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Quantum critical behavior in the presence of a random field

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Using the generalized Ginzburg-Landau-Wilson Hamiltonian with a random field, we have shown the absence of classical to quantum crossover near zero temperature. Our finding suggests that the critical exponents of a quantum system with a random field at $T=0$ can be obtained from those of the corresponding pure system by a dimensionality shift from d to $d-(\sigma+z)$, where σ is related to the range of interaction and z is the characteristic exponent of a given quantum system.

Quenched random fields can drastically modify the second-order phase transition and change the critical exponents.[1-5] If the interaction is short range, a random field shifts the upper critical dimensionality from 4 to 6, and the critical properties of a d-dimensional system in the presence of a random field are identical to those of a $(d-2)$-dimensional pure system. For systems with continuous symmetry and $m \geqslant 2$ (m is the number of components of order parameter), a random field destroys the long-range ordering for all $d \leqslant 4$ (Refs. 1 and 6). For $m=1$, there still exist controversial arguments regarding the lower critical dimensionality, although at the present it seems to be 2 (Ref. 7). Recently, it has been shown that in the case of transverse Ising model, a random field destroys the classical to quantum crossover.[8,9] Similar behavior of quantum effects has been discussed for the quantum Bose gas with a random field.[10]

In this Rapid Communication we show that a quenched random field can strongly affect the critical behvaior of quantum systems. Here we present the general arguments that a quenched random field can always remove the classical to quantum crossover at $T \rightarrow 0$, and that the critical exponents of a quantum system at $T=0$ can be obtained from those of a pure quantum system by the dimensionality shift from d to $d-(\sigma+z)$.

We begin with the generalized Ginzburg-Landau-Wilson (GLW) Hamiltonian for a m-component spin field plus a (small) time-independent random field $\vec{h}(\vec{r})$:

$$\bar{H} = -\frac{1}{2}\int_\lambda (r+q^\sigma+|\omega|^x q^{-y})\,\vec{\sigma}(\lambda)\cdot\vec{\sigma}(-\lambda) - \frac{u_0}{\beta}\int_{\lambda_1}\int_{\lambda_2}\int_{\lambda_3}\vec{\sigma}(\lambda_1)\cdot\vec{\sigma}(\lambda_2)\,\vec{\sigma}(\lambda_3)\cdot\vec{\sigma}(-\lambda_1-\lambda_2-\lambda_3) + \sqrt{\beta}\int_\lambda \vec{h}(\vec{q})\cdot\vec{\sigma}(\lambda)\delta_{\omega,0} \; , \tag{1}$$

where $\beta = 1/k_BT$, $\lambda=(\vec{q},\omega)$, $\omega = 2\pi s/\beta$ with $s=0, \pm 1, \pm 2, \ldots$ is the Matsubara frequency,

$$\int_\lambda = (2\pi)^{-d}\int d^dq \sum_\omega \; ,$$

and $\vec{h}(\vec{q})$ and $\vec{\sigma}(\vec{q},\omega)=\vec{\sigma}(\lambda)$ are, respectively, the Fourier transforms of $\vec{h}(\vec{r})$ and the (real) spin field $\vec{\sigma}(\vec{r},\tau)$ with $0 \leqslant \tau \leqslant \beta$. A quantum system is specified by σ for the range of interaction ($0<\sigma\leqslant 2$ for short-range forces), by $x \geqslant 1$, $y \geqslant 0$, and $z=(\sigma+y)/x$ for the dynamic exponent, and by m for the number of components. The Hamiltonian (1) without the random field has been discussed by Hertz.[11]

In this paper we consider the quenched random field with the properties $\langle h_i(\vec{q})\rangle_{\rm av}=0$ and

$$\langle h_i(\vec{q})h_j(\vec{q}\,')\rangle_{\rm av} = h^2\delta_{ij}\hat{\delta}(\vec{q}+\vec{q}\,')$$

for $i,j=1,2,\ldots,m$, where $\hat{\delta}(\vec{q})=(2\pi)^d\delta^d(\vec{q})$. Using the replica trick[12] and performing the average, we obtain from (1) the effective Hamiltonian

$$\bar{H}_{\rm eff}(\{\vec{\sigma}^\alpha\}) = -\frac{1}{2}\int_\lambda \sum_{\alpha\alpha'}[(r+q^\sigma+|\omega|^x q^{-y})\delta_{\alpha\alpha'} - \beta h^2\delta_{\omega,0}]\,\vec{\sigma}^\alpha(\lambda)\cdot\vec{\sigma}^{\alpha'}(-\lambda) - \frac{u_0}{\beta}\int_{\lambda_1}\int_{\lambda_2}\int_{\lambda_3}\sum_\alpha \vec{\sigma}^\alpha(\lambda_1)\cdot\vec{\sigma}^\alpha(\lambda_2)\,\vec{\sigma}^\alpha(\lambda_3)\cdot\vec{\sigma}^\alpha(-\lambda_1-\lambda_2-\lambda_3) \; , \tag{2}$$

where $\alpha=1,2,\ldots,n$ is the replica index. The propagator of the replicated system is given by

$$\langle\sigma_i^\alpha(\lambda)\sigma_j^{\alpha'}(\lambda')\rangle_0 = (r+q^\sigma+|\omega|^x q^{-y})^{-1}[\delta_{\alpha\alpha'} + \beta h^2(r+q^\sigma+|\omega|^x q^{-y} - n\beta h^2\delta_{\omega,0})^{-1}\delta_{\omega,0}]\delta_{ij}\hat{\delta}(\lambda+\lambda') \; , \tag{3}$$

where $\langle\cdots\rangle_0$ is an average over the quadratic part of (2).

We will use the Wilson-Kogut renormalization group[13,14] to construct the recursion relations, with the rescaling of wave vectors and frequencies $q=q'/b$, $\omega=\omega'/b^z$, $\beta=b^z\beta'$, and

$$\vec{\sigma}(\vec{q},\omega) = b^{(d+2-\eta)/2}\vec{\sigma}\,'(\vec{q}\,',\omega') \; ,$$

where $b \geqslant 1$ is the spatial rescaling factor. Since for $d>2\sigma-z$, u_0 is irrelevant, but h^2 is relevant and the recursion relations contain the product term u_0h^2, we introduce a new variable $v_0=u_0h^2$. In the lowest order of ϵ expansion, after taking the limit $n\to 0$ and setting $\eta=2-\sigma$, we obtain the recursion relations

$$r'=b^{\sigma}[r+4(m+2)u_0f_1+4(m+2)v_0f_2] \ , \tag{4a}$$

$$v_0'=b^{3\sigma-d}[v_0-4(m+8)u_0v_0f_3-8(m+8)v_0^2f_4] \ , \tag{4b}$$

$$u_0'=b^{2\sigma-(d+z)}[u_0-4(m+8)u_0^2f_3-8(m+8)u_0v_0f_4] \ , \tag{4c}$$

$$\beta'=b^{-z}\beta \ , \tag{4d}$$

where, with the notation $\int_p^>$ for integration in a domain $1/b<|\vec{p}|<1$,

$$f_{2s-1}=\frac{1}{\beta}\int_p^> \sum_{\omega}(r+p^{\sigma}+|\omega|^x p^{-y})^{-s}; \ s=1,2 \ , \tag{5a}$$

$$f_{2s}=\int_p^> (r+p^{\sigma})^{-s-1}; \ s=1,2 \ . \tag{5b}$$

Now we consider the cases $T\neq 0$ and $T=0$ separately.

A. $T\neq 0$ case

It is readily seen that for any finite temperature, after carrying out renormalization l times, the spacing between the Matsubara frequencies $\Delta\omega=2\pi/\beta(l)$ tends to ∞ as $l\to\infty$. Consequently, only the $\omega=0$ contributes to the critical behavior. In terms of the new variables $\tilde{u}=u_0/\beta$ and $v=(u_0/\beta)(\beta h^2)$, and setting $b=e^l$, we obtain the differential equations in the asymptotic form for $l>>1$

$$\frac{dr}{dl}=\sigma r+4K_d(m+2)[\tilde{u}(1+r)+v](l+r)^{-2} \ , \tag{6a}$$

$$\frac{dv}{dl}=\epsilon v-4K_d(m+8)v[\tilde{u}(1+r)+2v](1+r)^{-3} \ , \tag{6b}$$

$$\frac{d\tilde{u}}{dl}=(\epsilon-\sigma)\tilde{u}-4K_d(m+8)\tilde{u}[\tilde{u}(1+r)+2v](1+r)^{-3} \ , \tag{6c}$$

where $K_d=2^{1-d}\pi^{-d/2}/\Gamma(d/2)$ and $\epsilon=3\sigma-d$.

These equations are the same as those obtained for classical random field systems.[2] $\tilde{u}$ is irrelevant if $d>2\sigma$. For $d>3\sigma$ the Gaussian fixed point is stable. When $2\sigma<d<3\sigma$, the critical behavior is controlled by the random fixed point at

$$r^*=-\frac{m+2}{2\sigma(m+8)}\epsilon, \quad v^*=\frac{1}{8K_d(m+8)}\epsilon, \text{ and } \tilde{u}^*=0 \ ;$$

with the eigenvalues

$$\lambda_r=\sigma-\frac{m+2}{m+8}\epsilon, \quad \lambda_v=-\epsilon, \text{ and } \lambda_{\tilde{u}}=-\sigma \ ,$$

and the critical exponents can be obtained from the pure system by a dimensionality shift from d to $d-\sigma$. For $\sigma=2$ this is true in all orders of ϵ expansion around $d=6-\epsilon$ dimensions.[3-5] However, for $\sigma\neq 2$, this is known not to be the case to $O(\epsilon^2)$ as proved in Ref. 4. The modification of the scaling laws, i.e., a shift $d\to d+\lambda_{\tilde{u}}$ in the scaling relation $\nu d=2-\alpha$, is the only effect of $\tilde{u}$, which is a dangerous irrelevant variable. Therefore, in the presence of a random field, for any $T\neq 0$, the quantum fluctuations are ineffective as well as the thermal fluctuations.

B. $T=0$ case

In the limit $T\to 0$, ω becomes a continuous variable and all the Matsubara frequencies contribute to the critical behavior. However, we obtain the same recursion relations for r', u_0', and v_0' as those given by (4a)–(4c), but with the $1/\beta\sum_\omega$ in (5a) replaced by $\int d\omega/2\pi$. Therefore, u_0 always decays to 0 for $2\sigma-z<d<3\sigma$, and the random fixed point r^*, v^*, and u^* is the same as that for the classical system with a random field. Thus, the random field destroys dimensional crossover as $T\to 0$, and the asymptotic critical behavior of a quantum random field system stays the same as that of a classical one.

For the rest of this paper, we will present quantitative discussions separately for the case ($x=2$; $y=0$; $z=\sigma/2$), where we do not require a cutoff of the Matsubara frequencies, and for the case ($x=1$; $y=0,1$; $z=\sigma+y$), where a cutoff of frequencies is usually imposed.

(1) The differential recursion relations for $x=2$, $y=0$, and $z=\sigma/2$ can be derived as follows:

$$\frac{dr}{dl}=\sigma r+2K_d(m+2)[u(1+r)^{-1/2}+2v^2(1+r)^{-2}] \ , \tag{7a}$$

$$\frac{dv}{dl}=\epsilon v-K_d(m+8)v[u(1+r)^{-3/2}+8v(1+r)^{-3}] \ , \tag{7b}$$

$$\frac{du}{dl}=[\epsilon-(z+\sigma)]u-K_d(m+8)u[u(1+r)^{-3/2}+8v(1+r)^{-3}] \ , \tag{7c}$$

with the initial values $r(0)=r$, $v(0)=v_0$, and $u(0)=u_0$. We should point out that in the absence of the random field, the above equations are slightly different from their $d+z$ classical counterparts. Nevertheless, to the leading order of the $\tilde{\epsilon}=2\sigma-(d+z)$ expansion, they lead to the same critical behavior.

Equations (7a)–(7c) have a Gaussian fixed point with the eigenvalues $\lambda(G)_r=\sigma$, $\lambda(G)_v=\epsilon$, and $\lambda(G)_u=\epsilon-(z+\sigma)$, as well as a random fixed point with the eigenvalues $\lambda_r=\sigma-[(m+2)/(m+8)]\epsilon$, $\lambda_v=-\epsilon$, and $\lambda_u=-(z+\sigma)$. In the absence of the random field, (7a)–(7c) yield precisely the same eigenvalues for the Gaussian and the nontrivial fixed points but with $\epsilon\to\tilde{\epsilon}$ and $v\to u$, though the positions of the fixed points and the eigenvectors are different.

Therefore, we conclude that the critical exponents for a quantum random field system at $T=0$ can be obtained from a pure system by a dimensionality shift from d to $d-(\sigma+z)$. Simultaneously, the scaling laws have to be modified with the same shift since u is a dangerous irrelevant variable. This conclusion is valid for dimensionalities $2\sigma<d+z<3\sigma$ and arbitrary values of m. Perhaps we should mention that regarding the lower critical dimensionality, the above calculations suggest a value $\sigma+z$ for random field quantum systems.

(2) For $x=1$, $z=\sigma+y$ with $y=0, 1$, we can follow the discussion of Hertz on the paramagnon model (Ref. 11). Assuming a cutoff of frequencies $0<|\omega|<1$ and $0<|\vec{q}|<1$, in each step of renormalization we scale $\omega'=\omega e^{zl}$ and $q'=qe^l$, and take integration first over a horizontal strip ($|\omega|\simeq 1; 0<|\vec{q}|<1$) and then over a vertical strip

($|\vec{q}| \simeq 1; 0 < |\omega| < 1$). This yields

$$\frac{dr}{dl} = \sigma r + 4K_d(m+2)[uF_1(r,\sigma,z) + v(1+r)^{-2}] \ , \qquad (8a)$$

$$\frac{dv}{dl} = \epsilon v - 4K_d(m+8)v[uF_2(r,\sigma,z) + 2v(1+r)^{-3}] \ , \qquad (8b)$$

$$\frac{du}{dl} = [\epsilon - (\sigma+z)]u - 4K_d(m+8)u[uF_2(r,\sigma,z) + 2v(1+r)^{-3}] \ , \qquad (8c)$$

where

$$F_1(r,\sigma,z) = \frac{z}{\pi}\int_0^1 dx\, x^d[x(x^\sigma + r) + 1]^{-1} + \frac{1}{\pi}\ln\left(\frac{2+r}{1+r}\right) \ , \qquad (9a)$$

$$F_2(r,\sigma,z) = \frac{z}{\pi}\int_0^1 dx\, x^{d+1}[x(x^\sigma + r) + 1]^{-2} + \frac{1}{\pi}\frac{1}{(1+r)(2+r)} \ . \qquad (9b)$$

with the initial values $r(0) = r$, $v(0) = v_0$, $u(0) = u_0$, and $z = 1 + \sigma$.

As in the previous case, here the random fixed point also exists and is the same as the one for the classical random field system. An analysis of the above equations again shows that the critical exponents for the quantum random field system are the same as those for the pure quantum system after a dimensionality shift from d to $d - (\sigma+z)$, although there is a difference in the positions of the fixed points and the eigenvectors.

In this paper we have studied the critical behavior of quantum systems under the influence of a quenched random field. Our results on the absence of classical to quantum crossover as $T \to 0$ and the dimensionality shift by $\sigma + z$ can be applied to those quantum systems which are characterized by real order parameters and undergo a phase transition versus temperature and coupling constant. Some of them are ($\sigma = 2$): $z = 1$ ($x = 2, y = 0$), tranverse Ising model ($m = 1$), singlet-doublet system with the XY ($m = 2$), or the Heisenberg ($m = 3$) exchange; $z = 2$ ($x = 1, y = 0$), the itinerant antiferromagnet; $z = 3$ ($x = 1, y = 1$), the paramagnon model.

The concrete forms of the initial coupling constants in the GLW functional can be determined if we start from the microscopic Hamiltonian and then perform the Hubbard-Stratonovich transformation and the cumulant expansion. While such procedure can be generalized to the case of quenched random field, our choice (1) is the simplest form to study the universality of quantum critical phenomena with a random field. We would like to point out that the random field considered in this paper is short-range correlated, in contrast to the random field infinitely correlated in the $(d+1)$ time direction which appeared in an equivalent approach to the transverse Ising model,[8,9] where some of our results are already known.

Since what we have derived here is a one-loop result, for $\sigma \neq 2$, whether our finding is still correct to $O(\epsilon^2)$ is an open question.

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