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Weak-localization, near-magnetism, and triplet-pairing superconductivity in three dimensions

M. T. Béal-Monod
Laboratoire de Physique des Solides, Université de Paris-Sud, Bâtiment 510, 91405 Orsay, France

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In three-dimensional nearly magnetic Fermi liquids, disorder due to impurity scattering is shown to enhance the paramagnon strength and to weaken the triplet superconductivity pair-breaking parameter (through weak-localization quantum effects). As a result (a) "heavy fermion" superconductors are good candidates to exhibit triplet pairing and (b) normal liquid ^{3}He can be considered as both nearly magnetic and nearly localized.

It was recently proposed[1] that, close to a magnetic instability in itinerant two-dimensional (2D) fermion systems, a weak disorder could, at low temperature, favor the observation of triplet-pairing superconductivity. Indeed, the impurity pair-breaking parameter, preventing triplet pairing, was shown to be actually diminished by weak-localization quantum effects when strong paramagnons develop near a magnetic transition. The question of possibly applying such a result to explain the puzzling superconductivity observed in the so-called "heavy fermion systems,"[2] was raised in Ref. 1, although that paper was restricted to 2D, or possibly quasi-2D, systems.

In the present Rapid Communication, I first show that, in contrast to what was thought in Ref. 1, a similar conclusion can be drawn for *3D systems as well.*[3] Therefore, triplet-pairing superconductivity, which is known in pure systems[4] to be favored by the strong spin fluctuations ("paramagnons") arising near a magnetic instability, is not as strongly prevented by the presence of nonmagnetic impurities as one might fear *a priori.*[5] Moreover, and this is the second point that I emphasize in this paper, in itinerant 3D fermion systems (described by a simple parabolic band), a weak disorder actually *reinforces* the paramagnon strength. Thus, if the pure system is already close to a magnetic instability, the impurity scattering gives rise to localization corrections and simultaneously brings the system closer to the magnetic transition. Therefore, the paramagnons enhanced by impurity scattering all the more favor a transition to triplet-pairing superconductivity and add weight to the conjecture of Ref. 1 that the "heavy fermion systems"[2] might be triplet superconductors. Another important consequence concerns liquid ^{3}He for which the nearly localized (i.e., nearly solid) and the nearly magnetic characters, are not in contradiction, but, on the contrary, reinforce each other.

I consider first the extension of our conjecture in favor of triplet superconductivity to 3D fermion systems. I first note that in the same issue of Physical Review where Ref. 1 appeared, Anderson[6] also proposed, with emphasis on the nearly localized character "à la Brinkman-Rice," that heavy fermion systems are most likely triplet (or odd-pairing) superconductors. In view of the ideas developed in the second half of the present paper, I think that Anderson's arguments and mine are very close. Experimentally, these "heavy fermion systems,"[2] exhibit, just above their superconducting temperatures, all the characteristics of strongly interacting Fermi liquids. It was then tempting to associate this superconductivity with a triplet pairing induced by the strong spin fluctuations present in the normal phase, by analogy with the superfluidity of liquid ^{3}He.[4] But in contrast with liquid ^{3}He, the heavy fermion systems are dirty and this fact is known to prevent tiplet pairing from occurring.[5] Various attempts have been made[7] to propose a singlet-pairing superconductivity for these systems, mostly based on the hypothesis of a Kondo lattice ground state. However, if such an hypothesis is plausible for $CeCu_2Si_2$ and UBe_{13}, it seems unlikely for UPt_3, U_6Fe, and U_2PtC_2, which do not exhibit a Kondo-type behavior at high temperature. Moreover, Ott *et al.*[2] recently showed that the specific heat of UBe_{13} has a temperature variation, in the superconducting state, characteristic of triplet superconductivity rather than BCS singlet-pairing type; similarly, Bishop *et al.*[2] found that the ultrasonic attenuation in the superconducting phase of UPt_3 is inconsistent with a singlet pairing, but rather suggests a triplet one; nuclear-magnetic-resonance measurements[2] in $(U,\mathrm{Th})Be_{13}$ also exhibit odd-pairing-type behavior.

This is why it seems timely to reconsider the possibility of triplet pairing[1,6] for these materials and to first reexamine the impurity pair-breaking effect, as is done in Ref. 1 and in the following, *independently* of whether or not the system is Kondo-like at high T. I first show that one can generalize to 3D the results obtained in 2D,[1] namely, that the usual pair-breaking argument[5] against triplet superconductivity is much less stringent in systems which are close to a magnetic instability. Following Ref. 5, the pair-breaking parameter for triplet pairing is identified with the transport relaxation rate τ_{tr}^{-1} due to scattering on impurities. As in Ref. 1, I restrict my investigation to the simple case of a single parabolic band of free fermions, plus a strong Hubbard-type, instantaneous, contact repulsion among opposite spins (dimensionless $\bar{I}$), so that the Stoner enhancement of the spin susceptibility $(1-\bar{I})^{-1}$ is very large when the pure system is nearly magnetic ($\bar{I}\sim 1$). A small amount of randomly distributed impurities introduces a weak disorder through the usual phenomenological lifetime τ_0 ($\tau_0^{-1} << \epsilon_F$ in the weakly localized regime, ϵ_F being the Fermi energy of the fermions). Quantum corrections enter in the temperature variation of τ_{tr}^{-1} through a dimensionless coupling constant g which includes impurity scattering as well as mutual interactions among the fermions. The general form[8] for τ_{tr}^{-1}

(escaping all unimportant constants) behaves as follows. For $T < \tau_0^{-1} \ll \epsilon_F$,

$$\frac{\tau_{tr}^{-1}}{\tau_0^{-1}} = \begin{cases} 1+[g/(\epsilon_F\tau_0)][\ln(\tau_0 T)^{-1}], & d=2 \quad (1a) \\ 1+[g/(\epsilon_F\tau_0)^2](1-\sqrt{\tau_0 T}), & d=3 \quad (1b) \end{cases}$$

For positive g, τ_{tr}^{-1} increases when T decreases; in that case the pair-breaking parameter is very efficient to prevent triplet pairing. However, for a Hubbard-type contact repulsion among opposite spins in the nearly magnetic systems considered in Ref. 1 and here, *g happens to be negative and large;* such g's were calculated in Ref. 9 for 2D and in Ref. 10 for 3D. Retaining the most important contributions when $\bar{I} \to 1$, one gets the following. For $(1-\bar{I})^{-1} > 1$, $[|g|/(\epsilon_F\tau_0)] < 1$,

$$g \sim \begin{cases} -(3/\bar{I})\ln[(1-\bar{I})^{-1}], & d=2 \quad (2a) \\ -(4/\bar{I})\sqrt{(1-\bar{I})^{-1}}, & d=3 \quad (2b) \end{cases}$$

(I recall the 2D case[1] here for comparison with the 3D one, for which g is stronger.) Putting (2) in (1) results, when T lowers, into a *decrease* of τ_{tr}^{-1} and thus a decrease *of the pair-breaking parameter* for triplet superconductivity, both *in 2D and in 3D.* [Note that such an increase in the conductivity, for decreasing T, is observed in the metallic phase of Si:P,[11] when the condition $|g| < (\epsilon_F\tau_0)$ can apply, i.e., not close to the metal-insulator transitions.]

It was also recently shown,[12] in the 2D case, that the shift in the superconducting temperature $(T_c - T_{c0})$, where T_{c0} corresponds to the pure system, is not simply given by the pair-breaking parameter as in Ref. 5, because the relaxation time is here strongly energy dependent; i.e., the shift in T_c is not reduced by the same amount as the pair-breaking parameter. However, Ref. 12 showed that quantum corrections associated with weak-localization and Stoner enhancement effects, indeed, yield a reduction of the shift $(T_c - T_{c0})$ although the effect is weaker than on the pair-breaking parameter. The same kind of calculation performed in 3D would result in a similar reduced shift. Therefore, the conjecture of Ref. 1 that triplet pairing is not forbidden in the heavy fermion systems, even if dirty, applies as such, without having to invoke a possible quasi-2D character since it is also true in 3D.

The above conjecture is of course only qualitative. The over simplified paramagnon model reduced to a single parabolic band of spin-$\frac{1}{2}$ fermions and involving only one interaction cannot apply, *stricto sensu,* to heavy fermion superconductors which are much more complicated objects. However, experimentally, just above their superconducting temperature, the key feature is the enhanced Fermi-liquid behavior with strong spin fluctuations, as in liquid ^{3}He; therefore, we believe with the authors of Ref. 6, that the dominant contribution to such a behavior can be mimicked by the effect of a strong contact repulsion among opposite spins as involved in the paramagnon model. Indeed, as emphasized by Anderson,[6] "the Fermi liquid parameters are mostly determined by general sum rules" and "band formation is not expected to modify very much the Friedel sum rule which determines the phase shifts at the Fermi surface."

The second part of the present paper shows that, in itinerant 3D fermion systems, the tendency to localization (through its quantum effects) and the tendency to ferromagnetism (through paramagnon effects) *reinforce each other;* more precisely, for the same model as above, a weak disorder renders the system closer to become magnetic, and, as a corollary, these two quantum effects help each other in favoring the observation of triplet superconductivity.

To be more specific, I examine the behavior of the paramagnon peak (measuring the paramagnon strength); it is given by the maximum value, versus frequency ω, of Imχ, where $\chi(q,\omega,\tau_0^{-1})$ is the dynamical spin-spin correlation function in presence of a weak disorder. To the lowest order in ω

$$\mathrm{Im}\chi = \mathrm{Im}\left(\frac{\chi_0}{1-I\chi_0}\right) \simeq \frac{N(\epsilon_F)\lambda\omega}{[1-I\,\mathrm{Re}\chi^0(q,\omega=0,\tau_0^{-1})]^2 + (\bar{I}\lambda\omega)^2} \quad . \tag{3}$$

$\chi^0(q,\omega,\tau_0^{-1})$ is the correlation function in absence of the interaction I and $N(\epsilon_F)$ the density of states at the Fermi level; $\bar{I} \equiv IN(\epsilon_F)$, λ depends on q and τ_0^{-1}. The position of the paramagnon peak $\omega = \omega_0$ and its height, are, respectively, given by

$$\omega_0 = [1 - I\,\mathrm{Re}\chi^0(q,\omega=0,\tau_0^{-1})]/(\bar{I}\lambda) \quad , \tag{4}$$

$$[\mathrm{Im}\chi]_{\omega=\omega_0} = [N(\epsilon_F)/(2I)][1 - I\,\mathrm{Re}\chi^0(q,\omega=0,\tau_0^{-1})]^{-1} \quad . \tag{5}$$

λ, i.e., Imχ^0 is well known, in the diffusive regime;[8] for $k_F q\tau_0 \ll 1$, $\omega\tau_0 \ll 1$, it reads (in a.u.)

$$\mathrm{Im}\chi^0 \simeq N(\epsilon_F)\lambda\omega = N(\epsilon_F)3\omega/(k_F^2 q^2 \tau_0) \quad . \tag{6}$$

k_F is the Fermi momentum. On the other hand, Re$\chi^0(\omega=0)$ is usually approximated by its first constant term $N(\epsilon_F)$. For our purposes, such an approximation is not sufficient since it would imply that the peak height (5) remains unchanged when τ_0^{-1} increases, although the peak frequency (4) would decrease. Such a result would violate causality and the f sum rule.[13] However, de Gennes calculated,[14] for other purposes, Re$\chi^0(q,\omega=0,\tau_0^{-1})$ under the form of an integral [see formula (39) of Ref. 14]. It is straightforward, although tedious, to calculate a small q and small τ_0^{-1} analytical expansion of that integral. I thus found, in agreement with the curves drawn in Ref. 14, that, for $q/(2k_F) \ll 1$ and $\tau_0^{-1} \ll \epsilon_F$, Re$\chi^0$ *increases* with disorder:

$$\mathrm{Re}\chi^0(q,\omega=0,\tau_0^{-1}) \simeq N(\epsilon_F)\left[1 - \frac{q^2}{12k_F^2}\left(1 - \frac{1}{3(4\epsilon_F\tau_0)^2}\right)\right] \quad . \tag{7}$$

Note that, in general, the coefficients entering in that formula would be band-structure dependent; the ones displayed here are those corresponding to the parabolic band that I have assumed throughout the present paper. It then follows that, when the disorder increases, i.e., when τ_0^{-1} increases, the position of the paramagnon peak (4) shifts to lower frequencies, and its height (5) *increases;*[15] i.e., one gets stronger paramagnons. This last effect, in turn, will modify the effective enhancement. As commented already in Ref. 14, nonmagnetic impurities cannot, *a priori,* modify the uniform static susceptibility; the value of $\chi^0(q=0,\omega=0)$ and thus of $[1-I\,\mathrm{Re}\chi^0(q=0,\omega=0)]^{-1}$ is unchanged in the presence of a finite τ_0^{-1}. However, it has been shown,[16] in absence of disorder, that the *dynamics* of the paramagnons strongly modify the finite T dependence of the spin susceptibility $\chi(T)$; but they also modify the zero-temperature value $\chi(0)$ (although in a minor way); therefore, the effective enhancement $[1-\bar{I}_{\text{eff}}]^{-1}$ is different from the Stoner one $[1-\bar{I}]^{-1}$ that one started with, and $\bar{I}_{\text{eff}}$ may be computed by perturbation.[17] The procedure is self-consistent: a strong enhancement induces strong paramagnons, which, in turn, modify the effective enhancement. The same kind of argument may apply in presence of weak disorder.[18] Although it would be out of the scope of the present Communication to calculate in detail $\bar{I}_{\text{eff}}$, the mere fact that paramagnons were found stronger in the presence of weak disorder yields one to expect a stronger effective enhancement, so that the system would actually be closer to becoming magnetic. In other words, in nearly magnetic fermion systems which can be described by a simple parabolic band,[19] one can draw the following conclusion: introducing a weak disorder, and thus a tendency to localization, acts as if one would increase the paramagnon strength which, in turn, would increase the effective enhancement[20] and, as a consequence, favor[4] the possibility of observing triplet-pairing superconductivity. This is in agreement with various recent proposals[6] that an almost localized strongly interacting Fermi liquid has a superconducting transition toward a p-wave state. Another consequence is that the old debate concerning whether liquid ^{3}He is a nearly ferromagnetic Fermi liquid or an almost localized fermion system (i.e., nearly solid) is most likely irrelevant. The two behaviors are *not in contradiction,* but, on the contrary, represent two aspects of the same character.[21]

To conclude, I have shown that in 3D strongly interacting fermions, a weak disorder, due to impurities, on one hand, enhances the spin fluctuations arising near a magnetic instability and, on the other hand, introduces quantum corrections which diminish the strength of the triplet superconductivity pair-breaking parameter. This combined effect is proposed to qualitatively favor triplet pairing in heavy fermion superconductors and to render, in liquid ^{3}He at increasing pressure, the tendency to become localized and the tendency to become magnetic, complementary rather than exclusive.

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[1]M. T. Béal-Monod, H. Ebisawa, and H. Fukuyama, Phys. Rev. B **30**, 1563 (1984).

[2]So far in $CeCu_2Si_2$, see F. Steglich *et al.*, Phys. Rev. Lett. **43**, 1892 (1979); **49**, 1448 (1982); UBe_{13}, see H. R. Ott *et al.*, *ibid.* **50**, 1595 (1983); **52**, 1915 (1984), $(U,Th)Be_{13}$, see D. E. MacLaughlin *et al.*, *ibid.* **53**, 1833 (1984); U_6Fe, see L. E. Delong *et al.*, *ibid.* **51**, 312 (1983); UPt_3, see G. R. Stewart *et al.*, *ibid.* **52**, 679 (1984) and D. J. Bishop *et al.*, *ibid.* **53**, 1009 (1984); U_2PtC_2, see G. P. Meisner *et al.*, *ibid.* **53**, 1829 (1984).

[3]The following sentence in the abstract of Ref. 1 must be revised: "It might then, in principle, become easier to observe triplet-pairing superconductivity in dirty two-dimensional, or quasi-two-dimensional metals, than in three-dimensional ones;" the end of that sentence ("than in three-dimensional ones") should be suppressed and replaced by "when they are close to a magnetic instability."

[4]A. Layzer and D. Fay, Int. J. Magn. **1**, 135 (1971); P. W. Anderson and W. F. Brinkman, Phys. Rev. Lett. **30**, 1108 (1973); S. Nakajima, Prog. Theor. Phys. Jpn. **50**, 1101 (1973).

[5]A. I. Larkin, Pis'ma Zh. Eksp. Teor. Fiz. **2**, 205 (1965) [JETP Lett. **2**, 130 (1965)]; I. F. Foulkes and B. L. Gyorffy, Phys. Rev. B **15**, 1395 (1977).

[6]P. W. Anderson, Phys. Rev. B **30**, 1549 (1984); see also C. M. Varma, Bull. Am. Phys. Soc. **29**, 404 (1984); O. T. Valls and Z. Tesanovic, Phys. Rev. Lett. **53**, 1497 (1984); K. S. Bedell and K. F. Quader (unpublished); T. M. Rice *et al.*(unpublished).

[7]H. Razafimandiby, P. Fulde, and J. Keller, Z. Phys. B **54**, 111 (1984); N. Grewe, *ibid.* **56**, 111 (1984); M. Tachiki and S. Maekawa, Phys. Rev. B **29**, 2497 (1984); anisotropic single pairing was also proposed: J. Ohkawa and H. Fukuyama (unpublished); A. W. Overhauser and J. Appel, Phys. Rev. B (to be published).

[8]See, for instance, the reviews by H. Fukuyama, and by B. L. Altshuler and A. G. Aronov, in *Electron-Electron Interaction in Disordered Systems,* edited by A. L. Efros and M. Pollak (North-Holland, Amsterdam, in press).

[9]H. Fukuyama, Y. Isawa, and H. Yasuhara, J. Phys. Soc. Jpn. **52**, 16 (1983).

[10]Y. Isawa and H. Fukuyama, J. Phys. Soc. Jpn. **53**, 1415 (1984).

[11]T. F. Rosenbaum *et al.*, Phys. Rev. Lett. **46**, 568 (1981); **47**, 1758 (1981).

[12]H. Ebisawa, H. Fukuyama, and M. T. Béal-Monod, J. Phys. Soc. Jpn. **53**, 2370 (1984).

[13]See, for instance, D. Pines and P. Nozières, *The Theory of Quantum Liquids* (Benjamin, New York, 1966).

[14]P. G. de Gennes, J. Phys. Rad. **23**, 630 (1962); note an obvious misprint in the expression of S'' in formula (38) of that paper where two (+) signs should appear in the numerator instead of two (−) ones.

[15]In the respect, the numerical computation of R. Jullien, J. Low Temp. Phys. **42**, 207 (1981), is erroneous since it found that the position of the peak *and* its height decrease when τ_0^{-1} increases.

[16]M. T. Béal-Monod, S. K. Ma, and D. R. Fredkin, Phys. Rev. Lett. **20**, 929 (1968); T. Moriya and A. Kawabata, J. Phys. Soc. Jpn. **34**, 639 (1973); M. T. Béal-Monod Phys. Rev. B **28**, 1630 (1983).

[17]Indeed it has been shown [M. T. Béal-Monod and K. Maki, Phys. Rev. Lett. **34**, 1461 (1975); J. A. Hertz, Phys. Rev. B **14**, 1165 (1976)] that quantum effects yield 3D paramagnon theory to assume an effective dimensionality greater than 4, so that on one hand, mean field applies to give the critical exponents, i.e., the susceptibility enhancement will appear to the power $\gamma=1{:}1/[1-\bar{I}_{\text{eff}}]^{\gamma=1}$, and on the other hand, $\bar{I}_{\text{eff}}$, as compared with the value $\bar{I}$ one started with, may be computed by perturbation theory.

[18]Except that the "effective dimensionality" is seven in presence of disorder, instead of six in the pure case (see Hertz in Ref. 17).

[19]For a more general band structure, such an effect or the reverse one may occur as well: thus, for instance, the sign of the low T^2 dependence of the spin susceptibility, in the pure case, has been shown to be band-structure dependent [M.T. Béal-Monod and J. M. Lawrence, Phys. Rev. B **21**, 5400 (1980)]. J. Appel *et al.* (unpublished) starting from a different view point, also find modifications in the Stoner factor due to disorder, which are band-structure dependent.

[20]The current study relies on the perturbative expansion of Refs. 9 and 10. However, more sophisticated studies [A. M. Finkel'stein, Z. Phys. B **56**, 189 (1984); C. Castellani *et al.*, Phys. Rev. B **30**,

1596 (1984); R. Opperman, Solid State Commun., **44**, 1297 (1982); in *International Conference on Localization Interaction and Transport Phenomena in Impure Metals,* edited by L. Schweitzer and B. Kraner (PTB, Braunschweig, Germany, 1984), p. 121] qualitatively agree with my conclusions: in the presence of disorder and interactions, spin alignment develops and strong spin fluctuations are generated; moreover, the q^2 dependence of the susceptibility *weakens,* as in my formula (7); thus, the susceptibility tends to become q independent as in an "à la Brinkman Rice" nearly localized description; the uniform spin fluctuations switch to *local* ones.

[21]This followed already from the microscopic work of K. Levin and O. T. Valls, Phys. Rep. **98**, 1 (1983).