Effect of surface polaritons on the lateral displacement of a light beam at a dielectric interface

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We consider the effect of surface polaritons on the shift of a beam of light of finite cross section on reflection from a two-layered semi-infinite medium. The beam is incident upon the first layer at an angle greater than the critical angle; the evanescent wave (in the first layer) interacts with the surface polariton at the interface between the first layer and the second layer. The plane-wave components of the incident "finite-cross-section" beam are absorbed and phase shifted by various amounts. The resultant emerging (reflected) beam, therefore, undergoes a shift. It is found that the calculated shift can assume very large values.

The problem of the lateral displacement (or shift) of a (finite-cross-section) beam that is totally reflected at the interface between two media has a long history. This shift is associated with the names of two German scientists and is now referred to as a Goos-Hänchen shift. For a list of early references, the paper by Horowitz and Tamir may be consulted.[1]

In a 1971 paper by Tamir and Bertoni,[2] they discussed the lateral displacement of optical beams by multilayered structures. Their emphasis is on leaky waves. In our Brief Report, we discuss the effect on stationary eigenmodes of the layered system, i.e., surface polaritons when the dielectric constant of the active medium is negative.

We rely heavily on the formalism developed by Horowitz and Tamir[1] and therefore we discuss now briefly the underlying physics of their method and why it is of interest to apply their method to a different geometry. They choose an incident beam having a Gaussian variation in intensity across its width. The only reason for this choice is that the integrals can be carried out fairly easily within a certain approximation. The "Gaussian beam" is then Fourier analyzed to give a continuum of infinite plane waves. Each incident plane wave is, on reflection at the interface, multiplied by the Fresnel reflectance Γ, which is a function of the angle of incidence. For angles greater than the critical angle, Γ is a complex number, of modulus unity.

Each component plane wave in the Gaussian beam strikes the interface at an angle of incidence slightly different from every other component. Therefore, each component is multiplied by a slightly different phase factor and when all the plane waves are superimposed again after reflection we obtain a displacement (or shift) in the maximum of the Gaussian.

In 1968, Otto[3] used total internal reflection at a dielectric-air interface to couple p-polarized light to surface plasmon waves by bringing the base of a totally reflecting quartz prism close to a silver surface. The evanescent field produced in the air space between the base of the prism and the silver surface coupled to the surface plasmon waves and excited them. This is revealed as a dip in the reflected radiation. It is natural to suspect that the Goos-Hänchen shift could be greatly affected at angles of incidence close to the angles of incidence for which the dips in reflectivity occur. The results and discussion are given in the following paragraphs.

The geometry that is pertinent to our discussion is shown in Fig. 1. Our work parallels very closely the paper by Horowitz and Tamir.[1] Instead of working near the critical angle we confine our attention to a small range of angle near the angle at which we obtain surface polariton absorption. The mathematics in this case becomes quite elementary and we obtain fairly rapidly an expression for the shift. We start our analysis with Eq. (10) of Ref. 1 which we

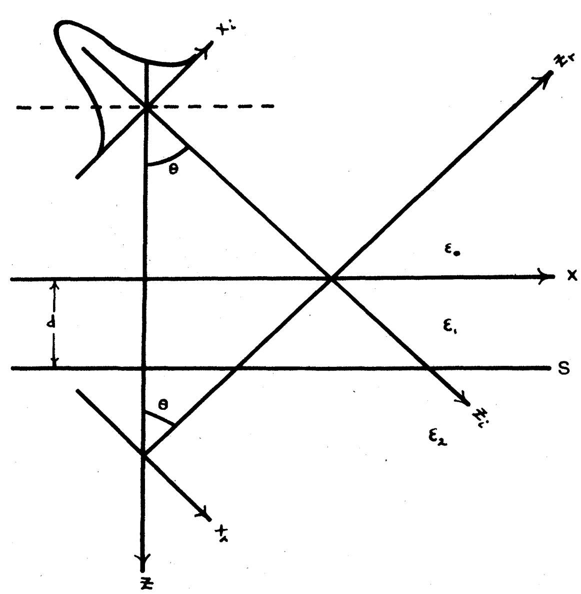

FIG. 1. Geometry of the reflected-beam problem. The surface polariton is excited at the interface S. If there were no shift, the peak of the incident Gaussian would travel along z_i and then along z_r after reflection at the x-y plane.

reproduce here:

$$H_{\text{refl}}=\frac{1}{2\pi\cos\theta}\int_{-\infty}^{+\infty}\Gamma(k_x)\exp\left[-\left(\frac{k_x-k\sin\theta}{2\cos\theta}W\right)^2\right]\exp\{i[k_xx-k_z(z-h)]\}\,dk_x\ , \tag{1}$$

where $k\sin\theta$ is the component of the wave vector parallel to the surface, W is essentially the width of the Gaussian beam, and $\Gamma(k_x)$ is the Fresnel reflectance. In our case, $\Gamma(k_x)$, for transverse magnetic waves, is given by[4]

$$\Gamma(k_x)=\frac{(k_z'/\epsilon_1-k_z/\epsilon_0)(k_z''/\epsilon_2+k_z'/\epsilon_1)\exp(-ik_z'd)-(k_z/\epsilon_0+k_z'/\epsilon_1)(-k_z''/\epsilon_2+k_z'/\epsilon_1)\exp(ik_z'd)}{(k_z'/\epsilon_1+k_z/\epsilon_0)(-k_z''/\epsilon_2-k_z'/\epsilon_1)\exp(-ik_z'd)+(k_z'/\epsilon_1-k_z/\epsilon_0)(-k_z''/\epsilon_2+k_z'/\epsilon_1)\exp(ik_z'd)}\ . \tag{2}$$

The time dependence, $\exp(i\omega t)$, has been suppressed in Eq. (1). The definitions of the terms appearing in Eq. (2) are

$$k_z=(\epsilon_0\omega^2/c^2-k_x^2)^{1/2}>0\ , \tag{3}$$

$$k_z'=i(k_x^2-\omega^2/c^2)^{1/2}\ , \tag{4}$$

$$k_z''=(\epsilon_2\omega^2/c^2-k_x^2)^{1/2}\ , \tag{5}$$

with $(k_0^2-\omega^2/c^2)$ greater than zero.

In order to have absorption of the incident beam by surface polaritons, we must have that ϵ_2 is complex. The square root in Eq. (5) is chosen so that the imaginary part of k_z'' is greater than zero in order to have decaying waves into medium 2. The media labeled 0, 1, and 2 have, respectively, the dielectric constants ϵ_0, ϵ_1, and ϵ_2. $\epsilon_1=1.0$.

We now assume that the main Fourier component of our Gaussian beam is incident on the top surface at an angle given by $\sin\theta=k_0/k$. We also assume that $k_0w>>1$. The latter assumption implies that the range of Fourier components, Δk, contributing to the makeup of the beam is such that $(\Delta k)(w)\equiv 1$ or $\Delta k<<k_0$. We can therefore expand $\Gamma(k_x)$ around $k_x=k_0$. A straightforward expansion results in the following equation:

$$\Gamma(k_x)=\Gamma(k_0)\left\{1+\frac{R'(k_0)F'(k_0)-E'(k_0)S'(k_0)}{E'(k_0)F'(k_0)(k_0d)}\delta d\right\}$$

$$=\Gamma(k_0)[1+\Gamma'(k_0)(\delta d)]\ , \tag{6}$$

where $E'(k_0)$ and $F'(k_0)$ are, respectively, the numerator and denominator of Eq. (2) evaluated at $k_x=k_0$, $R'(k_0)=(\partial E'(k_x)/\partial k_x)_{k_x=k_0}$, $S'(k_0)=(\partial F'(k_x)/\partial k_x)_{k_x=k_0}$, and $\delta=(k_x-k_0)$.

The approximate form for $\Gamma(k_x)$ given in Eq. (6) is substituted into Eq. (1) and the integrals over k_x are carried out. We obtain

$$H_r=\frac{H_{0i}}{2\pi\cos\theta}\int_{-\infty}^{+\infty}\Gamma(k_0)[1+\Gamma'(k_0)(d\delta)]\exp[-(\delta w/2\cos\theta)^2]\exp\{i[k_xx-k_z(z-h)]\}dk_x\ . \tag{7}$$

Following Horowitz and Tamir, we separate the reflected field into two parts, H_r^1 and H_r^2, given by

$$H_r^1=\frac{H_{0i}E(k_0)\Gamma(k_0)d}{2\pi\cos\theta F(k_0)}\int_{-\infty}^{+\infty}\exp(-w^2\delta^2/4\cos^2\theta)\exp i[k_xx-k_z(z-h)]dk_x\ , \tag{8}$$

$$H_r^2=\frac{H_{0i}E(k_0)\Gamma(k_0)d}{2\pi\cos\theta F(k_0)}\int_{-\infty}^{+\infty}\delta\exp(-w^2\delta^2/4\cos^2\theta)\exp i[k_xx-k_z(z-h)]dk_x\ . \tag{9}$$

k_z is expanded around $k_x=k_0$ retaining up to quadratic terms in k_x. This yields

$$k_z\simeq k\cos\theta-(k_x-k\sin\theta)\tan\theta$$

$$-[(k_x-k\sin\theta)^2/2k\cos^3\theta]\ . \tag{10}$$

On substituting k_z from Eq. (10) into Eqs. (8) and (9) and performing the resulting elementary integrals, we obtain

$$H_r=\frac{H_{0i}E(k_0)}{2\pi\cos\theta F(k_0)}\exp(-x_r^2/w^2)$$

$$\times[1+\Gamma'(k_0)2id\cos\theta x_r/w^2]\ , \tag{11}$$

where

$$x_r=x\cos\theta+(z-h)\sin\theta\ . \tag{12}$$

The coordinate x_r is related to the coordinates x and z by a rotation through the angle θ. See Fig. 1 which shows the various coordinate systems pertinent to our geometry.

If the absolute value of the second term in the last parenthesis of Eq. (11) is much smaller than one, we can exponentiate the quantity in this parenthesis to give the following expression for the reflected wave:

$$H_r=H_{ro}\exp(-x_r^2/w^2)\exp(i\Delta x_r/w^2)$$

$$=H_{ro}\exp[-(x_r-i\Delta/2)^2/w^2]\exp(-\Delta^2/4w^2)\ . \tag{13}$$

The Goos-Hänchen shift is given by the real part of $i\Delta/2$,

$$D=\text{Re}[i\Gamma'(k_0)2d\cos\theta]=\text{Re}[i\Delta/2]\ . \tag{14}$$

The properties of D are most conveniently expressed by plotting on a graph the imaginary part of $\Gamma'(k_0)$. It should not be surprising to expect that if the parameters for our geometry are chosen such that k_0 varies over a resonance in the reflected energy due to absorption by a surface polariton that one could obtain dramatic variations in the shift. This is what is found as can be seen by the curves plotted in Figs. 2 and 3.

When the imaginary part of the dielectric constant, ϵ_2, is taken to be zero, the reflection coefficient is equal to one and all of the incident energy is reflected, as expected. However, the shift goes through a maximum even for this case. It is most interesting that in this example we have the

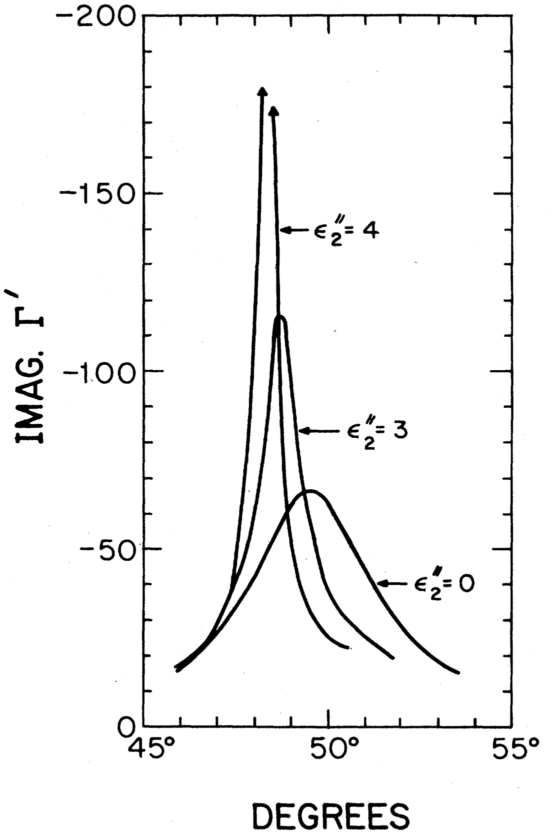

FIG. 2. The imaginary part of Γ', which is proportional to the shift, is plotted against the angle of incidence for several values of the imaginary part of the dielectric constant ϵ_2. The real part of ϵ_2 is −9.

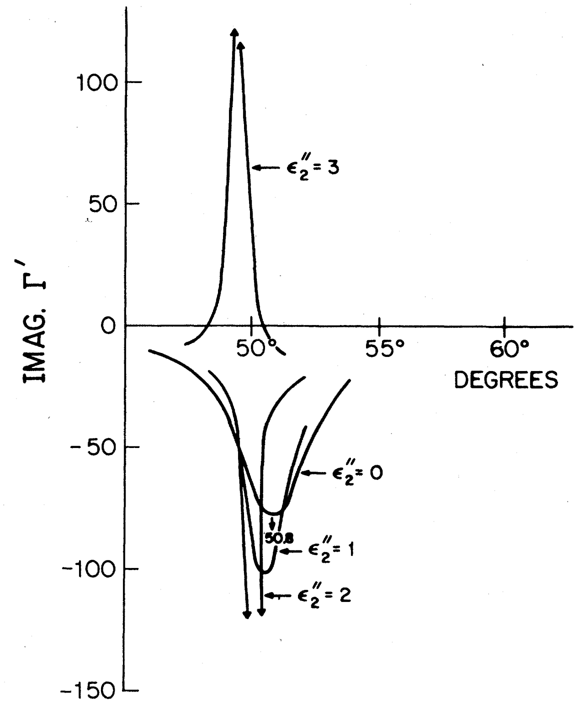

FIG. 3. The same plot as in Fig. 2 with the real part of ϵ_2 equal to −7. Note the apparent sudden reversal in sign of the shift near the imaginary part of $\epsilon_2''=2$.

means of detecting a surface polariton without the necessity of exchanging energy with it. For ϵ_2 real and with $\epsilon_2 < -1$, the dispersion relation for surface polaritons is well known and given by

$$\omega^2/c^2 = (k_{\parallel}^2/|\epsilon_2|)(|\epsilon_2|-1) \ . \tag{15}$$

If one chooses the incident radiation so that the angular frequency ω, and the angle of incidence θ, satisfies $k^2=\epsilon_0\omega^2/c^2$ and $k_{\parallel}=k\sin\theta$ and substitutes these expressions into Eq. (15), one obtains

$$\sin^2\theta = |\epsilon_2|/(|\epsilon_2|-1)\epsilon_0 \ . \tag{16}$$

One observes in Fig. 2 that for ϵ_2 real and equal to −9, that the angle at maximum shift is located approximately at 49.5°. Equation (16) gives the value 49.25°. As one increases the imaginary part of the dielectric constant, the maximum in the shift moves to smaller incident angles. For a particular value of the imaginary part of the dielectric constant, seen in Fig. 3, the shift jumps discontinuously from a large positive value to a large negative value. However, as can be seen from its derivation, our expression for the shift becomes invalid for arbitrarily large values of $\Gamma'(k_0)$.

We have deliberately chosen our geometry so that the interaction of the incident light with the surface polariton does not occur near the critical angle. This avoids the analysis of Horowitz and Tamir at the branch point in $\Gamma(k_x)$. For the single-surface geometry they have shown that the maximum shift occurs at an angle very near the critical angle. We have shown that arbitrarily large shifts can occur at angles where surface polariton absorption may take place, even at angles far from the critical angle. In this case, part of the incident beam is absorbed by the surface polariton and the shift is the result of the combined effect of a "phase shifting" of the Fourier components in the incident beam plus the removal by absorption of some of the Fourier components through interaction with the surface polariton.

It would appear that similar effects should occur with elastic waves. In this case the incident elastic wave would interact with a surface elastic wave (e.g., a Rayleigh wave). On reflection, the incident elastic wave might then undergo a Goos-Hänchen–type shift.

In cases where our calculations give very large shifts, the exponentiation which we used to obtain the shift is actually invalid. One has to do better in approximating the integral or, in the last resort, perform a numerical integration on a computer. In this case the reflected beam may experience considerable distortion rather than just a simple shift of the "center" of the beam.

Finally, we mention that the so-called classical value for the shift is obtained from our formulas by letting d go to infinity. In this limit, the influence of the crystal disappears and we return to the one-surface geometry at total reflection.

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