High-field electron mobility and temperature in bulk semiconductors

Vijay K. Arora
Department of Physics, King Saud University,
P.O. Box 2455, Riyadh, Saudi Arabia
(Received 11 June 1984)

An electron distribution function for acoustic-phonon scattering is described starting from first principles, for arbitrary values of the electric field. General expressions for the electron temperature and mobility are obtained using this distribution function. At high electric fields, the electron mobility is limited by the field-broadening effect which gives saturation of the drift velocity.

In the high-field regime, the intracollisional field effect due to the finite extent of an electron plays an important role.[1] Given this finite extent, the position of an electron is uncertain by an amount $\chi_D=\hbar/p$, with the associated time uncertainty $\tau_F=\hbar/(e\mathscr{E}\chi_D)$ obtained from the Heisenberg uncertainty principle, where $e\mathscr{E}\chi_D$ is the energy uncertainty of an electron in an electric field. In the low-field limit ($\mathscr{E}\rightarrow 0$), this field broadening $\hbar\tau_F^{-1}=e\mathscr{E}\chi_D$ is small and the electron transport is dictated by scattering interactions. But, in high electric fields, $\hbar\tau_F^{-1}$ may become very large. When $\hbar\tau_F^{-1}\sim\hbar\tau^{-1}$, where τ is the relaxation time, deviations from Ohmic transport are clearly visible.[1] In *n*-germanium, where acoustic-phonon scattering[2] is the dominant mechanism of scattering[3] (especially at intermediate temperatures), $h\tau^{-1}\sim 0.35$ meV can become comparable to $\hbar\tau_F^{-1}$ at a field of $\mathscr{E}\sim 700$ V/cm, an easily attainable field in modern devices.

Various approaches[4] (also, see Ref. 1 for important references) have been used to include higher-order terms in order to explain the experimentally observed results. Nag[4] has given an excellent review on the principles and basic assumptions on which earlier theories of hot carrier conduction have been developed. The concensus of the various theories[4] seems to be that the mobility in high electric fields is limited by the optical-phonon scattering which yields a saturation drift velocity. Various discrepancies associated with these theories are pointed out by Nag.[4] The author is of the opinion that these discrepancies arise due to the neglect of the field-broadening effect.

By determining the proper selection of states from the principle of equal *a priori* probabilities,[5] the distribution function for nondegenerate semiconductors is obtained as

$$f(\vec{p})=\exp[(\zeta-\epsilon)/k_BT-\delta\cos(\theta)] \ , \tag{1}$$

with

$$\delta=e\mathscr{E}l_a/k_BT \ , \tag{2}$$

$$l_a=\pi\hbar^4\rho_d u^2/m^{*2}E_1^2k_BT \ , \tag{3}$$

$$\zeta=k_BT[\ln(n_e/N_c)+\ln(\delta/\sinh\delta)] \ , \tag{4a}$$

$$N_c=2(2\pi m^*k_BT/h^2)^{3/2} \ . \tag{4b}$$

Here θ is the angle between the carrier vector $\vec{p}$ and the electric field $\vec{\mathscr{E}}$. ζ is the Fermi energy in an electric field (electrochemical potential), l_a the mean free path for acoustic phonons,[6] ρ_d the crystal density, u the longitudinal speed of sound, E_1 the deformation potential constant, m^* the electron effective mass, n_e the carrier concentration, and $\epsilon=p^2/2m^*$ the random thermal energy of the carriers. The parameter δ in Eq. (2) defines a transition from the Ohmic to the hot-electron regime. If δ is small (or equivalently $\hbar\tau_F^{-1}\ll\hbar\tau^{-1}$), the distribution function of Eq. (1) reduces to the linearized distribution function in the Ohmic limit. But, when δ is large (or $\delta\gg 1$) it is strongly protruded in the direction of carrier travel.

The average energy of an electron in the presence of an electric field is obtained by averaging $\langle\epsilon-\delta k_BT\cos\theta\rangle$, which when equated to $(\frac{3}{2})k_BT_e$ gives the effective electron temperature T_e as a function of an electric field:

$$T_e=T[1+(\tfrac{2}{3})\delta\mathscr{L}(\delta)] \ . \tag{5}$$

Here, $\epsilon=p^2/2m^*$ is the random carrier's kinetic energy and $V=-e\vec{\mathscr{E}}\cdot\vec{l}_a$ is the carrier's dipole potential energy in an electric field. $\mathscr{L}(\delta)=\coth(\delta)-\delta^{-1}$ is the Langevin function. In the Ohmic limit ($\mathscr{E}\rightarrow 0$), $T_e=T$ as expected. In the warm-electron regime ($\delta<1$), Eq. (5) gives $\Delta T/T=(T_e-T)/T=2\delta^2/9$; and in the hot-electron regime $\Delta T/T=2\delta/3$ is obtained. The experimental data[7] on various devices seems to follow this general trend.[7] The comparison of Eq. (5) with experiments is complicated because the results are very much sensitive to the method used to extract the electron temperature. Noise temperature measurements of Erlbach and Gunn[8] follow the trend of Eq. (5): a quadratic rise at low fields to a linear rise at high fields. Similarly, the mobility μ, using the distribution function of Eq. (1) is obtained as

$$\mu=\mu_0(3/\delta)\mathscr{L}(\delta) \ , \tag{6}$$

with

$$\mu_0=4el_a/3\sqrt{\pi}(2m^*k_BT)^{1/2} \ . \tag{7}$$

Equation (6) gives $\mu=\mu_0$ in the Ohmic limit ($\delta\rightarrow 0$), $\mu=\mu_0(1-\delta^2/15)$ in the warm-electron limit ($\delta<1$), and $\mu=3\mu_0/\delta$ in the hot-electron limit ($\delta\ll 1$).

In Figs. 1 and 2, we show the variation of relative mobility μ/μ_0 and relative temperature $\Delta T/T_0$ with normalized electric field $\delta=\mathscr{E}/\mathscr{E}^*$, where

$$\mathscr{E}^*=k_BT/el_a \tag{8}$$

is the critical field ($\delta=1$) at which onset of nonlinearity is expected to take place. It is clear from these figures that nonlinear effects come into play only as $\mathscr{E}\gtrsim\mathscr{E}^*\cdot\mathscr{E}^*$ increases as lattice temperature increases or mobility decreases. This is consistent with the earlier experimental data of Ryder and Shockley.[9]

 ©1984 The American Physical Society

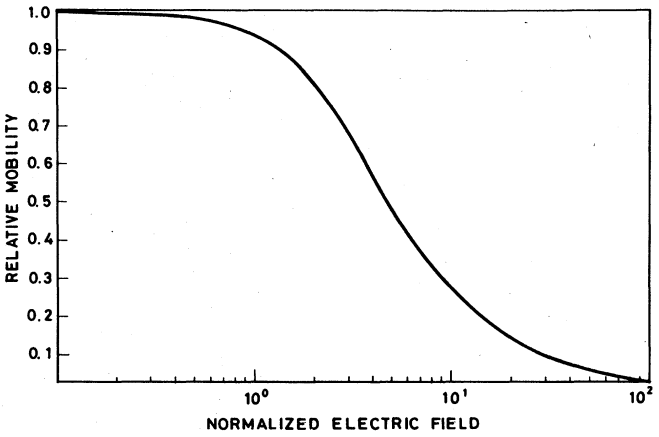

FIG. 1. Relative mobility vs normalized electric field $\delta = \mathscr{E}/\mathscr{E}^*$.

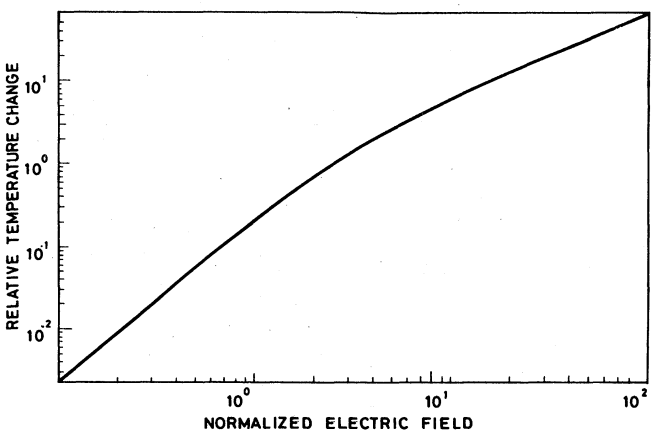

FIG. 2. Relative change in temperature $\Delta T/T$ of quasi hot electrons as a function of normalized electric field $\delta = \mathscr{E}/\mathscr{E}^*$.

To explain the importance of the field-broadening effect, we apply the results to the simple model of n-germanium, where anisotropic effects due to the band structure may be assumed to be absorbed in l_a which we obtain by comparing the electric field $\mathscr{E}_{1/2}$ at which mobility falls to half of its Ohmic value. $\delta = 4.78$ at $\mathscr{E} = \mathscr{E}_{1/2}$. At room temperature ($T = 300$ K) $\mathscr{E}^* = 0.59$ kV/cm corresponding to $l_a = 442$ nm. The relative mobilities obtained from Eq. (6) are 0.85, 0.62, 0.47, and 0.38 for $\mathscr{E} = 1$, 2, 3, and 4 kV/cm, respectively. These are in excellent agreement with the median experimental values of 0.82, 0.62, 0.46, and 0.38, respectively, compiled by Nag.[4] Similarly at 77 K, if we take the experimental data of Zücker,[10] for example, with $\mathscr{E}_{1/2} = 0.52$ kV/cm, the experimental values of μ/μ_0 are 0.52, 0.29, 0.16, and 0.10 at $\mathscr{E} = 0.5$, 1, 2, and 3 kV/cm, respectively. The corresponding theoretical values calculated from Eq. (6) are 0.51, 0.29, 0.16, and 0.10, respectively. In the warm-electron regime at room temperature, the warm-electron coefficient $\beta[\mu = \mu_0(1+\beta\mathscr{E}^2)]$ is found to be $\beta = (15\mathscr{E}^{*2})^{-1} = -1.98\times10^{-7}$ cm^2/V^2, which compares well with $\beta_{\text{expt}} = -1.5\times10^{-7}$ cm^2/V^2 reported by Gunn.[11] The saturation velocity $v_s = 6.2\times10^6$ cm/s so obtained agrees with experimental value[4] of $v_s = 5.5\times10^6$ cm/s and is virtually independent of scattering parameters:

$$v_s(\mathscr{E}) = (8k_BT/\pi m^*)^{1/2}(1-\delta^{-1}) \ , \tag{9}$$

which accounts for the small temperature variation of the saturation velocity. In n-Ge at room temperature $\delta = 7$ for $\mathscr{E} = 4$ kV/cm.

In order to explain the anisotropic effects in many-valley semiconductors, the effective mass tensor needs to be used in a manner similar to that included in Refs. 3 and 12. When optical-phonon scattering is also included, it effectively makes l_a smaller. But, at sufficiently high electric fields, the mobility becomes virtually independent of scattering parameters and is comparable to the thermal velocity of the carriers. The carriers are thus localized in a distance l_a, the mean free path, with conduction taking place from one localized site to the other by thermal hopping. The presence of the electric field thus establishes some order in the motion of the carriers, which is otherwise fully disordered.

To conclude, we have presented above a simplified distribution function which explains very well the general features of the high-field transport by including in it the field-broadening effect which is found to be very important at high electric fields in limiting the mobility.

[1]V. K. Arora, J. Appl. Phys. **54**, 824 (1983).
[2]H. N. Spector, Solid State Phys. **19**, 291 (1966).
[3]V. K. Arora, Phys. Rev. B **26**, 7046 (1982).
[4]B. R. Nag, Solid State Electron. **10**, 385 (1967).
[5]S. Zukotynski and W. Howlett, Solid State Electron. **21**, 35 (1978).
[6]E. M. Conwell, *High Field Transport in Semiconductors* (Academic, New York, 1967), p. 109.
[7]G. Bauer, in *Determination of Electron Temperatures and of Hot Electron Distribution,* edited by G. Hohler, Springer Tracts in Modern Physics, Vol. 74 (Springer-Verlag, Berlin, 1974).
[8]E. Erlbach and J. B. Gunn, Phys. Rev. Lett. **8**, 280 (1962).
[9]E. J. Ryder and W. Shockley, Phys. Rev. **81**, 139 (1951); W. Shockley, Bell Syst. Tech. J **30**, 990 (1951).
[10]J. Zücker, J. Phys. Chem. Solids **12**, 350 (1959).
[11]J. B. Gunn, Prog. Semicond. **2**, 213 (1957).
[12]V. K. Arora, C. Munera, and M. Jaafarian, J. Phys. Chem. Solids **38**, 469 (1977).