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Channeling radiation from relativistic electrons and positrons

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With the use of one- and two-dimensional hydrogen models (with quantum defects) for planar and axial channeling of electrons and harmonic-oscillator models (with anharmonicity) for positron channeling, some analytical results have been obtained for channeling radiation frequencies. The effects of dislocations in these simple models are then considered and suitable experiments suggested to test the results.

INTRODUCTION

It is well known in electrodynamics that any charged particle passing through matter gives rise to bremsshtrahlung radiation because of random acceleration or deceleration experienced by the particle due to the charges of the medium. However, when this motion is a correlated one, owing to steering by major crystallographic axes or planes in channeling, the processes of acceleration and deceleration, as well as the oscillatory motion of the particle, are regular and, to a good approximation, emit the same frequency or frequencies along their trajectories. For relativistically fast particles, the frequency of the emitted radiation in the forward direction is Doppler shifted. For example, the MeV electrons and positrons have been found to give rise to keV photons.[1–3] After the first observation of these radiations, several workers have become involved with this problem, mostly because of its potential application in studies of channeling phenomena itself, the crystal properties including defects, and possibility of using the phenomena to construct a radiation source. There exist, for example, already some review articles on this subject.[4,5] Of course, as for the channeling phenomena themselves, the observed channeling radiation characteristics have already been used to determine the parameters of some assumed form of the interatomic potential, by using data fitting on a computer.[6] The purpose of present investigations is, however, to try to solve the problem analytically as far as possible, within reasonable approximations so that the applications in defect studies may be feasible. For this reason we investigate the characteristics of these radiations for a model potential of power law type which has been successfully and analytically used[7] in studies of the effects of defects on channeling.

FORMALISM

Planar case: Electrons

The potential presented by a plane to any incoming negatively charged particle is taken as[7]

$$V(y)=\frac{2V_0a}{|y|+a} \;, \tag{1}$$

where $V_0=\pi Z_1Z_2e^2Nd_pCa$, Z_1 and Z_2 are the charge numbers of the projectile and target atom, respectively, N the bulk density of atoms in the target, $d_p=2l$ the interplanar separation, C a constant ($\cong\sqrt{3}$), and y the distance measured from the plane. The Thomas-Fermi screening radius a is given by

$$a=0.8853a_0/(Z_1^{2/3}+Z_2^{2/3})^{1/2} \;,$$

where a_0 is the Bohr radius. The one-dimensional Schrödinger equation with potential (1) is essentially the one-dimensional hydrogen atom with quantum-defect correction so that the energy levels for transverse motion of the channeled electron are given by

$$E_n=\frac{2mV_0^2a^2}{\hbar^2(n+\delta n)^2} \;, \tag{2}$$

where the quantum-defect correction[8] δn for odd states is given by $\delta n=2a/a_0$. The temperature effects are included by replacing a in the denominator of Eq. (1) by $a_T=(a^2+u^2)^{1/2}$ where u^2 is the mean-square vibrational amplitude perpendicular to the plane. This means $\delta n=2a_T/a_0$.

Planar case: Positrons

The positrons, being positively charged particles, move in the potential field of two planes and for the form (1) of single plane potential, we get

$$V(x)=\frac{4V_0La}{L^2-x^2} \;, \tag{3}$$

where $x(=l-y)$ is now measured from the midplane between two planes and $L=l+a$. As is clear for $x<L$, one can expand in powers of x to write

$$V(x)=\frac{4V_0a}{L}+\frac{4V_0a}{L^3}x^2+\frac{4V_0a}{L^5}x^4+\cdots \;. \tag{4}$$

Using harmonic oscillator results, the forward direction observed radiation frequency is found to be

$$\omega_{\rm obs}=2\gamma^{3/2}\left[\frac{8V_0a}{mL^3}\right]^{1/2} \;, \tag{5}$$

where γ is the relativistic factor given by $\gamma=1/(1-v^2/c^2)^{1/2}$. The linewidth due to anharmonicity is also easily calculated and compares well with the experimental results.[9]

If we use the results obtained with potential (3) for distorted (due to dislocations) planes[10] in an otherwise perfect crystal, we expect some additional radiation frequencies depending upon the dislocation parameter. For example, a well channeled particle (which will not emit any channeling radiation) will start oscillating after passing through a distorted channel, at distance r_0 away from dislocation core with Burger's vector b, with an average amplitude[10]

$$\bar{x}_{\rm amp} = 0.49 L^3 Eb / 4 V_0 a \pi^2 r_0^2 \quad , \tag{6}$$

and will have a period

$$T = \left[\frac{2mL(L^2 - \bar{x}_{\rm amp}^2)}{V_0 a} \right]^{1/2} F\left[\frac{\bar{x}_{\rm amp}}{L} \right] \quad , \tag{7}$$

where F is the complete elliptic integral of the second kind. The observed additional channeling radiation frequency in the forward direction should, therefore, be

$$\omega_{\rm obs} = 4\pi\gamma^2 / T \quad . \tag{8}$$

Axial case: Electrons and positrons

The analysis for the axial situation is basically the same as for the planar case except that one has to deal with a two-dimensional, hydrogen-like model with a quantum-defect correction (for electrons) or harmonic oscillatorlike, with anharmonicity corrections (for positrons). For instance, taking the positron axial channeling case, the harmonic potential seen by the positron for not too large r has been shown to be of the form[11]

$$V(r) = U_0 \left[1 + \frac{r^2}{R_0^2} \right] \quad , \tag{9}$$

where R_0 is the axial channel size, r the distance measured from the central axis, $U_0 = n_s Z_1 Z_2 e^2 C^2 a^2 / dR_0^2$ for n_s strings surrounding the channel. This problem of circular oscillator gives the energy levels

$$E_n = (n_1 + n_2 + 1)\hbar\omega \quad , \tag{10}$$

where n_1 and n_2 are the two quantum numbers, $\omega = (2U_0 / m\gamma R_0^2)^{1/2}$, and the observed radiation frequency in the forward direction is

$$\omega_{\rm obs} = 2\gamma^{3/2} (2U_0 / mR_0^2)^{1/2} \quad . \tag{11}$$

Again, for a small concentration of dislocations of Burger's vector b, the amplitude acquired by the initially well channeled particle is calculated to be

$$\bar{r}_{\rm amp} = 4R_0^2 Eb / 3\pi^3 d_0^2 U_0 \quad , \tag{12}$$

where d_0 is the distance of the channel from the dislocation core. The corresponding period and resulting frequency is obtained as before.

DISCUSSIONS AND CONCLUSIONS

The main point of the present work was to explore the possibilities of the application of channeling radiation to defect studies. The simple potential used for analytical calculations has been used before for defect studies[7] and also the channeling radiation problem.[9,12] From the present preliminary results on the application of channeling radiation to the dislocation problem, it seems possible to observe these effects in the emitted spectrum. For example, if a uniform distribution of screw dislocations (with density N_d per cm^2) is introduced in an otherwise perfect crystal, the least affected planar channel situated approximately halfway between two dislocation cores will induce minimum oscillation amplitude $\bar{x}_{\rm amp}$ for initially well channeled positrons. This amplitude is obtained from expression (6) with r_0^2 replaced by $1/N_d$ and the corresponding minimum frequency of channeling radiation (above which an enhancement should be expected) is obtained from the expression (8). The part of the spectrum above this frequency will contain the effects of dislocations. However, the dislocation density should be below a critical value such that the distance between consecutive dislocation cores is larger than the diameter of the dechanneling cylinder defined by Quere,[13] which for the planar case and screw dislocations is given by

$$\left[\frac{Eb}{8.6 Z_1 Z_2 e^2 N d_p} \right]^{1/2} \quad .$$

This has a numerical value of the order of 800 Å for 56-MeV positrons channeling in (110) planes of silicon crystals. The corresponding critical density of dislocations at which all the particles will be dechanneled and, hence, no channeling radiation, is found to be of the order of 10^{11} dislocations/cm^2.

For dislocation densities lower than this critical value, channeling radiation will still be observed but the spectra will be modified. Firstly, the particles responsible for the radiation peak are the ones which were oscillating in a perfect crystal and the majority of these (except the ones reaching distorted channel in right phase) will be dechanneled. On the other hand, the particles which were well channeled (or oscillated with a very small amplitude) will acquire an amplitude given by expression (6) and, hence, will be responsible for channeling radiation. The corresponding radiation frequency emitted by particles acquiring an amplitude $\bar{x}_{\rm amp} = L/2$, due to a dislocation density $= 3.4 \times 10^9$ per cm^2, is given by 49.2 keV [for 56-MeV positrons channeling along (110) planes in a silicon crystal]. The detailed calculation of the entire spectrum is yet to be done but the present preliminary and approximate results are expected to motivate some experiments in this direction.

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