

Excitation energies of multilevel tunneling states in the presence of large local strain fields: A possible model for the glassy state

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The density of states, $P_0(E)$, of the elementary excitation energies E is obtained for a single two-level and four-level tunneling state in the presence of a distribution $P_1(\xi)$ of random local strain fields ξ and a fixed tunneling matrix element Δ . Whereas the two-level states have low-energy excitations only when both Δ and ξ are small, the four-level (or six-level) states have very-low-energy excitations, $E \propto \Delta^2/\xi$, even for large values of Δ , provided the strain fields ξ are sufficiently large. We thus obtain the unexpected result that the excitation energies become smaller with increasing local strain fields for the multilevel states but not for the two-level states. For the case when $P_1(\xi) \propto |\xi|^{-k}$ for large ξ , the density of the low-energy excitations for the four-level states is $P_0(E) \propto E^{k-2}$ for $E \ll \Delta$. For $k=2$, $P_0(E)$ is a constant for low E for the four-level states. We also show that for a number of physically interesting probability distributions of strain fields $P_0(E)$ is approximately constant for the four-level states but not for the two-level states. In particular we find that the probability for low ξ is not relevant in determining the low-temperature thermodynamic properties of the four-level (and six-level) states. The possible significance of our results to the low-temperature thermodynamic properties of glasses is discussed.

I. INTRODUCTION

It has been suggested that the low-temperature properties of glasses,¹⁻³ amorphous solids,⁴ and of certain impurities in alkali halides^{5,6} all arise from tunneling states. For this reason, it is important to understand the nature of the low-energy excitations from tunneling units. Recently, the author⁷ considered the statistical mechanics of pairs of two-level and four-level tunneling units which interact via a potential $J(r)$ of the form $J(r) = \pm a/r^n$, where a is a constant and r is the distance between the tunneling units. He obtained the density $P_0(E)$ of the low-energy excitation energy E , the specific heat $C(T)$ for temperature T , and the thermal conductivity $\kappa(T)$. The calculation was limited to the physical situation in which (a) the depth of each of the potential wells of any single tunneling unit was equal and (b) where the pair interaction alone was a good description of the physics of the problem (terms in the free energy only up to the second virial coefficient were kept). The effective interaction \mathcal{H}_{ij} between the pairs of tunneling units was assumed to have the form $\mathcal{H}_{ij} = -J(r)\sigma_i^z\sigma_j^z$, where σ_i^z and σ_j^z are the z components of the Pauli spin matrices for the two-level states and $\mathcal{H}_{ij} = -J(r)\vec{\mu}_i \cdot \vec{\mu}_j$, where $\vec{\mu}_i$ and $\vec{\mu}_j$ are classical dipoles which may take values of $\vec{\mu}_i = (\mu_x, \mu_y) = (\pm 1, 0)$ and $(0, \pm 1)$ for the four-level tunneling states.⁷ It was found that for a fixed value of the tunneling matrix element Δ , the density $P_0(E)$ of the excitation energy E is, $P_0(E) \propto E^{(3-n)/n}$ for $E \ll \Delta$, and that $C(T) \propto T^{n/3}$ and $\kappa(T) \propto T^{(3n-3)/n}$. For $n=3$, we obtain that $P_0(E)$ is approximately constant for low E and that $C(T) \propto T$ and $\kappa(T) \propto T^2$ for very low T , not unlike the results observed for glassy systems.⁴ These results are interesting, particularly since the strain-strain interaction calculated in a continuum model^{8,9} for pair of strain dipoles give $n=3$.

The purpose of this paper is to consider the low-energy excitations from two-level and four-level (also six-level) tunneling units in the presence of a large local strain field which allows the distortion of the depth of the wells. In previous work⁷ on two-level states such a distortion was not allowed, for it resulted in an exponential decrease of the specific heat from its linear dependence on T . The strain interaction is assumed to be "volume conserving," in the sense that in two dimensions, an increased strain energy associated with the strain dipole when it is oriented in the x direction will result in a decreased strain energy when it is oriented in the y direction. Our calculations show a completely different behavior of the two-level tunneling units from those of the four-level units. Whereas, the two-level tunneling units contribute to the low-energy excitations only when both, the local strain field ξ and the tunneling matrix element Δ are small, the four-level (and six-level) tunneling units contribute to the low-energy excitations for large local strain fields ξ , even for the case when Δ is not too small. We find that the low-energy excitation $E \propto \Delta^2/\xi$ for sufficiently large ξ , and the larger the local strain field the smaller the excitation energy. We therefore obtain the important result that the density of low-energy excitations for the four-level (or six-level) tunneling units is determined by the probability for large local strain fields.

The contents of this paper is as follows. We first derive the relationship between the probability density $P_1(\xi)$ of the local strain fields ξ and the probability density $P_0(E)$ of the excitation energies E . Next we obtain $P_0(E)$ for a number of physically interesting distributions $P_1(\xi)$.

Case 1. $P_1(\xi)$ is a Lorentzian centered about ξ_0 , with width δ and cutoff in the maximum local field ξ_M . For this case, we obtain that $P_0(E)$ is approximately constant for $E \ll \Delta$ and $E > [(\xi_M^2 + 4\Delta^2)^{1/2} - \xi_M]/2 = A_-$. $P_0(E)$

=0 for $E < A_-$.

Case 2. $P_1(\xi) \propto \xi^2 / (\xi^2 + \delta^2)^2$. Again, $P_0(E)$ is approximately constant for $A_- < E \ll \Delta$.

Case 3. $P_1(\xi) \propto |\xi|^{-k}$. For this case we show that $P_0(E) \propto E^{k-2}$ for low E , thus for $k=2$, $P_0(E) \approx \text{const}$.

Case 4. Gaussian distribution of strain fields. For this case $P_0(E)$ is exponentially small for low values of E .

Case 5. Here, we consider the case where the local strain field results from the interaction potential $J(r) = \pm a/r^n$ with a single tunneling unit, where r is the random distance between the tunneling units. For the four-level (and six-level) states we obtain $P_0(E) \propto T^{(3-n)/n}$ and the corresponding specific heat $C(T) \propto T^{n/3}$ and the thermal conductivity $\kappa(T) \propto T^{(3n-3)/n}$. Thus, for $n=3$ (i.e., for $1/r^3$ interaction between the tunneling units), which arises when the strain field is calculated in a continuum model,^{8,9} we obtain $P_0(E) \propto E^0$, $C(T) \propto T$ and $\kappa(T) \propto T^2$. These results are not unlike those observed in glasses.¹⁻³

Examining more closely cases 1-5 discussed above, we find that a local field at the multilevel (but not two-level) tunneling site which arises from an r^{-3} potential gives a constant density of states for low E , and deviations from the r^{-3} field gives deviations from the constant density of states. These results suggest that the constant density of states may arise from the strain fields which have an r^{-3} dependence.⁸

Even though the predicted results for $n=3$ agree with those observed in some glasses, we explicitly avoid a detailed comparison of our work with experiments on the thermodynamic properties of glasses at low temperatures. One reason for this is that the nature of the tunneling states and the realistic distribution of $P_1(\xi)$ are not available for glasses at the present. We still believe, however, that our results present an interesting microscopic model for a constant density of excitation energies for asymmetric potential wells, provided the tunneling units are multilevel systems and have sufficiently large local strain fields.

In Appendix A, we consider the Hamiltonian for a six-level tunneling unit with a volume conserving traceless strain tensor such that the sum of the strain energies in the x , y , and z directions equal zero. The orientation of the strain dipole $\vec{\mu}$ is assumed to be $\vec{\mu} = (\mu_x, \mu_y, \mu_z) = (\pm 1, 0, 0)$, $(0, \pm 1, 0)$, and $(0, 0, \pm 1)$. We find that the low-energy excitations of the six-level states are qualitatively similar to those of the four-level states.

II. DERIVATION OF THE DENSITY OF STATES

A. Hamiltonian for a single tunneling unit

The Hamiltonian and the free energy of a single four-level tunneling unit at site i in the presence of a local strain field ξ_i is now obtained. In Appendix A we obtain the two lowest-energy levels for a set of six-level tunneling units for which the low-energy excitation is qualitatively very similar to those of the four-level states.

To compare with the results for the four-level tunneling states (FLS), we first write down the Hamiltonian H_i and the free energy F_i for the two-level tunneling states (TLS)

in the presence of a local strain field ξ_i and a fixed tunneling matrix element Δ_i . We have,

$$H_i = \begin{pmatrix} -\xi_i & -\Delta_i \\ -\Delta_i & \xi_i \end{pmatrix} \quad (2.1)$$

and

$$F_i = -\beta^{-1} \ln \{ 2 \cosh[\beta(\xi_i^2 + \Delta_i^2)^{1/2}] \}, \quad (2.2)$$

where $\beta = (k_B T)^{-1}$ with T representing the temperature.

We next consider the four-level state. For convenience we let the potential minima of the four levels be of the form shown in Fig. 1(a) and the four states of the system are labeled by $|x\rangle$, $|-x\rangle$, $|y\rangle$, and $|-y\rangle$ as shown in Fig. 1(b). The tunneling states are assumed to have an electric dipole moment of magnitude p_e and a strain dipole moment of magnitude p_s . The vector electric dipole moment is then \vec{p}_e ,

$$\vec{p}_e = p_e \vec{\mu} \quad (2.3a)$$

and the strain dipole moment \vec{p}_s ,

$$\vec{p}_s = p_s \vec{\mu}, \quad (2.3b)$$

where the magnitude of $\vec{\mu}$ is unity and $\vec{\mu} = (\mu_x, \mu_y)$. The allowed values for $\vec{\mu}$ are assumed to be

$$\vec{\mu} = (\pm 1, 0), (0, \pm 1). \quad (2.4)$$

The longitudinal mean-field Hamiltonian H_i^L arising from a local strain field at site i is obtained in Appendix B, with the result,

$$H_i^L = -p_e (\vec{\mu}_i \cdot \vec{E}_0) - \xi_i (\mu_{ix}^2 - \mu_{iy}^2), \quad (2.5)$$

where E_0 is an externally applied electric field having x and y as components E_x and E_y , respectively. We note that the strain field has the volume conserving term $-\xi_i (\mu_{ix}^2 - \mu_{iy}^2)$, i.e., a strain field decreases the energy in the x direction and increases the energy in the y direction.

In the absence of strain interaction the four states of the (FLT) unit are given by $|\pm x\rangle$ and $|\pm y\rangle$, with the operator equations,

$$\mu_x |\pm x\rangle = \pm |\pm x\rangle, \quad \mu_y |\pm y\rangle = \pm |\pm y\rangle. \quad (2.6)$$

The transverse part of the Hamiltonian H^T is obtained by connecting the $|\pm x\rangle$ states with the $|\pm y\rangle$ states via

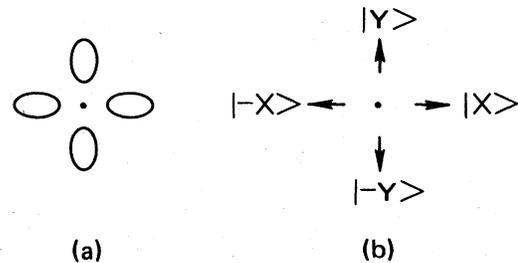


FIG. 1. (a) Shows the assumed potential minima for the four-level tunneling unit. (b) Graphical representation of the four states. These are conveniently labeled by $|x\rangle$, $|-x\rangle$, $|y\rangle$, and $|-y\rangle$.

the tunneling matrix element Δ_i . We only allow 90° tunneling and neglect 180° tunneling from $|-x\rangle$ to $|x\rangle$ and $|-y\rangle$ to $|y\rangle$ states. The total mean-field Hamiltonian of a FLS at site i is

$$H_i = p_e(\vec{\mu}_i \cdot \vec{E}_0) - \xi_i(\mu_{ix}^2 - \mu_{iy}^2) - \Delta_i[(|x_i\rangle + |-x_i\rangle)(\langle y_i| + \langle -y_i|) + H_c], \quad (2.7)$$

where H_c is the Hermitian conjugate of the previous term.

The Hamiltonian, Eq. (2.7), is conveniently written in matrix form,

$$H_i = \begin{bmatrix} a_i & -b_i \\ -b_i & c_i \end{bmatrix}, \quad (2.8a)$$

where

$$a_i = \begin{bmatrix} -\xi_i - E_x & 0 \\ 0 & -\xi_i + E_x \end{bmatrix}, \quad b_i = \begin{bmatrix} \Delta_i & \Delta_i \\ \Delta_i & \Delta_i \end{bmatrix}, \quad (2.8b)$$

$$c_i = \begin{bmatrix} \xi_i - E_y & 0 \\ 0 & \xi_i + E_y \end{bmatrix}.$$

The eigenvalues λ_ν^0 ($\nu=1,4$) of Eq. (2.8) in the absence of an externally applied electric field are

$$\lambda_1^0 = -(\xi_i^2 + 4\Delta_i^2)^{1/2}, \quad (2.9a)$$

$$\lambda_2^0 = -\xi_i, \quad (2.9b)$$

$$\lambda_3^0 = \xi_i, \quad (2.9c)$$

$$\lambda_4^0 = (\xi_i^2 + 4\Delta_i^2)^{1/2}. \quad (2.9d)$$

The free energy F for N tunneling states is given by

$$F = -\beta^{-1} \sum_{i=1}^N \ln \{ 2 \cosh(\beta \xi_i) + \cosh[\beta(\xi_i^2 + 4\Delta_i^2)^{1/2}] \} \quad (2.10a)$$

$$= -\beta^{-1} \sum_i (\ln \{ 2 \cosh[\beta E_+(i)] \} + \ln \{ 2 \cosh[\beta E_-(i)] \}), \quad (2.10b)$$

where

$$E_\pm(i) = \frac{(\xi_i^2 + 4\Delta_i^2)^{1/2} \pm \xi_i}{2}. \quad (2.11)$$

The four eigenvalues of the Hamiltonian as a function of the local strain ξ_i and for a fixed value of Δ_i is shown in Fig. 2. We note that for positive ξ_i the gap, $E_-(\xi_i)$, between the ground state and the first excited state is $2\Delta_i$ for $\xi_i=0$, and $E_-(\xi_i)$ is monotonically decreasing and approaching zero, $E_-(\xi) \propto \Delta^2/\xi$, for large ξ . This result shows a radical departure of the ξ dependence of the low-energy excitations E for the two-level states given by Eq. (2.2) and those of the four-level states given by Eq. (2.11). Whereas in the former the excitation energy E is proportional to $(\xi_i^2 + \Delta_i^2)^{1/2}$ and is small (compared to $k_B T$) only when both ξ_i and Δ_i are small, in the latter the excitation energy $E_-(\xi)$ is proportional to Δ_i^2/ξ_i (for positive ξ_i) and $E_-(\xi)$ is small for sufficiently large ξ even when Δ_i

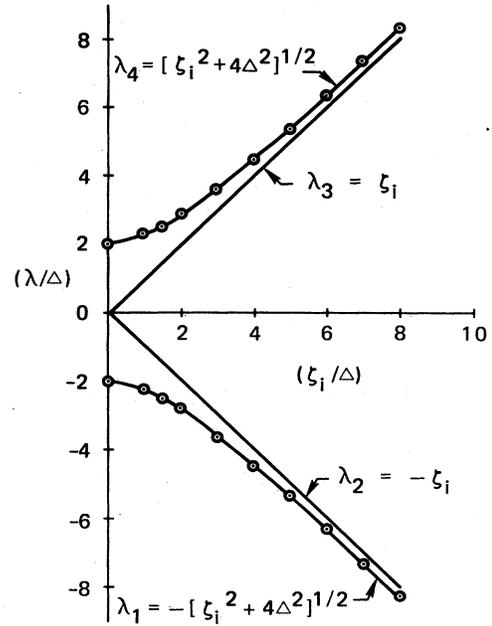


FIG. 2. Shows the four eigenvalues of the Hamiltonian, Eq. (2.8), as a function of the local strain field ξ_i .

is not small. Therefore, TLS contribute to the low-temperature thermal properties of the system only when both ξ and Δ are small, whereas the major contribution to the low- T properties of the FLS arises from large strain fields. We, furthermore, show that several physically interesting probability distributions give an approximately constant density of states for the excitation energies for the four-level (and six-level) states, but not for the two-level states.

B. Calculation of density of states

We next relate the probability density, $P_1(\xi)$, of the local strain fields to the density $P_0(E)$ of the excitation energy E . Suppressing the subscript i , we have,

$$P_0(E_\pm) = P_1(\xi) \left| \frac{d\xi}{dE_\pm} \right|. \quad (2.12)$$

Since

$$\xi = \pm(E_\pm^2 - \Delta^2)^{1/2}, \quad (2.13)$$

Eq. (2.12) becomes

$$P_0(E_\pm) = \frac{E_\pm^2 + \Delta^2}{E_\pm^2} P_1 \left(\frac{\pm(E_\pm^2 - \Delta^2)}{E_\pm} \right), \quad (2.14)$$

where $P_0(E_+)$ and $P_0(E_-)$ give the density of excitation energies for the first and second terms of the free energy given by Eq. (2.10b). The lowest excitation energies are contributed by $E_-(\xi)$ for positive ξ and by $E_+(\xi)$ for negative ξ .

We next calculate the density of states $P_0(E)$ for several interesting probability distributions for the local field ξ .

Case 1. Lorentzian distribution of fields. We consider a Lorentzian distribution of strain fields ξ , centered about ξ_0 and having a cutoff ξ_M in the maximum field. Thus,

$$P(\xi) = \begin{cases} N_0 \frac{\delta}{\delta^2 + (\xi - \xi_0)^2}, & |\xi| \leq \xi_m \\ 0, & \xi > \xi_M, \end{cases} \quad (2.15)$$

where

$$N_0 = \{ \tan^{-1}[(\xi_M - \xi_0)/\delta] + \tan^{-1}[(\xi_M + \xi_0)/\delta] \}^{-1} \quad (2.16)$$

is the normalization constant for $P_1(\xi)$.

The cutoff in the maximum value of ξ will restrict the maximum and minimum values of E_{\pm} . Let

$$A_{\pm} = [(\xi_M^2 + 4\Delta^2)^{1/2} \pm \xi_M]/2. \quad (2.17)$$

Using Eq. (2.15) in Eq. (2.14) gives

$$P_0(E_{\pm}) = \begin{cases} N_0 \delta \frac{E_{\pm}^2 + \Delta^2}{\delta^2 E_{\pm}^2 + [\pm(E_{\pm}^2 - \Delta^2) - \xi_0 E_{\pm}]^2}, & A_- < E_-, A_+ > E_+ \\ 0, & E_- < A_-, E_+ > A_+. \end{cases} \quad (2.18)$$

Graphs of $P_0(E/\Delta)$, are shown in Figs. 3 and 4. For a typical near-neighbor strain interaction energy of the order of 500 K, ξ_M is of the order of 2000 K.⁷ For values of $E_- \leq \Delta^2/\xi_M$, $P_0(E_+) = 0$.

For small values of E_{\pm} (i.e., $E_{\pm} \ll \Delta$) we obtain from Eq. (2.18) for $E_- \geq A_-$ and $E_+ < A_+$,

$$P_0(E_{\pm}) = \frac{N_0 \delta}{\Delta^2} \left[1 \mp 2 \left(\frac{\xi_0}{\Delta} \right) \left(\frac{E}{\Delta} \right) + \frac{(3\xi_0^2 + 3\Delta^2 - \delta^2)}{\Delta^2} \left(\frac{E}{\Delta} \right)^2 + \dots \right] = 0, \quad E_- < A_-, \quad E_+ > A_+. \quad (2.19)$$

We thus obtain that $P_0(E_+)$ is approximately constant for $(E_{\pm}/\Delta) \ll 1$ and $E_- > A_-$. For reasonable experimental values^{10,11} of Δ of order 1 K, $A_- = \Delta^2/\xi_M \approx (1/2000)$ K, thus $P_0(E_-)$ is approximately constant for $E_- > 10^{-3}$ K. To calculate the thermodynamic quantities given in Eq. (2.10b) for low T we are interested in $[P^+(E_+) + P^-(E_-)]/2$. Thus, the term proportional to ξ_0 in Eq. (2.19) cancels to give a lowest correction of order E^2 to the constant density of states. The coefficient of the E^2 term is positive for $3(\xi_0^2 + \Delta^2) < \delta^2$, and negative when $3(\xi_0^2 + \Delta^2) > \delta^2$. The sign of the coefficient of the E^2 term determines the sign of the corrections to the thermodynamic quantities from their $T \rightarrow 0$ values.

Before we end this section, it is interesting to consider the case of a set of randomly, uniformly, and independently distributed tunneling units in which the local field at site i , ξ_i , has the following form:

$$\xi_i = \sum_j (\pm a / r_{ij}^3),$$

where a is a constant, the plus and minus sign has equal

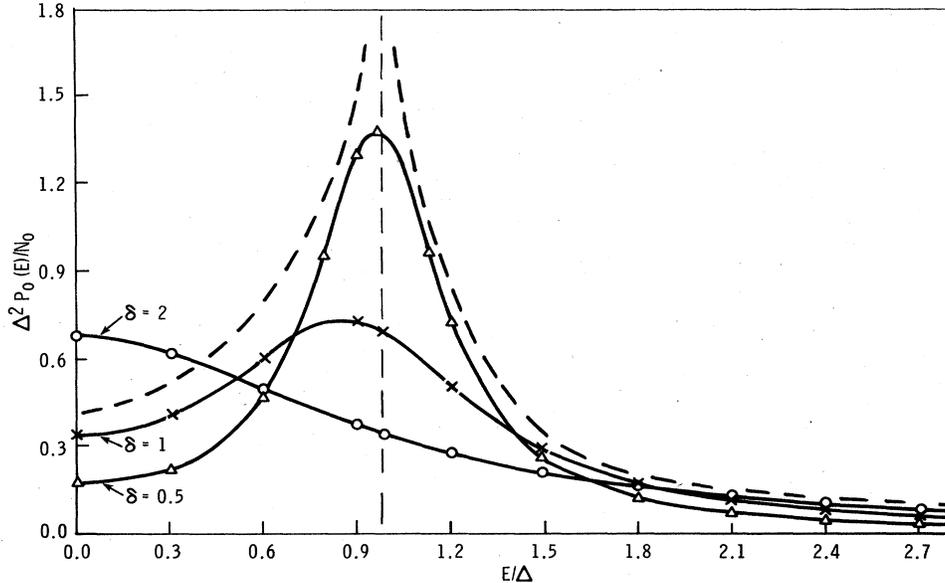


FIG. 3. Density $P_0(E)$ as a function of the excitation energy E for a Lorentzian distribution of strain fields ξ and for several width δ of the Lorentzian, $\xi_0 = 0$. The graph shows $\Delta^2 P_0(E/\Delta)$ vs E/Δ for several values of E/Δ . The dotted line shows the approximate shape of $P_0(E)$ when the local field arises from a single r^{-3} potential. Note the change in curvature of $P_0(E)$ for small E for small and large values of δ .

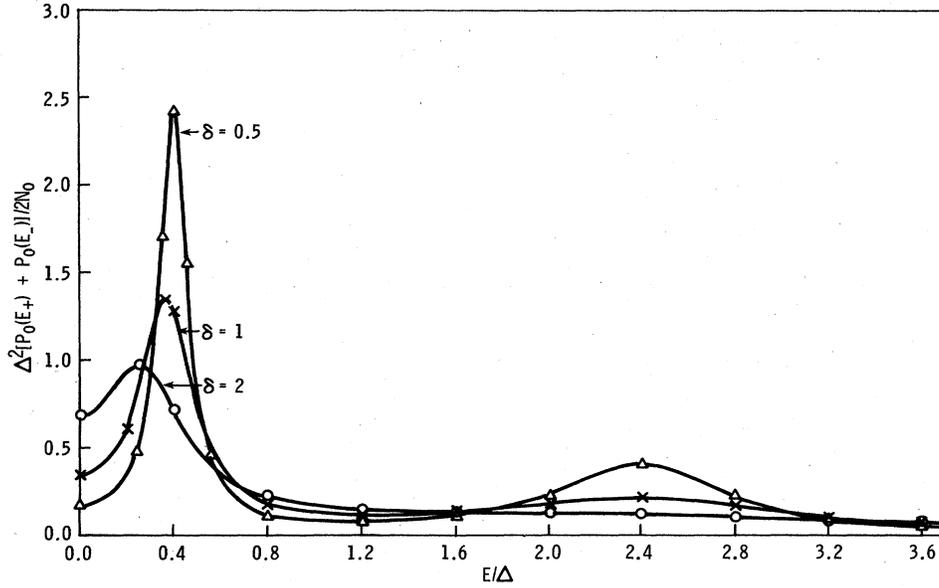


FIG. 4. Density of states $[P_0(E_+) + P_0(E_-)]/2$ for the case when $\xi_0/\Delta = 2$. Notation is the same as in Fig. 3. When $\xi_0 \neq 0$, there is a second peak in $P_0(E)$ as is shown in the figure above.

probability, and r_{ij} is the distance between the tunneling units. For this case one obtains¹²

$$P(\xi) = \frac{1}{\pi} \frac{\delta}{\delta^2 + \xi^2}, \quad (2.20)$$

with $\delta = (2\pi^2/3) |a| n_0 c$, where c is the fractional concentration of the tunneling units and n_0 is the number of sites per unit cell. Thus, Eq. (2.15) is an *ad hoc* simulation of Eq. (2.20) for the case when the distance of closest approach between the tunneling units is limited by a near-neighbor distance in the solid. The ξ^{-2} dependence of $P_1(\xi)$ for large ξ is a consequence of the $1/r^3$ form of the local strain field.

Case 2. Three-dimensional vector fields. We next consider the density of states arising from a probability distribution of strain fields of the form,

$$P_1(\xi) = \begin{cases} N_1 \frac{\xi^2}{(\xi^2 + \delta^2)^2}, & |\xi| < \xi_M \\ 0, & |\xi| > \xi_M, \end{cases} \quad (2.21)$$

with $N_1 = 4\delta [\tan^{-1}(\xi_M/\delta) - \xi_M \delta / (\xi_M^2 + \Delta^2)]^{-1}$.

Using Eq. (2.19) in Eq. (2.14) gives

$$P_0(E_{\pm}) = \begin{cases} N_1 \delta \frac{(E_{\pm}^2 + \Delta^2)(E_{\pm}^2 - \Delta^2)^2}{[\delta^2 E_{\pm}^2 + (E_{\pm}^2 - \Delta^2)^2]^2}, & A_- < E_-, \quad A_+ > E_+ \\ 0 & \text{otherwise.} \end{cases} \quad (2.22)$$

where A_{\pm} is given by Eq. (2.17). For $E \ll \Delta$ we obtain

$$P_0(E_{\pm}) \approx \frac{N_1 \delta}{\Delta^2} \left[1 + [3 - 2(\delta/\Delta)^2] \left[\frac{E_{\pm}}{\Delta} \right]^2 \right]. \quad (2.23)$$

We have explicitly chosen $P_1(\xi)$ given by Eq. (2.21) to

consider a distribution in which $P_1(\xi) \propto \xi^2$ for low ξ , in order to show that even though an internal field distribution of the form given by Eq. (2.21) gives a density $P_1(\xi)$ for small ξ which is proportional to ξ^2 , when this distribution is used to calculate the density of states for a four-level tunneling state, we obtain (for energies $E_- > A_-$) that $P_0(E)$ is approximately constant. The coefficient of the E^2 term may be positive or negative depending on whether $3\Delta^2$ is greater or less than $2\delta^2$. $P_0(E)$ for low E is approximately constant for the four-level tunneling units, but not for the two-level ones. For the two-level states Eq. (2.13) is replaced by the expression $\xi = (E^2 - \Delta^2)^{1/2}$, and $P_0(E) \propto E$ for $E > \Delta$ for the distributions $P(\xi)$ given by Eq. (2.20), $P_0(E) = 0$ for $E < \Delta$. This result shows explicitly that the constant density of states arises only in the multilevel, but not the two-level tunneling units.

It is of interest to examine the case when

$$\xi_i = \sum_j (\pm a/r_{ij}^3) \hat{r}_j,$$

where \hat{r}_j is a three-dimensional unit vector which is equally likely to be oriented in any one direction, and r_{ij} is the distance between the tunneling units at sites r_i and r_j . With randomly, uniformly, and independently distributed tunneling units, the probability density for ξ_i is similar to that assumed in Eq. (2.21). See for example, C. Held and M. W. Klein, Phys Rev. Lett. 35, 1783 (1975). The spin system discussed in this reference is of no interest here, we just use a local field whose distribution $P_1(\xi) \propto \xi^2$ for small ξ and $P_1(\xi=0) = 0$. In spite of this, $P_0(E)$ is approximately constant for low E . This constant $P_0(E)$ results from the r^{-3} potential, which gives that $P_1(\xi) \propto \xi^{-2}$ for large ξ .

The probability density $P_0(E)$ for this case is shown in Fig. 5. Note that for $3\Delta^2 > 2\delta^2$ the initial slope of $P_0(E)$

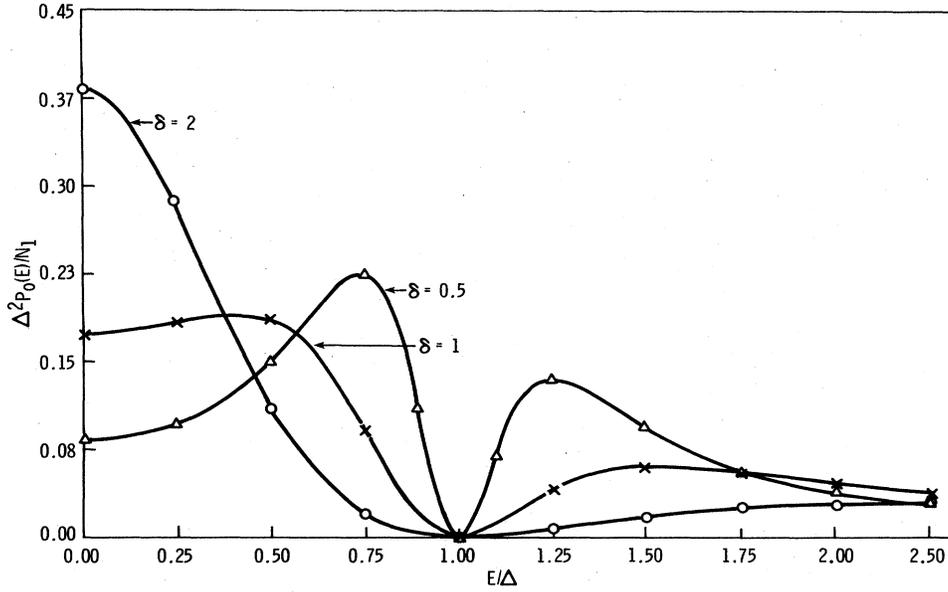


FIG. 5. Density of states $P_0(E)$ as a function of E , for a distribution of strain fields given by Eq. (2.21) of the text. The graph shows $\Delta^2 P_0(E/\Delta)$ vs E/Δ for several values of δ/Δ and $\xi_0=0$.

is positive, and that $P_0(E) \rightarrow 0$ as $E \rightarrow \Delta$.

Case 3. Consider the case where for large ξ :

$$P_1(\xi) \begin{cases} \propto |\xi|^{-k}, & |\xi| < \xi_M \\ = 0 & \text{otherwise.} \end{cases} \quad (2.24)$$

Using Eq. (2.24) in Eq. (2.14) gives

$$P_0(E_{\pm}) \propto \frac{E_{\pm}^2 + \Delta^2}{|(E_{\pm}^2 - \Delta^2)|^k} \left[\frac{E}{\Delta} \right]^{k-2}. \quad (2.25)$$

For $k=2$, $P_0(E)$ is approximately constant for low E . The probability densities given in Eq. (2.15) and Eq. (2.21), both have $P_1(\xi) \propto \xi^{-2}$ for large ξ and therefore give approximately constant densities of state for low E .

Case 4. Gaussian distribution of strain fields. We next consider the case where

$$P_1(\xi) = (2\pi\delta^2)^{-1/2} \exp[-\xi^2/(2\delta^2)]. \quad (2.26)$$

Using Eq. (2.26) in Eq. (2.14) gives

$$P_0(E) = \frac{1}{\sqrt{2\pi\delta}} \exp \left[-\frac{1}{2} \left[\frac{E_{\pm}^2 - \Delta^2}{\delta E_{\pm}} \right]^2 \right] \left[\frac{\Delta^2 + E_{\pm}^2}{E_{\pm}^2} \right] \quad (2.27)$$

For this case $P_0(E)$ for $E \ll \Delta$ is

$$P_0(E) \propto E^{-2} \exp \left[-\frac{1}{2} \frac{\Delta^4}{\delta^2 E^2} \right]. \quad (2.28)$$

Thus, the density of states for small energies E is exponentially small for Gaussian distribution of strain fields ξ .

Case 5. Finally we consider the case where the field at site 0 arises from the interaction with a single tunneling unit separated by a distance r from the tunneling unit under consideration. We consider the case where the strain field ξ ,

$$\xi = \pm a/r^n, \quad (2.29)$$

with the plus and minus signs each having equal probabilities. We let the second tunneling unit be randomly and uniformly distributed in the effective volume, V_0 of the pair of tunneling units. This case was treated previously⁷ with the result that

$$P_0(E) \propto \frac{(E^2 + \Delta^2)(E^{(3-n)/n})}{|(E^2 - \Delta^2)|^{(3+n)/n}}, \quad E \ll \Delta. \quad (2.30)$$

For a continuum model,⁸ it was found that $n=3$, and we obtain for low E ,

$$P_0(E) \propto [1 + 3E^2/\Delta^2 + O(E^4)]. \quad (2.31)$$

We also note, that the very-low-temperature specific heat $C \propto T^{n/3}$ from the pair model and is linear in T for $n=3$, the low-temperature thermal conductivity⁷ $\kappa \propto T^{(3n-3)/n}$ and is proportional to T^2 for $n=3$. We also found⁷ that the low-temperature specific heat $C \propto \exp[-\text{const}/T]$ for $T \ll 1/A_-$. However, the very-low-temperature ($T \ll 1/A_-$) thermal conductivity is determined by boundary and impurity scattering and is not proportional to $\exp[-\text{const}/T]$ as is suggested in Ref. (7). The reason for this is that $P_0^{-1}(E)$, which enters the denominator of Eq. (3.25) in Ref. 7, becomes infinite for $E_- < A_-$.

A rough sketch of $P_0(E)$ versus E from Eq. (2.29) for $n=3$ is shown by the dashed line in Fig. 3. Note that the density of states represented by Eq. (2.29) starts from a constant and increases with E until $P_0(E)$ has a seeming divergence at $E=\Delta$. For small E the behavior of $P_0(E)$ is approximately constant for the case where the strain fields arise from a single random potential of the form $\xi_i = \pm a/r_{ij}^3$ as well as for the case when the strain field is given by the sum

$$\xi_i = \sum_j (\pm a) / r_{ij}^3.$$

In each case it is only the $1/r^3$ interaction that determines the constant density of states for low E , and it does not matter whether the local strain field arises from a single or many tunneling units.

III. CONCLUSION

We consider the low-energy excitations from a set of two-level, four-level (and six-level) tunneling states in the presence of large local strain fields. Whereas, the excitation energies E for the two-level states is $E = (\xi^2 + \Delta^2)^{1/2}$, for the four-level and six-level tunneling states $E \propto \Delta^2/\xi$ for large strain fields ξ . Therefore, for a fixed value of Δ , the excitation energy decreases for increasing strain fields. We calculate the density of states arising from a number of physically interesting distributions of strain fields and find that the density of states is approximately a constant for low energies. In particular, we find that when $P_1(\xi) \propto |\xi|^{-k}$ for large ξ , $P_0(E) \propto E^{k-2}$ for small E . For $k=2$, the density of the excitation energies is approximately a constant for low E . It is interesting to note that for an effective interaction between the tunneling states proportional to r^{-3} , $P_1(\xi) \propto \xi^{-2}$ for large ξ , thus a $1/r^3$ potential results in a constant density of states. This is very suggestive that the approximately constant density of states of glassy and amorphous materials arise from the strain interactions in the disordered solid.

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APPENDIX A: SIX ORIENTATIONAL TUNNELING STATES IN THE MEAN-FIELD APPROXIMATION

We consider a volume-conserving strain field, with a (1,1,0) stress,^{13,14} such that the trace of the strain tensor equals zero.¹⁴ Let the strain field in the x direction be -2ξ and the strain field in the y and z direction be $+\xi$. We show that the energy difference E between the ground state and the first excited state for large ξ is proportional to Δ^2/ξ . Thus $(d\xi/dE) \propto 1/E^2$ for small E (large ξ) just like in Eq. (2.13). Hence, the density of states for the various probability distributions has the same qualitative behavior for the six-level as the four-level tunneling states.

The Hamiltonian for the six-level state is equal to

$$\begin{pmatrix} A & B \\ B & C \end{pmatrix} \quad (\text{A1})$$

where

$$A = \begin{pmatrix} 2\xi & 0 & -\Delta \\ 0 & 2\xi & -\Delta \\ -\Delta & -\Delta & -\xi \end{pmatrix}, B = \begin{pmatrix} -\Delta & -\Delta & -\Delta \\ -\Delta & -\Delta & -\Delta \\ 0 & -\Delta & -\Delta \end{pmatrix}, \quad (\text{A2})$$

$$C = \begin{pmatrix} -\xi & -\Delta & -\Delta \\ -\Delta & -\xi & 0 \\ -\Delta & 0 & -\xi \end{pmatrix}.$$

The ground-state and first excited state energies of the six-level states are

$$\lambda_0 \approx -2J - 8\Delta^2/\xi, \quad \lambda_1 = -2J. \quad (\text{A3})$$

The lowest excitation energy

$$E = \lambda_1 - \lambda_0 \approx 8\Delta^2/\xi + O(\xi^{-2}).$$

Thus, for small values of E , we have

$$\frac{d\xi}{dE} \propto \frac{1}{E^2}. \quad (\text{A4})$$

It is useful to compare this result with the one obtained in Eqs. (2.12) to (2.14). We thus find that the qualitative results for the low-energy excitations from the four-level and six-level states are the same and depend only on $P_1(\xi)$ for large ξ .

APPENDIX B: EFFECTIVE STRAIN HAMILTONIAN IN THE MEAN-FIELD APPROXIMATION

In this appendix we obtain an effective strain Hamiltonian in the mean-field approximation. We start with a Hamiltonian of the form

$$\mathcal{H} = - \sum J_{ij} (\vec{\mu}_i \cdot \vec{\mu}_j)^2 - \sum K_{ij} (\vec{\mu}_i \cdot \vec{\mu}_j). \quad (\text{B1})$$

The first sum in Eq. (B1) represents a "quadrupolar" term which depends upon the square of the vector dipole moment $\vec{\mu}$, the second sum represents an effective "dipolar" term. Equation (2.4) gives for each $\vec{\mu}_i$ that $\mu_{ix}^2 + \mu_{iy}^2 = 1$.

Using the result that $\mu_{ix}\mu_{iy} = 0$, we have

$$(\vec{\mu}_i \cdot \vec{\mu}_j)^2 = \frac{1}{2} \{ [(\mu_{ix}^2 - \mu_{iy}^2)(\mu_{jx}^2 - \mu_{jy}^2)] + 1 \}. \quad (\text{B2})$$

Using Eq. (B2) and adding a constant term to Eq. (B1) gives

$$\mathcal{H} = - \frac{1}{2} \sum_{i,j} J_{ij} (\mu_{ix}^2 - \mu_{iy}^2)(\mu_{jx}^2 - \mu_{jy}^2) - \sum K_{ij} (\vec{\mu}_i \cdot \vec{\mu}_j). \quad (\text{B3})$$

Defining

$$\xi_i = \frac{1}{2} \sum_j J_{ij} \langle |\mu_{jx}^2 - \mu_{jy}^2| \rangle, \quad (\text{B4})$$

where the angular brackets in Eq. (B4) represent thermal averages, we obtain,

$$\mathcal{H} = - \sum_i \xi_i (\mu_{ix}^2 - \mu_{iy}^2) - \sum_i \vec{E}_i \cdot \vec{\mu}_i - \sum_i (\vec{E}_0 \cdot \vec{\mu}_i) - \xi_0 \sum_i (\mu_{ix}^2 - \mu_{iy}^2), \quad (\text{B5})$$

where E_0 and E_1 are external and internal electric fields, ξ_0 is an external pressure or strain field. For the case when $K_{ij}=0$ and $\xi_0=0$, the longitudinal mean-field Hamiltonian is given by Eq. (2.5) of the text.

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