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Distinct modes in the first zero-field current step of Josephson tunnel junctions

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Using numerical simulations of the sine-Gordon equation with boundary conditions appropriate to real experimental configurations, we find a coupling between the vortex and Swihart-type modes which gives rise to novel features in the junction dc current-voltage characteristics observed in experiment.

The relative local pair phase difference across a Josephson tunnel junction obeys the sine-Gordon equation[1] and, in principle, offers an excellent testing ground for a quantitative study of the rich nonlinear dynamics of such systems. A number of theoretical and experimental results have, in fact, already been obtained.[2,3] However, to make detailed quantitative comparisons with experiment which will allow us to develop and test in depth our physical understanding, it is essential that the theoretical calculations use the boundary conditions appropriate to the experimental situation. Related to this, the external drive must also be coupled in a realistic fashion and, in addition, appropriate damping terms must be included. Here, we report results obtained from a computer simulation of a model in which these features are taken into account. In particular, we focus here on the effects of the mixing of vortex[4] (2π kink) and Swihart mode[5] (oscillation). We find that two distinct modes of excitation can exist in a one-dimensional Josephson tunnel junction biased on a zero-field current step.

In most theoretical investigations of the Josephson junction dynamics, it is assumed that the biased current is distributed uniformly in space.[3,6] Only recently has the fact that current is fed into the junction from edges been considered in describing long in-line junctions.[7,8] However, in overlap junctions, the biased current may be concentrated within a distance λ_L from the edges of the junction. This is seen experimentally in the observation[9] that the low-temperature zero-voltage critical current of a long overlap junction is much smaller than the quasiparticle–current-step height at the voltage equivalent to twice the energy gap. Furthermore, the recently developed scanning focused-laser-beam technique[10] has convincingly demonstrated that the static current distribution in an overlap junction biased at its critical-current value agrees qualitatively with the calculated results of Owen and Scalapino,[11] who have assumed a biased current distribution concentrated on the junction ends. Therefore, it is important to take this boundary condition seriously and to study it in further detail.

Using this boundary condition, we have made a computer simulation of one-dimensional junctions of length $L=5\lambda_J$ and $L=10\lambda_J$. Here, λ_J is the Josephson length. We find that because of the large biased current concentrated on the ends, the spatial derivatives of the phase difference at the ends are large. Therefore, when a vortex (antivortex) collides with the junction ends and gets reflected, in addition to having an antivortex (vortex) created, other excitations such as Swihart modes are also excited. These excitations then mix with the vortex (or antivortex) in the junction. Depending upon the damping factor in the system, this mixing can cause the junction to switch from a vortex-propagation mode to a standing-wave mode and vice versa. We believe that this has been observed experimentally as the fine structures observed on the first zero-field current steps.[9,12] Although the latter was already proposed in Ref. 12, our simulation made it clear that the origin of the Swihart-mode excitations is the collision of the vortex or antivortex against the junction ends.

We consider a long narrow Josephson tunnel junction. The width of the junction is assumed to be much smaller than both the Josephson length λ_J and the physical length of the junction L. Then the space and time dependence of the relative pair phase ϕ is governed by the well-known Josephson equation[1]

$$\left(\frac{\partial^2}{\partial y^2}-\frac{\partial^2}{\partial t^2}-\eta\frac{\partial}{\partial t}\right)\phi(y,t)=\sin\phi(y,t) \quad . \tag{1}$$

Here, we measure distance y in units of λ_J and time t in units of the inverse of the plasma frequency $\omega_J^{-1}=\lambda_J/c$ with c being the speed of the electromagnetic wave in the junction. η is a damping factor which depends on the temperature. The boundary conditions are the ones used in Ref. 11:

$$\frac{\partial\phi(y,t)}{\partial y}=H_a+I, \quad \text{for } y=L \tag{2a}$$

and

$$\frac{\partial\phi(y,t)}{\partial y}=H_a-I, \quad \text{for } y=0 \quad . \tag{2b}$$

Here, H_a is the applied magnetic field in units of $H_c\equiv\hbar c/(2ed\lambda_J)$, I the bias current in units of $(c/2\pi)H_c=2\lambda_J J$, and d twice the London penetration depth.

Equations (1) and (2) are integrated using a fourth-order Runge-Kutta method.[13]

In Fig. 1 we demonstrate that for a fixed biased current $I=1.80$, there are two distinct steady-state modes. The three left-hand figures are for what we will call the vortex-propagation mode. The function $\phi(y,t)$ is not symmetric with respect to the center of the junction, and the existence of the 2π kink and its propagation is evident. Furthermore, the period for $V(0,t)$ is twice that of the space-averaged voltage as expected for the vortex-propagated picture. There are also structures in $V(0,t)$ between the sharp peaks. This is a manifestation of the Swihart modes. These modes propagate at close to the speed of light as is evident in the case of low biased current where the voltage peaks associated with the vortex are well separated in time, and this better time resolution separates the Swihart modes from the vortex mode. The corresponding figures for the standing-wave mode are shown on the right-hand side. The function $\phi(y,t)$ is symmetric with respect to the center of the junction (for the case of zero applied field); the periods of $V(0,t)$, $V(L,t)$, and the space-averaged voltage $\langle V(y,t)\rangle_y$ are the same. All these characters are distinctively different from those of the vortex-propagation model.

The switching between these two modes, as mentioned before, can be either continuous or discontinuous. In Fig. 2, we show the calculated current-voltage (I-V) characteristics for voltages below the asymptotic voltage of the first zero-field step for two different junctions. The circles represent the vortex-propagation mode while the crosses represent the standing-wave mode. The solid curves are drawn as a guide to the eye, and the dashed curves represent the I-V characteristics due to just the quasiparticle current. The transition in the short ($L=5\lambda_J$) lossy ($\eta=0.5$) junction is continuous[14] and in the long ($L=10\lambda_J$) junction with small loss ($\eta=0.2$) is discontinuous.

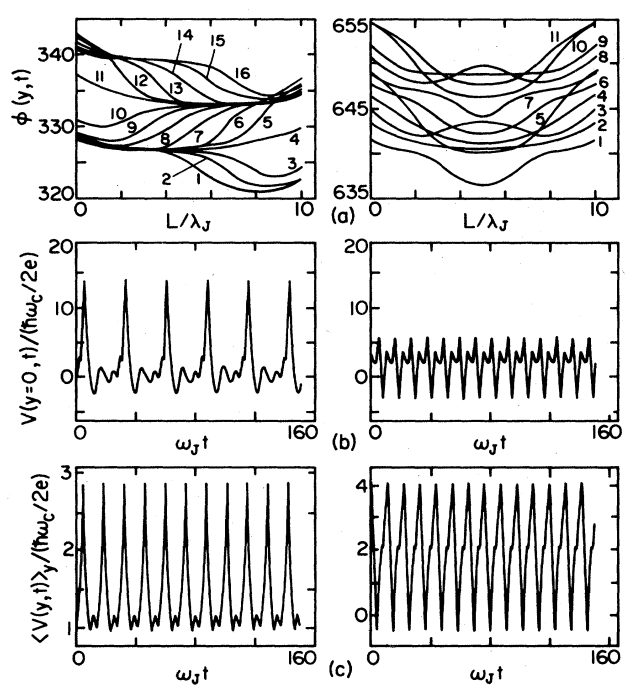

FIG. 1. Simulated results for a junction with $L=10\lambda_J$, $H_a=0$, and $\eta=0.2$. (a) Spatial dependencies of the phase-difference function for consecutive time separated by $\Delta t=2\omega_J^{-1}$. (b) Time dependencies of $V(0,t)$. (c) Time dependencies of the space-averaged junction voltage in units of $\hbar\omega_c/2e$ with $\omega_c\equiv\bar{c}/2L$.

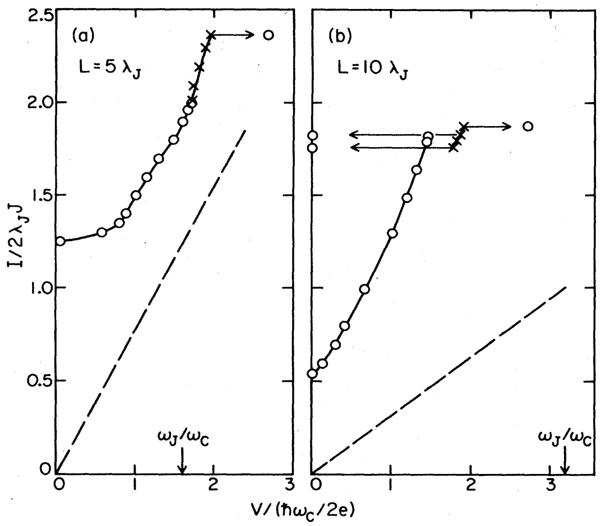

FIG. 2. I-V characteristics of the first zero-field step for two junctions obtained from simulation. The arrows indicate the way the junctions switch.

Figure 3(a) shows the experimental I-V characteristics of an Sn-SnO-Sn junction of length $L\simeq 10\lambda_J$ at various temperatures. The discontinuous transition is evident. In Fig. 3(b) we compare the temperature dependences of the maximum first zero-field current step to those of two other junctions. One is long with $L=5.7(2\pi\lambda_J)$ and the other is short with $L=0.5(2\pi\lambda_J)$. A transition has clearly occurred in the $L\simeq 10\lambda_J$ junction as the ambient temperature is varied.

We emphasize that our standing-wave mode is different from the symmetric mode that occurs on the *second* zero-field step.[15] The symmetric mode is due to the propagation of a vortex and an antivortex in opposite directions and hence gives rise to an asymptotic voltage twice that of our

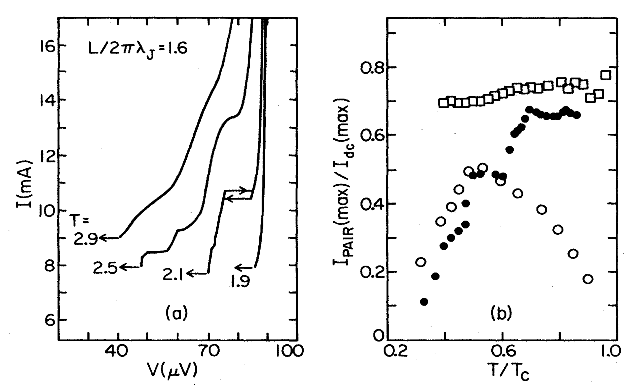

FIG. 3. Experimental data: (a) I-V characteristics of the first zero-field step for various temperatures, and (b) the temperature dependencies of the normalized maximum pair current for junctions with $L/2\pi\lambda_J=5.7$ (□), 1.6 (●), 0.5 (○).

standing-wave mode. The standing-wave mode, on the other hand, is closely related to the Swihart modes. Clearly, a detailed experimental study of these different modes is of interest. This could be achieved by measuring the power spectrum of the emitted electromagnetic waves[16] or by studying the effect of a focused laser beam on the I-V curve of the junction.[10]

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