Polar optical interface phonons and Fröhlich interaction in double heterostructures

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The phonon dispersion relations and the Fröhlich interaction in polar double heterostructures are derived, by application of a modified image-charge ansatz. The interface phonons couple to form a symmetric and antisymmetric normal mode. The effective interaction is described by two-dimensional form factors, which represent a reasonable measure for the interaction strength. The theoretical results are in good agreement with recent magnetophonon experiments.

I. INTRODUCTION

In recent years there has been an increasing interest in the interaction between low-energy electrons and the elementary excitation of solids in the presence of interfaces. In the case of polar semiconductor layers the dispersion relation of polar-optical phonons and their effective interaction with two-dimensionally confined electrons can be strongly modified compared to the three-dimensional case. Additionally, to the restricted number of bulklike modes there appears a series of corresponding two-dimensional, so-called interface phonon modes.

In a fundamental paper Fuchs and Kliewer[1] derived the optical vibration modes in an ionic slab. They found a symmetrical and an antisymmetrical surface mode (decaying exponentially from the interface) and sinusoidal bulk LO phonon modes with nodes at the interfaces.

For a semiconductor-insulator structure and a completely two-dimensional electron gas (2D EG), the interaction Hamiltonian with interface optic phonons was derived by Tzoar.[2]

In this paper a generalized approach is presented for a double heterostructure (DHS) made of arbitrary polar semiconductors and for realistic electronic wave functions. The DHS represents an intermediate step between single heterostructures and superlattices. Therefore quite general physical features are derived.

In order to obtain the Fröhlich interaction in a DHS we apply the energy-loss method: The Poisson equation for the potential of a point charge moving in the DHS layer is solved by introducing a modified image-charge ansatz. The bulk dielectric functions of the two materials are used as an input. The classical result for the energy loss of the test charge is then compared with the quantum-mechanical rate of energy loss, yielding the interface phonon energies and the interaction strengths. Mapping the so-obtained local electron-phonon Hamiltonian with the electronic wave functions, the effective two-dimensional Fröhlich potentials are calculated.

The dependence of the interface phonon energies and the effective interaction on the phonon wave vector and the system parameters is discussed and used to explain recent magnetophonon experiments in 2D systems.[3]

II. CLASSICAL ENERGY LOSS

The DHS interfaces are taken parallel to the (x,y) plane at $z=\pm d/2$ and the bulk dielectric functions of the layer and the surrounding material are denoted by ϵ_1 and ϵ_2, respectively. The symmetry of the problem allows working in the $(q_\perp,z)$ space, which makes the calculations simple and leads to analytical results.

At first the classical rate of energy loss of a test charge moving with velocity $v_\perp$ in the DHS layer is calculated from the imaginary part of the potential $\phi(\vec{x},t)$:

$$\frac{dW}{dt}=\lim_{\vec{x}\to\vec{v}_\perp t} e\vec{v}_\perp\cdot\vec{\nabla}\phi(\vec{x},t)=2e\int d^2q_\perp\int d\omega\,\omega\,\mathrm{Im}\phi(q_\perp,\omega). \tag{1}$$

By a comparison with the quantum-mechanical expression for the energy loss:

$$-\frac{dW}{dt}=\frac{2\pi}{\hbar}\sum_q\omega_q\,|\langle H_{int}\rangle|^2\delta(\omega_q-q_\perp v_\perp)\,, \tag{2}$$

the phonon energies $\hbar\omega_q$ as well as the matrix elements $\langle H_{int}\rangle$ for one-phonon emission are obtained in a second step.

From Poisson's equation, the general relation between potential ϕ, charge density ρ, and dielectric function ϵ (including a small imaginary part $i\delta$) is given by

$$\phi(q,\omega)=4\pi\rho(q,\omega)/q^2[\epsilon(q,\omega)+i\delta]\,. \tag{3}$$

For a point charge moving in the $z=z_0$ plane with velocity $v_\perp$ one obtains

$$\phi(q_\perp,\omega,z-z_0)=\frac{e\delta(\omega-q_\perp v_\perp)}{8\pi^2q_\perp\epsilon(q,\omega)}e^{-q_\perp|z-z_0|}\,. \tag{4}$$

In a DHS this relation is modified due to the presence of the interfaces. The discontinuities of the dielectric lattice properties are described by introducing a modified image-charge ansatz for the potential to solve the Maxwell continuity conditions at $z=\pm d/2$. Based on symmetry considerations and analogous to the single-interface case in classical electrodynamics, four different image charges $\sigma_1(q_\perp),\ldots,\sigma_4(q_\perp)$ are introduced for a test charge at $|z_0|<d/2$:

$$\phi(q_\perp,\omega)=\frac{e\delta(\omega-q_\perp v_\perp)}{8\pi^2 q_\perp}\times\begin{cases}[(1+\sigma_1)e^{-q_\perp(z-z_0)}+\sigma_2 e^{-q_\perp(d+z+z_0)}]/(\epsilon_2+i\delta), & z>d/2\\ [e^{-q_\perp|z-z_0|}+\sigma_2 e^{-q_\perp(d+z+z_0)}+\sigma_3 e^{-q_\perp(d-z-z_0)}]/(\epsilon_1+i\delta), & |z|<d/2\\ [(1+\sigma_4)e^{-q_\perp(z_0-z)}+\sigma_3 e^{-q_\perp(d-z-z_0)}]/(\epsilon_2+i\delta), & z<-d/2\,.\end{cases} \tag{5}$$

As usual, the value and the position of the image charges depend on the position of the observer at z. With respect to the observer, they are always placed on the opposite side of the interface. For example, if $z>d/2$, the interface at $d/2$ is between the observer and the test charge and an image charge $\sigma_1(q_\perp)$ is set on the same place as the real charge. On the other hand, the test charge is between the observer and the interface at $(-d/2)$, and the corresponding image charge $\sigma_2(q_\perp)$ is placed at $(-d-z_0)$.

However, it is important to point out that the image charges $\sigma_1(q_\perp),\ldots,\sigma_4(q_\perp)$ are not classical image charges, since their value depends on the Fourier wave vector $q_\perp$ parallel to the interface. With this remaining degree of freedom in Eq. (5), the Maxwell continuity conditions for both the potential and the dielectric displacement at the interfaces can be fulfilled. In contrast to solutions with conventional image charges, which require an infinite series to describe two interfaces, this new approach allows exact solutions with a small number of "modified" image charges. After some algebraic manipulations they are obtained in the following form:

$$\sigma_1=\frac{[(\epsilon_2^2-\epsilon_1^2)-\gamma_+(\epsilon_2-\epsilon_1)^2]}{[(\epsilon_1+\epsilon_2)^2-\gamma^2(\epsilon_2-\epsilon_1)^2]}\,,$$
$$\sigma_4=\frac{[(\epsilon_2^2-\epsilon_1^2)-\gamma_-(\epsilon_2-\epsilon_1)^2]}{[(\epsilon_1+\epsilon_2)^2-\gamma^2(\epsilon_2-\epsilon_1)^2]}\,,$$
$$\sigma_2=-\sigma_4,\quad \sigma_3=-\sigma_1,\quad \gamma=\exp(-q_\perp d)\,,\quad \gamma_\pm=\exp[-q_\perp(d\pm 2z_0)]\,. \tag{6}$$

For the determination of the energy loss [Eq. (1)] only the imaginary part of the potential contributes, which is obtained by setting $z=z_0$ and using the relation $\mathrm{Im}[1/(x-i\delta)]=\pi\delta(x)$ for the dielectric function:

$$\mathrm{Im}\phi(q_\perp,\omega)=\frac{-e\delta(\omega-q_\perp v_\perp)}{8\pi q_\perp}\left[\frac{(1-\gamma_+-\gamma_-+\gamma^2)}{(1-\gamma^2)}\delta(\epsilon_1)+\frac{4\gamma\cosh^2(q_\perp z_0)}{(1+\gamma)}\delta[\epsilon_2(1+\gamma)+\epsilon_1(1-\gamma)]\right.$$
$$\left.+\frac{4\gamma\sinh^2(q_\perp z_0)}{(1-\gamma)}\delta[\epsilon_1(1+\gamma)-\epsilon_2(1-\gamma)]\right]. \tag{7}$$

It is seen that the test charge loses energy only for ω values where either $\epsilon_1(\omega)$ or the mixtures of $\epsilon_1(\omega)$ and $\epsilon_2(\omega)$ in the δ functions are equal to zero. These frequencies are attributed to the phonon normal modes of the system. The prefactors before the δ functions correspond to interaction strengths.

III. LOCAL PHONON STRUCTURE AND ELECTRON-PHONON INTERACTION

The classical DHS potential Eq. (7) contains all the information on the phonon structure and the electron-phonon interaction. The first term in the large parentheses corresponds to the interaction with bulklike phonons of the layer material (ϵ_1). The restricted number of modes is expressed by the prefactor. The second and the third term represent interface phonon modes, with a weighted mixture of both dielectric functions (ϵ_1,ϵ_2) appearing in the δ functions. The prefactors indicate that the modes at the two interfaces couple to form a symmetric and an antisymmetric normal mode.

The quantum-mechanical expression for the electron-phonon interaction is therefore written in the following form, which already includes the specific phonon wave functions (for $|z|<d/2$):

$$V_{\mathrm{ph}}^{(2s)}=\sum_{q_\perp}e^{i\vec{q}_\perp\vec{x}}(e^{-q_\perp(d/2-z)}+e^{-q_\perp(d/2+z)})$$
$$\times v_{2s}b_{2s}/\sqrt{1+\gamma}+\mathrm{H.c.}\,, \tag{8}$$

$$V_{\mathrm{ph}}^{(2a)}=\sum_{q_\perp}e^{i\vec{q}_\perp\vec{x}}(e^{-q_\perp(d/2-z)}-e^{-q_\perp(d/2+z)})$$
$$\times v_{2a}b_{2a}/\sqrt{1-\gamma}+\mathrm{H.c.}\,, \tag{9}$$

$$V_{\mathrm{ph}}^{(b)}=\sum_{q_\perp}e^{i\vec{q}_\perp\vec{x}}\sum_{n=1}^{\infty}\sin[(z/d+\tfrac{1}{2})n\pi]v_b b_3+\mathrm{H.c}\,, \tag{10}$$

where b_{2s}, b_{2a}, and b_3 denote destruction operators of the interface phonons and the bulk mode. The bulklike potential $V_{\mathrm{ph}}^{(b)}$ is characterized by a restricted number of modes with nodes at $z=\pm d/2$. $V_{\mathrm{ph}}^{(2s)}$ and $V_{\mathrm{ph}}^{(2a)}$ describe the symmetric and the antisymmetric two-dimensional DHS interface phonon modes.

For simplicity, we treat in the following only the situation when the layer as well the surrounding material are of single-mode type, characterized by

$$\epsilon_{1,2}(\omega)=\epsilon_{1,2}^{\infty}(\hbar^2\omega^2-L_{1,2}^2)/(\hbar^2\omega^2-T_{1,2}^2)\,. \tag{11}$$

ϵ^∞, L, and T denote the high-frequency dielectric constant, the longitudinal, and the transverse bulk optic phonon energy. In this case one obtains two interface phonon modes corresponding to the two bulk materials involved, with a symmetric and an antisymmetric form. The classical potential [Eq. (7)] is inserted in Eq. (1) for the energy loss and the ω integration is performed:

$$-\frac{dW}{dt}=\frac{e^2}{8\pi\hbar}\int\frac{d^2q_\perp}{q_\perp}\left[\delta(L_1-\hbar q_\perp v_\perp)(L_1^2-T_1^2)\frac{(1-\gamma_+-\gamma_-+\gamma^2)}{(1-\gamma^2)}\right.$$
$$\left.+4\gamma\sum_{\pm}\left[\delta(\omega_\pm^s-q_\perp v_\perp)f_\pm^s\frac{\cosh^2(q_\perp z_0)}{(1+\gamma)}+\delta(\omega_\pm^a-q_\perp v_\perp)f_\pm^a\frac{\sinh^2(q_\perp z_0)}{(1-\gamma)}\right]\right], \quad (12)$$

where the interface phonon energies $\hbar\omega_\pm^{s,a}$ are given by

$$(\hbar\omega_\pm^i)^2=\frac{\{p\pm[p^2-(\epsilon_1^i+\epsilon_2^i)(\epsilon_1^iL_1^2T_2^2+\epsilon_2^iL_2^2T_1^2)]^{1/2}\}}{(\epsilon_1^i+\epsilon_2^i)}, \quad (13)$$

$$p=[\epsilon_1^i(T_2^2+L_1^2)+\epsilon_2^i(T_1^2+L_2^2)]/2, \quad i=s,a \quad (14)$$

$$\epsilon_1^s=\epsilon_1^\infty(1-\gamma), \quad \epsilon_1^a=\epsilon_1^\infty(1+\gamma),$$
$$\epsilon_2^s=\epsilon_2^\infty(1+\gamma), \quad \epsilon_2^a=\epsilon_2^\infty(1-\gamma), \quad (15)$$

$$f_\pm^i=\pm\frac{[\hbar^2(\omega_\pm^i)^2-T_1^2][\hbar^2(\omega_\pm^i)^2-T_2^2]}{\hbar^2[(\omega_+^i)^2-(\omega_-^i)^2](\epsilon_1^i+\epsilon_2^i)}. \quad (16)$$

A comparison of Eq. (12) with the quantum-mechanical energy loss [Eq. (2)] using the electron-phonon interaction [Eqs. (8)—(10)] yields the following two-dimensional interaction strengths:

$$v_{2i}^\pm=(e^2f_\pm^i/4\hbar\omega_\pm^iAq_\perp)^{1/2}, \quad (17)$$

where the normalization area is denoted by A. The index $(\pm)$ describes the two interface phonon modes corresponding to the two bulk materials, while the index (i) denotes the symmetric or antisymmetric form.

It can be seen that the two-dimensional phonon energies $\hbar\omega_\pm^i$ (as well as the interaction strengths $v_{2i}^\pm$) depend on the layer thickness d and on the phonon momentum $q_\perp$ parallel to the interface. This is a new feature compared to the practically constant dispersion relation of three-dimensional optic phonons.

The bulk interaction v_b is given by

$$v_{b,n}^2=e^2L_1(1-T_1^2/L_1^2)/Ad\epsilon_1^\infty[q_\perp^2+(n\pi/d)^2], \quad (18)$$

which, as expected, is quite similar to the three-dimensional Fröhlich interaction.

IV. TWO-DIMENSIONAL EFFECTIVE INTERACTION

In a 2D system the motion of the electrons normal to the interface is restricted and practically replaced by discrete transitions between the electric subbands. In most cases only scattering within the zeroth subband or between the zeroth and the first electric subband is important. Therefore it is useful to define an effective 2D interaction for the most important cases, which means that the interaction $V_{\rm ph}^\mu$ is written as a sum over 2D momentum transfers, with appropriate form factors:

$$V_{\rm ph}^\mu=\sum_{q_\perp}e^{i\vec{q}_\perp\cdot\vec{x}}v_{2D}F_\mu(q_\perp d), \quad (19)$$

$$v_{2D}^2=e^2L(1-T^2/L^2)/4Aq_\perp\epsilon^\infty. \quad (20)$$

The index μ includes the specific transition (e.g., 0-0) and the type of phonon involved. v_{2D} is the idealized Fröhlich interaction of a point charge moving in a plane and coupling to unperturbed three-dimensional phonons. The form factors F_μ carry all the information on the z dependence of the system.

The electronic wave functions in a DHS are given by

$$\psi_r(z)=\sqrt{2/d}\sin[(z/d+\tfrac{1}{2})r\pi]. \quad (21)$$

Mapping the local electron-phonon interaction Eqs. (8)—(10) with the electronic wave functions, we obtain the effective 2D potentials and form factors F_μ. It is evident that only the symmetric DHS mode contributes to the (0-0) transition, whereas the antisymmetric mode introduces a coupling between the zeroth and the first electric subband:

$$F_{(0\text{-}0)}^\pm=\frac{2(1-\gamma)}{\alpha(1+\alpha^2/4\pi^2)}\left[\frac{f_\pm^s\epsilon^\infty L}{(L^2+T^2)\hbar\omega_\pm^s(1+\gamma)}\right]^{1/2}, \quad \alpha=q_\perp d, \quad (22)$$

$$F_{(0\text{-}1)}^\pm=\left[\frac{f_\pm^a\epsilon^\infty L}{(L^2-T^2)\hbar\omega_\pm^a(1-\gamma)}\right]^{1/2}\times2\alpha(1+\gamma)\left[\frac{1}{\alpha^2+\pi^2}-\frac{1}{\alpha^2+9\pi^2}\right]. \quad (23)$$

For the bulklike phonons in the layer, the effective interaction can be approximated within a few percent by the modes with $n=1$ (0-0) and $n=2$ (0-1):

$$F_{(0\text{-}0)}^b=\frac{16}{3\pi}[\alpha/(\alpha^2+\pi^2)]^{1/2}, \quad (24)$$

$$F_{(0\text{-}1)}^b=\frac{64}{15\pi}p\alpha/[\alpha^2+4\pi^2)]^{1/2}. \quad (25)$$

For a comparison, the effective interaction with undisturbed three-dimensional phonons is calculated as

$$F_{(0\text{-}0)}^{3D}=\left[\frac{2}{\alpha}+\frac{\alpha}{\alpha^2+4\pi^2}-\frac{2(1-\gamma)}{\alpha^2(1+\alpha^2/4\pi^2)^2}\right]^{1/2}. \quad (26)$$

V. RESULTS AND DISCUSSION

In this paper a theoretical approach has been developed to determine the properties of LO phonons in polar double heterostructures. The phonon energy spectrum and the Fröhlich interaction strength are obtained by calculat-

ing the classical energy loss of a test charge. A comparison with the quantum-mechanical expression for the energy loss enables one to assign the different phonon energies and matrix elements to specific normal modes. The introduction of a modified image-charge ansatz simplifies the analytical calculations.

The phonon structure of a layered system is strongly modified as compared to a 3D system. The number of bulklike modes is restricted to those with zero amplitude at the material boundaries. Two-dimensional phonons are created at the interfaces.

In a DHS the 2D modes at the two interfaces couple to form a symmetric and an antisymmetric normal mode. The dependence of the DHS phonon energies on the wave vector $q_\perp$ is a new feature compared to the constant three-dimensional dispersion relation. The influence of the two dielectric materials (ϵ_1,ϵ_2) is weighted by the factor $\gamma=\exp(-q_\perp d)$, and the relative strength is reversed for the symmetric (antisymmetric) mode, as can be seen from Eq. (7).

It is interesting to compare our results with the calculations of Fuchs and Kliewer,[1] who calculated the potential vibration modes of an ionic slab. Using both lattice dynamics and using classical electrodynamics, they derived a set of coupled integral equations for the optic phonons. A solution was obtained by using trial wave functions: a symmetrical and an antisymmetrical surface mode (decaying exponentially from the interface) and sinusoidal bulk LO phonon modes with nodes at the interfaces. Fuchs and Kleiwer also found $q_\perp$-dependent surface mode dispersion relations.

In contrast to the calculations of Fuchs and Kliewer our approach uses the energy-loss method (described above), applying a modified image-charge ansatz for the potential of a test charge. The quantum-mechanical Fröhlich interaction Hamiltonian is directly derived. As an input bulk dielectric functions of the layer and the surrounding material are used. In this approach, the possibility of working in $(q_\perp,z)$ space makes the calculations quite simple, for arbitrary bulk dielectric materials. However, it should be mentioned that an extension of the method introduced by Fuchs and Kliewer is expected to yield analogous results.

In the following the theory is applied to a GaInAs-InP double heterostructure. The interface phonon energies and the 2D form factors for the Fröhlich interaction [Eq. (19)] are compared. The results are used to explain recent magnetophonon experiments.

A. Application to $Ga_{1-x}In_xAs$-InP

The numerical calculations are performed for a $Ga_{0.5}In_{0.5}As$ layer embedded in InP. Although it is well known[4] that GaInAs is a two-mode material, the lower mode is neglected for simplicity. This is justified since for the given composition it couples much weaker to the electrons than the upper mode. The material constants are taken to be $L_1=34$, $T_1=31.7$, $L_2=43$, $T_2=37.6$ (all in meV), $\epsilon_1^\infty=11.35$, and $\epsilon_2^\infty=9.56$.

Generally it can be said that the properties of the DHS phonons depend essentially on the dimensionless parameter $\alpha=q_\perp d$ describing the relation of the layer thickness to the phonon wavelength ($\sim 1/q_\perp$) parallel to the interface.

Figure 1 shows the phonon energy spectrum as a function of α, revealing the characteristical features of DHS phonons. The modes can be separated into an InP-like [Fig. 1(a)] and a GaInAs-like sector [Fig. 1(b)]. The solid lines correspond to the bulk LO and TO phonon energies (the InP bulk mode, however, does not couple directly to the electrons in the DHS and is shown only for completeness). The dashed lines indicate the symmetric and the dotted lines the antisymmetric modes, respectively.

For $\alpha \ll 1$, i.e., for long phonon wavelengths or small layer widths, the symmetric DHS phonon corresponding to the layer material (GaInAs) is damped down to the TO phonon energy T_1. The InP mode, on the other hand, approaches the bulk value L_2. This can be interpreted as a transition from 2D to 3D interaction with the surrounding material for long wavelengths.

The antisymmetric modes show the opposite behavior: For $\alpha \to 0$, the mode corresponding to the layer material converges to its bulk value, whereas the InP mode is damped down to the TO phonon energy T_2. This "antisymmetric" behavior is evident from the second and the third δ function in Eq. (7), which have been discussed above.

For short wavelengths ($\alpha \to \infty$) the DHS phonon energies converge to a constant value, which is the single interface limit. This behavior is easily explained by the ex-

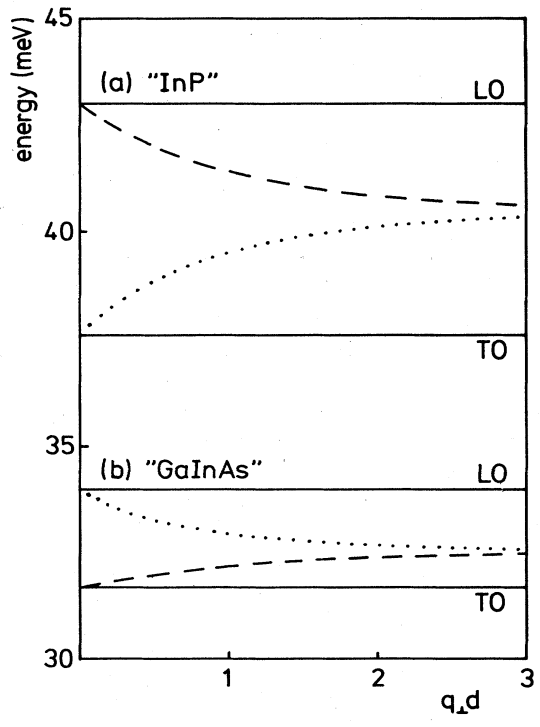

FIG. 1. DHS phonon dispersion relations versus $q_\perp d$, for a GaInAs-InP structure and correspondingly with an (a) InP and a (b) GaInAs phonon sector. The dashed lines indicate the symmetric and the dotted lines the antisymmetric interface phonon modes.

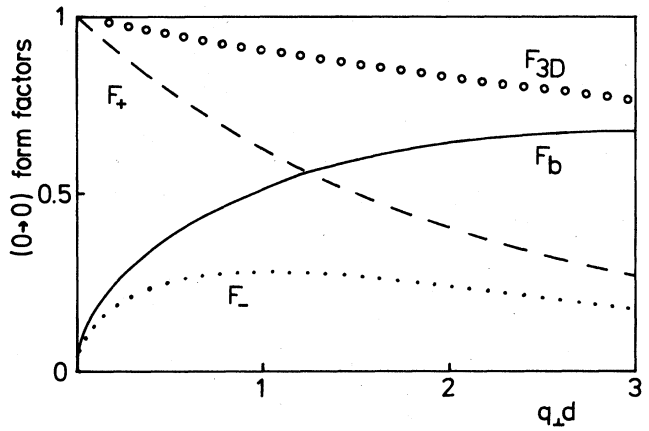

FIG. 2. Symmetric two-dimensional form factors $F^{\mu}_{(0\text{-}0)}$ for the Fröhlich interaction versus $q_{\perp}d$. The circles indicate the interaction with 3D phonons, the solid line the bulk GaInAs layer phonons, the dashed and the dotted lines the InP-like and GaInAs-like interface phonon interaction.

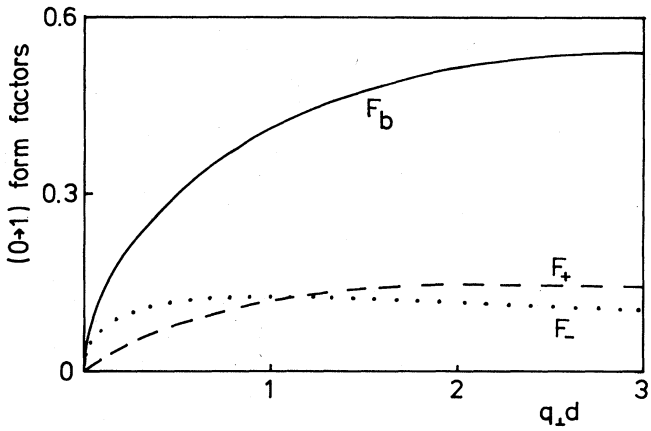

FIG. 3. Asymmetric two-dimensional form factors $F^{\mu}_{(0\text{-}1)}$ for the Fröhlich interaction versus $q_{\perp}d$. The solid line indicates the interaction with GaInAs bulk phonons, the dashed and the dotted line the interaction with InP and GaInAs antisymmetric interface phonons.

ponential decay of the 2D phonon amplitude with the distance from the interface, which implies decreasing influence of the second interface for $\gamma=\exp(-q_{\perp}d)\rightarrow 0$.

Figure 2 shows the symmetric 2D form factors $F^{\mu}_{(0\text{-}0)}$ as defined in Sec. IV, which represent a reasonable measure for the effective coupling strength. For comparison, the circles indicate the corresponding interaction with 3D phonons.

The interaction with the InP-like mode (F_+) approaches the 3D value for $\alpha\rightarrow 0$, describing the 2D→3D transition. For the GaInAs modes it can be seen that over the whole regime the bulk interaction (solid line, F_b) is considerably stronger than the interface phonon interaction (dotted line, F_-). This is due to the fact that the DHS phonon energy is closer to the TO phonon energy, which corresponds to weaker polarity.

For $\alpha>1.5$, the interaction with the layer bulk phonons becomes relatively larger than the coupling to interface modes from the surrounding material. For still higher phonon momenta, the interaction with bulk layer phonons approaches the 3D value, whereas the interface form factors decrease more rapidly. For absolute estimations, however, it should be mentioned that in the total interaction strength also the basic polarity of the material enters via v_{2D}. The form factors describe the general properties and the influence of the third space direction.

In Fig. 3 the antisymmetric form factors for the intersubband coupling ($F^{\mu}_{(0\text{-}1)}$) are shown as a function of α. Over the whole regime, the coupling with GaInAs bulk modes (solid line) is much stronger than the interaction with the two GaInAs (dotted line) and InP (dashed line) interface phonons. In contrast to the intrasubband interaction (Fig. 2), the (0-1) form factors $F_{\pm}$ are of the same order.

B. Magnetophonon effect

The magnetophonon effect describes the resonant optical phonon scattering between Landau levels. It represents an exact tool for the determination of polar-optical phonon energies in solids. Tsui *et al.*[5] were the first to observe the magnetophonon effect in 2D electron systems. The experimental results were quantitatively well explained by the theory of Lassnig and Zawadzki.[6]

Portal *et al.*[3] detected two series of magnetophonon oscillations in $Ga_{0.47}In_{0.53}As$-InP superlattices with $d=80$, 100, and 150 Å and InP layers of 400 Å. For the lower "GaInAs" series they found $\hbar\omega=33.6$ (32.9) meV for a 150 (80) Å sample. The energy decrease for lower d values can be well explained by the increasing influence of interface phonons for $d\rightarrow 0$. One observes a mixture of the bulk and interface phonon interaction, which is weighted by the symmetric form factors shown in Fig. 2.

For the higher "InP" mode the energy was found around 43 meV. The coupling mechanism of InP-like modes from the surrounding material into the layer has been well explained in the present work. However, theoretically a somewhat lower interface phonon energy is expected,[7] as can be seen from Fig. 1(a). The remaining discrepancy is possibly due to an incorrect description of nonparabolicity.

The asymmetric phonon modes can, in principle, only be detected in intersubband experiments, but have to our knowledge not been observed until now.

The effect of interface grading or surface roughness does not essentially alter the results obtained for the ideal interface, at least for phonon wavelengths larger than the irregularity dimension. In addition, the penetration of the electronic wave function into the InP, which allows a direct interaction with InP bulk modes, is weak since the bulk phonon amplitude close to the interface is very small.

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