag{24}$$

where $\{\hat{u}_1, \hat{u}_2, \hat{u}_3\}$ is an orthonormal triad of vectors that rotate about $\hat{z}$ at the Larmor frequency. We refer to this triad as the rotating coordinate system (axes) and $\{\hat{x}, \hat{y}, \hat{z}\}$ as the laboratory coordinate system (axes). The orientation of the rotating axes is fixed by requiring that the components of Eqs. (20), projected along the $\hat{u}_i$ directions, satisfy linear equations with time-independent coefficients. In particular, the rotating coordinate system defined by

$$\hat{u}_1 = \sin(\theta_H)\hat{z} - \cos(\theta_H)[\cos(\Omega_0 t)\hat{x} + \sin(\Omega_0 t)\hat{y}] \,, \tag{25a}$$

$$\hat{u}_2 = \sin(\Omega_0 t)\hat{x} - \cos(\Omega_0 t)\hat{y} \,, \tag{25b}$$

$$\hat{u}_3 = \cos(\theta_H)\hat{z} + \sin(\theta_H)[\cos(\Omega_0 t)\hat{x} + \sin(\Omega_0 t)\hat{y}] \,, \tag{25c}$$

leads to equations of motion for the longitudinal and transverse variables, defined with respect to $\hat{u}_3$, that are equivalent to those for $\theta_H = 0$. Note that $\hat{u}_3 = \hat{\mu}$ is the direction of the unperturbed rotating magnetization. It is convenient to introduce circularly polarized transverse variables,

$$B^{\pm}=(B^1\pm iB^2)/\sqrt{2}\ , \tag{26}$$

so that the equations of motion in the rotating frame become

$$\partial_t\sigma^{\pm}_{\hat{p}}\pm i\Omega_{\rm eq}(F^a_0\sigma^{\pm}_{\hat{p}}-\delta h^{\pm}_{\hat{p}}+\eta_a\zeta^{\pm}_{\hat{p}}) + \vec{v}_{\hat{p}}\cdot\vec{\partial}_{\vec{r}}(\sigma^{\pm}_{\hat{p}}+\delta h^{\pm}_{\hat{p}}+\eta_v\zeta^{\pm}_{\hat{p}}) = \pm i\Omega_{\rm eq}h^{\pm}_{\rm xt}-\vec{v}_{\hat{p}}\cdot\vec{\partial}_{\vec{r}}h^{\pm}_{\rm xt}\ , \tag{27a}$$

$$\partial_t\zeta^{\pm}_{\hat{p}}\pm i\Omega_{\rm eq}F^a_0\zeta^{\pm}_{\hat{p}}+\vec{v}_{\hat{p}}\cdot\vec{\partial}_{\vec{r}}\zeta^{\pm}_{\hat{p}}=0\ , \tag{27b}$$

$$\partial_t\sigma^3_{\hat{p}}+\vec{v}_{\hat{p}}\cdot\vec{\partial}_{\vec{r}}(\sigma^3_{\hat{p}}+\delta h^3_{\hat{p}}+\eta_v\zeta^3_{\hat{p}}+A_{\hat{p}}) = -\vec{v}_{\hat{p}}\cdot\vec{\partial}_{\vec{r}}(h^3_{\rm xt}+\eta_v\epsilon_{\rm xt})\ , \tag{27c}$$

$$\partial_t\zeta^3_{\hat{p}}+\vec{v}_{\hat{p}}\cdot\vec{\partial}_{\vec{r}}(\zeta^3_{\hat{p}}+\delta\epsilon_{\hat{p}}+\eta_a\beta_{\hat{p}})=-\vec{v}_{\hat{p}}\cdot\vec{\partial}_{\vec{r}}\epsilon_{\rm xt}\ , \tag{27d}$$

$$\partial_t\nu_{\hat{p}}+\vec{v}_{\hat{p}}\cdot\partial_{\vec{r}}(\nu_{\hat{p}}+\delta\epsilon_{\hat{p}}+\eta_v\beta_{\hat{p}}+\Sigma^3_{\hat{p}}) = -\vec{v}_{\hat{p}}\cdot\vec{\partial}_{\vec{r}}(\epsilon_{\rm xt}+\eta_v h^3_{\rm xt})\ , \tag{27e}$$

$$\partial_t\beta_{\hat{p}}+\vec{v}_{\hat{p}}\cdot\vec{\partial}_{\vec{r}}(\beta_{\hat{p}}+\delta h^3_{\hat{p}}+\eta_a\zeta^3_{\hat{p}})=-\vec{v}_{\hat{p}}\cdot\partial_{\vec{r}}h^3_{\rm xt}\ . \tag{27f}$$

There are several points to be made before we consider special solutions to these equations. (i) Equation (27b) for $\zeta^{\pm}_{\hat{p}}$ is decoupled from all other variables, including the external fields. Thus, $\zeta^{\pm}_{\hat{p}}$ are irrelevant variables, which we hereafter omit. This means that there are no first-order PHA corrections to the equations of motion for the transverse components $\sigma^{\pm}_{\hat{p}}$ of the spin-density distribution function in the rotating frame. (ii) Since Eq. (27a) for $\sigma^{\pm}_{\hat{p}}$ is independent of $\theta_H$, the spin wave eigenfrequencies, for any wave vector $\vec{q}$, are also independent of the tipping angle $\theta_H$. This result differs from that of JK, who finds the dispersion relation for the $l=0$ spin wave to be $[\omega_{00}(\vec{q})-\Omega_0]\propto\sec(\theta_H)q^2$. (iii) Equations (27c)–(27f) show that the scalar distribution function $\nu_{\hat{p}}$, and therefore zero sound, is coupled by PHA terms to $\sigma^3_{\hat{p}}$, $\zeta^3_{\hat{p}}$, $\beta_{\hat{p}}$, and *not* to any transverse variable. Since there are no spin wave modes associated with these variables, zero sound cannot be used as a probe of the spin wave spectrum in $^3$He. (iv) In JK, Ketterson was motivated to look for a mode crossing with $\omega-\Omega_0=\omega_{00}(\vec{q})$, where $\omega$ is the zero sound frequency, $\Omega_0$ is the Larmor frequency of the uniformly precessing magnetization, and $\omega_{00}(\vec{q})$ is the frequency of transverse spin waves with $l=0$ and wave vector $\vec{q}$. The reason we believe that JK finds spurious first-order frequency shifts in the collective mode spectrum, as well as an unphysical spin-orbit coupling in $\omega_{00}(\vec{q})$ for $\theta_H\neq0$, is that he makes an incorrect Fourier decomposition of the kinetic equations. This leads to an incorrect identification of the spin wave eigenfunctions for $\theta_H\neq0$, their eigenfrequencies, and the coupling of zero sound to the spin-dependent variables when PHA is included. In the laboratory reference frame JK discards terms which evolve as $e^{-i(\omega+\Omega_0)t}$ and retains terms that evolve as $e^{-i(\omega-\Omega_0)t}$. This approximation is used in the theory of NMR, and is a good approximation provided there is a magnetic resonance at $\omega=\Omega_0$. In this case there is no such resonance; the approximation is not valid, and both time dependences must be retained. In the rotating frame there are no terms with different time dependences, and consequently our Eqs. (27) include all couplings between the scalar and spin-dependent components of the distribution function. These equations also show that the spin wave eigenfunctions are the components of $\vec{\sigma}_{\hat{p}}$ that are transverse to the tipped magnetization in the rotating frame.

The effects of PHA on the zero sound dispersion can be determined from Eqs. (27c)–(27f). As is also true for the transverse spin waves, there are *no* corrections to the zero sound dispersion relation which are first order in PHA. This result is obtained by solving for the zero sound dispersion relation perturbatively in terms of the PHA parameters. Here it is convenient to introduce a matrix notation for the Fourier transform of Eqs. (27c)–(27f),

$$\omega\psi(\hat{p}\mid\vec{q}\omega)-L*\psi(\hat{p}\mid\vec{q}\omega)=\psi_{\rm xt}(\hat{p}\mid\vec{q}\omega)\ , \tag{28}$$

$$\psi=\begin{bmatrix}\nu_{\hat{p}}\\ \zeta^3_{\hat{p}}\\ \sigma^3_{\hat{p}}\\ \beta_{\hat{p}}\end{bmatrix}\ , \tag{29}$$

$\psi_{\rm xt}$ is a column vector representing the external fields, and $L$ is the linear operator given below. We label the elements of the vector $\psi$ by the index $\mu$ and the elements of $L$ by a pair of indices for the row and column positions. The matrix product in Eq. (28) includes an angular integration, so that

$$L*\psi(\hat{p})^{\mu}=\int\frac{d\Omega_{\hat{p}'}}{4\pi}L^{\mu\nu}(\hat{p},\hat{p}')\psi^{\nu}(\hat{p}')\ , \tag{30a}$$

$$\psi*L(\hat{p})^{\mu}=\int\frac{d\Omega_{\hat{p}'}}{4\pi}\psi^{\nu}(\hat{p}')L^{\nu\mu}(\hat{p}',\hat{p})\ . \tag{30b}$$

The longitudinal and transverse zero sound modes correspond to the eigenfunctions and eigenfrequencies of the homogeneous Eq. (28),

$$\omega_\lambda\psi_\lambda=L*\psi_\lambda\ , \tag{31}$$

where $\lambda$ is a label for a particular mode. We can write $L=\vec{q}\cdot\vec{v}_{\hat{p}}S$, where $S$ is symmetric with respect to interchange of $\hat{p}$ and $\hat{p}'$. Furthermore, $L\,(S)$ is the sum of a zeroth-order term $L^0(S^0)$ and a perturbation $L^{\eta}(S^{\eta})$, which is first order in PHA parameters:

$$S^0=\begin{bmatrix}I+F^s & 0 & 0 & 0\\ F^s & I & 0 & 0\\ 0 & 0 & I+F^a & 0\\ 0 & 0 & F^a & I\end{bmatrix}\ , \tag{32a}$$

$$S^{\eta}=\begin{bmatrix} 0 & 0 & \eta_a I+H^a & \eta_v I+G^s \\ 0 & 0 & 0 & \eta_a I+G^s \\ \eta_a I+H^s & \eta_v I+G^a & 0 & 0 \\ 0 & \eta_a I+G^a & 0 & 0 \end{bmatrix}, \quad (32b)$$

where $I=4\pi\delta^{(2)}(\hat{p}-\hat{p}')$.

If PHA is neglected, then zero sound is an eigenmode of Eqs. (9a) and (10a); we denote the eigenfunction and eigenfrequency of these equations by $\phi_{\hat{p}}$ and $\omega_0$. The corresponding four-component eigenvector of

$$\omega_0\psi_0=L^0*\psi_0\ , \quad (33)$$

is

$$\psi_0=\phi_{\hat{p}}\begin{bmatrix}1\\1\\0\\0\end{bmatrix} . \quad (34)$$

The first-order PHA correction to the zero sound frequency is given by

$$\delta\omega=\frac{\tilde{\psi}_0*(L^{\eta}*\psi_0)}{(\tilde{\psi}_0*\psi_0)}\ , \quad (35)$$

where $\tilde{\psi}_0$ is the eigenfunction of the transposed zeroth-order equation,

$$\omega_0\tilde{\psi}_0=\tilde{\psi}_0*L^0\ . \quad (36)$$

The eigenfunction of Eq. (36) is easily constructed,

$$\tilde{\psi}_0=a(\vec{q}\cdot\vec{v}_{\hat{p}})^{-1}\phi_{\hat{p}}(1,0,0,0)\ , \quad (37)$$

where $a$ is a normalization constant. It follows immediately from Eqs. (32b), (34), and (37) that $\delta\omega=0$; there are no first-order PHA corrections to the zero sound velocity.

Although there are no first-order PHA corrections to any of the collective mode frequencies in $^3$He, there is an interesting coupling between zero sound and the magnetization which is first order in the PHA parameters and, therefore, possibly observable. The coupling between the scalar and longitudinal vector components of the distribution function, exhibited in Eqs. (27c)—(27f), implies that zero sound drives small oscillations of the magnetization along the direction of the unperturbed magnetization in the rotating frame. These oscillations are obtained from the first-order correction to the zero sound solution $\psi_0$. If we set $\psi=\psi_0+\delta\psi$, then we have from Eq. (28)

$$(\omega_0-L^0)*\delta\psi=L^{\eta}*\psi_0\ . \quad (38)$$

For longitudinal zero sound

$$s_0=\omega_0/qv_F=c_0/v_F\cong\tfrac{1}{3}(1+F_1^s/3)F_0^s\gg 1\ ,$$

so $s_0^{-1}$ is a good expansion parameter. The only precaution that must be taken is to make sure that terms which are higher order in $s_0^{-1}$, but with large coefficients $\propto F_0^s$, are included. We do this by setting $\delta\psi=\omega_0^{-1}L^{\eta}*\psi_0+\delta\psi'$, then

$$(\omega_0-L^0)*\delta\psi'=\omega_0^{-1}L^0*(L^{\eta}*\psi_0)\ . \quad (39)$$

Then to leading order in $s_0^{-1}$, $\delta\psi'$ can be calculated by neglecting $L^0$ on the left-hand side of Eq. (39). From the third component of $\delta\psi$ we obtain the induced spin density

$$\sigma_{\hat{p}}^3=\sum_{l\geq 0}a_l(\hat{p}\cdot\hat{q})\left\{s_0^{-1}P_l(\hat{p}\cdot\hat{q})+s_0^{-2}(2l+1)^{-1}\left[(l+1)\left[1+\frac{F_{l+1}^a}{(2l+3)}\right]P_{l+1}(\hat{p}\cdot\hat{q})+l\left[1+\frac{F_{l-1}^a}{(2l-1)}\right]P_{l-1}(\hat{p}\cdot\hat{q})\right]\right\},$$
$$a_l=[\eta_v+\eta_a+(H_l^s+G_l^a)/(2l+1)]\phi_l\ , \quad (40)$$
$$\phi_l=(2l+1)\int\frac{d\Omega_{\hat{p}}}{4\pi}P_l(\hat{p}\cdot\hat{q})\phi_{\hat{p}}(\vec{q},\omega_0)\ .$$

The $l=0$ components of both $\sigma_{\hat{p}}^3$ and $\zeta_{\hat{p}}^3=\phi_{\hat{p}}$ contribute to the induced magnetization,

$$\delta M^3(\vec{q},\omega_0)=2N(0)\frac{\gamma\hbar}{2}(\sigma_0^3+\eta_N\phi_0)\ . \quad (41)$$

The contribution from $\sigma_0^3$ is

$$\begin{aligned}\sigma_0^3=&\ \phi_0[\eta_v+\eta_a+(H_1^s+G_1^a)/3]/(1+F_1^s/3)\\ &+s_0^{-2}(\phi_0/3)(\eta_v+\eta_a+H_0^s+G_0^a)(1+F_1^a/3)\\ &+s_0^{-2}(2\phi_2/15)[\eta_v+\eta_a+(H_2^s+G_2^a)/5](1+F_1^a/3)\ ,\end{aligned} \quad (42)$$

where the continuity equation has been used to eliminate $\phi_1$ in favor of the amplitude of the density fluctuations $\phi_0$: $s_0^{-1}\phi_1/3=\phi_0(1+F_1^s/3)^{-1}$. It is easily shown from Eqs. (9a) and (10a) for $\phi_{\hat{p}}$ that $\phi_2\cong 2\phi_0$ for $s_0\gg 1$, thus the terms on the second and third lines of Eq. (42) are generally small compared with those on the first line, except for the term $\propto s_0^{-2}(\phi_0/3)H_0^s$. Since $H_0^s=\eta_v F_0^s+$ smaller terms, we have to leading order in $s_0^{-1}$,

$$\sigma_0^3\cong\phi_0[\eta_a+(2+F_1^a/3)\eta_v+(H_1^s+G_1^a)/3](1+F_1^s/3)^{-1}\ , \quad (43)$$

so we can write the induced magnetization entirely in terms of the density fluctuation amplitude $\delta n(\vec{q},\omega_0)=2N(0)\phi_0$,

$$\delta\vec{M}(\vec{r},t)=\eta_M\left[\frac{\gamma\hbar}{2}\right]\delta n(\vec{q},\omega_0)e^{i(\vec{q}\cdot\vec{r}-\omega_0 t)}\hat{u}_3(t)\ , \quad (44)$$

where

$$\eta_M=\eta_N+[\eta_a+(2+F_1^a/3)\eta_v+(H_1^s+G_1^a)/3]\times(1+F_1^s/3)^{-1}\ .$$

Thus, in the laboratory reference frame there is an induced magnetization wave, modulated by the uniformly rotating magnetization, given by Eqs. (1).

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## APPENDIX

In this appendix we discuss the constraints on the linear-response functions of normal $^3$He imposed by spin rotational invariance and approximate particle-hole symmetry of the underlying Hamiltonian $\hat{H}$. A brief version of this argument is given in Ref. 14, while Serene has previously investigated the consequences of symmetry for the order-parameter response functions in superfluid $^3$He.[4]

If $^3$He is prepared in a state with uniformly precessing magnetization,

$$\vec{M}(t)=\overleftrightarrow{R}[\hat{z},\Omega_0 t]\cdot\vec{M}_0 , \tag{A1}$$

$$\vec{M}_0=\overleftrightarrow{R}[\hat{y},\theta_H]\cdot\vec{M}_{eq} , \tag{A2}$$

where $\vec{M}_{eq}=-2N(0)h_{eq}\hat{z}\equiv\chi\vec{H}_0$, then the density matrix that describes this state, assuming it was prepared from an equilibrium ensemble described by $\hat{\rho}(\hat{H}_T)$ with $\hat{H}_T=\hat{H}-\gamma\vec{H}_0\cdot\hat{\vec{S}}$, is

$$\hat{\sigma}=\hat{\rho}[\hat{H}-\gamma\chi^{-1}\vec{M}(t)\cdot\hat{\vec{S}}] . \tag{A3}$$

The density matrix satisfies the Liouville equation, $\partial_t\hat{\sigma}=i[\hat{\sigma},\hat{H}_T]$, and gives $\vec{M}(t)$ as the expectation value for the magnetization to linear order in $\vec{M}$. The density ($\chi_{nn}$) and spin-density ($\vec{\chi}_{sn}$) responses to a scalar external potential, as well as the spin-density response ($\overleftrightarrow{\chi}_{ss}$) to an external magnetic field, are determined by

$$\chi_{nn}=-i\Theta(t-t')\langle[\hat{n}_{H_T}(\vec{x}t),\hat{n}_{H_T}(\vec{x}'t')]\rangle_{\sigma_{H_T}} , \tag{A4}$$

$$\vec{\chi}_{sn}=-i\Theta(t-t')\langle[\hat{\vec{s}}_{H_T}(\vec{x}t),\hat{n}_{H_T}(\vec{x}'t')]\rangle_{\sigma_{H_T}} , \tag{A5}$$

$$\overleftrightarrow{\chi}_{ss}=-i\Theta(t-t')\langle[\hat{\vec{s}}_{H_T}(\vec{x}t),\hat{\vec{s}}_{H_T}(\vec{x}'t')]\rangle_{\sigma_{H_T}} , \tag{A6}$$

where the density ($\hat{n}_{H_T}$) and spin-density ($\hat{s}_{H_T}$) operators are defined in the Heisenberg picture and carry the time dependence generated by the Hamiltonian $\hat{H}_T$. In addition the density matrix becomes

$$\hat{\sigma}_{H_T}=\hat{\rho}[\hat{H}-\gamma\chi^{-1}\vec{M}(t)\cdot\hat{\vec{S}}_{H_T}(t)] , \tag{A7}$$

which is independent of time since $\vec{M}(t)$ and $\hat{\vec{S}}_{H_T}(t)$ satisfy the same Bloch equation, $\partial_t\vec{M}=\gamma(\vec{M}\times\vec{H}_0)$. We neglect very small nuclear dipolar interactions in $^3$He, so that $[\hat{H},\hat{\vec{S}}]=0$. Because of the invariance of $\hat{H}$ under spin rotations, Eqs. (A4)–(A6) are more conveniently written in terms of operators that carry only the time dependence generated by $\hat{H}$,

$$\chi_{nn}[\vec{M}_0]=-i\Theta(t-t')\langle[\hat{n}_H(\vec{x}t),\hat{n}_H(\vec{x}'t')]\rangle_{\sigma_H} , \tag{A8}$$

$$\vec{\chi}_{sn}=\overleftrightarrow{R}[\hat{z},\Omega_0 t]\cdot\vec{\chi}^{\,0}_{sn} , \tag{A9}$$

$$\vec{\chi}^{\,0}_{sn}[\vec{M}_0]=-i\theta(t-t')\langle[\hat{\vec{s}}_H(\vec{x}t),\hat{n}_H(\vec{x}'t')]\rangle_{\sigma_H} , \tag{A10}$$

$$\overleftrightarrow{\chi}_{ss}=\overleftrightarrow{R}[\hat{z},\Omega_0 t]\cdot\overleftrightarrow{\chi}^{\,0}_{ss}\cdot\overleftrightarrow{R}[\hat{z},\Omega_0 t]^{-1} , \tag{A11}$$

$$\overleftrightarrow{\chi}^{\,0}_{ss}[\vec{M}_0]=-i\Theta(t-t')\langle[\hat{\vec{s}}_H(\vec{x}t),\hat{\vec{s}}_H(\vec{x}'t')]\rangle_{\sigma_H} . \tag{A12}$$

Equations (A9) and (A11) give the transformation between the spin-dependent response functions defined in a coordinate frame rotating at the Larmor frequency ($\vec{\chi}^{\,0}_{sn}$ and $\overleftrightarrow{\chi}^{\,0}_{ss}$) and the response functions defined with respect to a nonrotating coordinate system ($\vec{\chi}_{sn}$ and $\overleftrightarrow{\chi}_{ss}$). Also note that $\vec{\chi}^{\,0}_{sn}[\vec{M}_0]$ and $\overleftrightarrow{\chi}^{\,0}_{ss}[\vec{M}_0]$, calculated with the density matrix $\hat{\sigma}=\hat{\rho}(\hat{H}-\gamma\chi^{-1}\vec{M}_0\cdot\hat{\vec{S}}_H)$, are the equilibrium response functions (in the rotating frame) calculated in the tipped magnetic field $\chi^{-1}\vec{M}_0$. We then expand these functions in the basis $\{\hat{u}^{\,0}_1,\hat{u}^{\,0}_2,\hat{u}^{\,0}_3\}$, with $\hat{u}^{\,0}_3||\vec{M}_0$, given by Eqs. (25) for $t=0$; $\vec{\chi}^{\,0}_{sn}=\sum_{i=1}^{3}\chi^i_{sn}\hat{u}^{\,0}_i$ and

$$\overleftrightarrow{\chi}^{\,0}_{ss}=\sum_{i,j=1}^{3}\chi^{ij}_{ss}\hat{u}^{\,0}_i\hat{u}^{\,0}_j .$$

The coefficients $\chi^i_{sn}$ and $\chi^{ij}_{ss}$ are then the usual equilibrium linear-response functions evaluated in the field $\chi^{-1}\vec{M}_0$. Thus, in the nonrotating frame we obtain,

$$\vec{\chi}_{sn}=\sum_{i=1}^{3}\chi^i_{sn}(\vec{x}t,\vec{x}'t'\,|\,\vec{M}_0)\hat{u}_i , \tag{A13}$$

$$\overleftrightarrow{\chi}_{ss}=\sum_{i,j=1}^{3}\chi^{ij}_{ss}(\vec{x}t,\vec{x}'t'\,|\,\vec{M}_0)\hat{u}_i\hat{u}_j , \tag{A14}$$

where $\{\hat{u}_1,\hat{u}_2,\hat{u}_3\}$ is the rotating coordinate system defined by Eqs. (25).

Spin rotational invariance of $\hat{H}$ and approximate particle-hole symmetry of $\hat{\sigma}$ and $\hat{H}$ imply important constraints on the linear-response functions. Rotations in spin space are generated by the unitary transformation, $\hat{A}\rightarrow\hat{U}(\vec{\Lambda})\hat{A}\hat{U}(\vec{\Lambda})^\dagger$ with $\hat{U}(\vec{\Lambda})=e^{i\vec{\Lambda}\cdot\hat{\vec{s}}}$, so that

$$\hat{\vec{s}}_H\rightarrow\overleftrightarrow{R}[\vec{\Lambda}]\cdot\hat{\vec{s}}_H , \tag{A15}$$

$$\hat{n}_H\rightarrow\hat{n}_H , \tag{A16}$$

$$\hat{\sigma}_H[\vec{M}_0]\rightarrow\hat{\sigma}_H[\overleftrightarrow{R}[\vec{\Lambda}]^{-1}\cdot\vec{M}_0] . \tag{A17}$$

In consequence, the linear-response functions satisfy, for any rotation $\vec{\Lambda}$,

$$\chi_{nn}[\vec{M}_0]=\chi_{nn}[\overleftrightarrow{R}[\vec{\Lambda}]^{-1}\cdot\vec{M}_0] , \tag{A18}$$

$$\vec{\chi}^{\,0}_{sn}[\vec{M}_0]=\overleftrightarrow{R}[\vec{\Lambda}]\cdot\vec{\chi}^{\,0}_{sn}[\overleftrightarrow{R}[\vec{\Lambda}]^{-1}\cdot\vec{M}_0] , \tag{A19}$$

$$\overleftrightarrow{\chi}^{\,0}_{ss}[\vec{M}_0]=\overleftrightarrow{R}[\vec{\Lambda}]\cdot\overleftrightarrow{\chi}^{\,0}_{ss}[\overleftrightarrow{R}[\vec{\Lambda}]^{-1}\cdot\vec{M}_0]\cdot\overleftrightarrow{R}[\vec{\Lambda}]^{-1} . \tag{A20}$$

A rotation of $\pi/2$ about $\vec{M}_0$ gives $\hat{u}^{\,0}_1\rightarrow\hat{u}^{\,0}_2$, $\hat{u}^{\,2}_0\rightarrow-\hat{u}^{\,0}_1$; therefore, Eqs. (A13) and (A14) reduce to

$$\vec{\chi}_{sn} = \chi^3_{sn}\hat{u}_3 \;, \tag{A21}$$

$$\overleftrightarrow{\chi}_{ss} = \chi^{11}_{ss}(\hat{u}_1\hat{u}_1 + \hat{u}_2\hat{u}_2) + \chi^{33}_{ss}\hat{u}_3\hat{u}_3 + \chi^{12}_{ss}(\hat{u}_1\hat{u}_2 - \hat{u}_2\hat{u}_1) \;. \tag{A22}$$

A $\pi$ rotation about $\hat{u}^0_2$ implies that all four functions ($\chi^3_{sn}$, $\chi^{11}_{ss}$, $\chi^{33}_{ss}$, $\chi^{12}_{ss}$) are functions of $|\vec{M}_0|$. From Eq. (A21) we have that $\vec{\chi}^0_{sn}$ is a purely longitudinal magnetic response along the same tipped field $\vec{M}_0$ that enters the density matrix. Thus, $\chi^3_{sn}$ does not contain contributions from (or couplings to) any spin wave mode in $^3$He.

As was noted earlier, the particle-hole transformation, represented by the unitary operator $C$, is an approximate symmetry of the low-energy (quasiparticle) Hamiltonian $\hat{H}_T$;[22] $C\hat{H}_T C^\dagger = \hat{H}_T +$small PHA terms. If all particle-hole asymmetry terms are neglected, then $C\hat{\vec{s}}_H C^\dagger = \hat{\vec{s}}_H$, $C\hat{n}_H C^\dagger = -\hat{n}_H +$const, $C\hat{\sigma}_H C^\dagger = \hat{\sigma}_H$, and $\vec{\chi}_{sn} = -\vec{\chi}_{sn} = 0$. The PHA terms imply that $\vec{\chi}_{sn}$ is nonzero and first order in PHA; $\chi^3_{sn} \sim \eta\chi_{nn}$, where

$$\eta \sim \frac{N'(0)}{N(0)}(\hbar\Omega_0) \sim \left[\frac{\hbar\Omega_0}{\epsilon_F}\right] \ll 1 \;.$$

Thus, spin rotational invariance and approximate particle-hole symmetry imply that the magnetic response to a scalar field, $u(\vec{q}\omega)e^{i(\vec{q}\cdot\vec{x}-\omega t)}$, when written in terms of the density response, $\delta n(\vec{x}t) = \delta n(\vec{q}\omega)e^{i(\vec{q}\cdot\vec{x}-\omega t)}$, becomes

$$\delta\vec{M}(\vec{x}t) = (\gamma\hbar)(\chi^3_{sn}/\chi_{nn})\delta n(\vec{x}t)\hat{u}_3(t) \;, \tag{A23}$$

which gives Eqs. (1).


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$$C = \exp\left[-i\frac{\pi}{4}\sum_{\vec{p}}(a^\dagger_{\vec{p}\alpha}a^\dagger_{\underset{\sim}{p}\beta} + a_{\underset{\sim}{p}\alpha}a_{\vec{p}\beta})(\sigma^y)_{\alpha\beta}\right] \;.$$