# Localized surface moments and a magnetic phase transition: The source of the anomalous  $H_{c3}/H_{c2}$  temperature dependence for clean strain-free vanadium specimens

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It is suggested that localized surface moments are the source of the anomalous sheath-state superconductivity exhibited by clean, well annealed vanadium specimens in  $H_{c3}/H_{c2}$  vs T data. I propose a simple, heuristic combination of the normal surface-layer modification of the sheath state, as calculated by de Gennes, and Werthamer's theory of the proximity effect of magnetic impurities on superconducting films. A magnetic phase transition of localized surface moments is also assumed. This results in an expression for a temperature-dependent extrapolation length  $b(T)$  which shows good agreement with published values extracted from "linearized"  $H_{c3}/H_{c2}$  data. The values of the parameters of the model used in the fit are consistent with (1) the low-temperature susceptibility data of Akoh and Tasaki for vanadium micropowders, (2) the results of the microscopic calculation of fluctuation-induced surface moments by Grempel and Ying, and (3) the estimate for the diffusion length of Cooper pairs into a magnetic layer coating a clean superconducting surface by Hauser, Theurer, and Werthamer.

#### I. INTRODUCTION

Akoh and Tasaki' have reported experimental evidence of magnetic moments localized in the free surface (I—<sup>2</sup> atomic layers) of vanadium micropowder specimens. Subsequently, Grempel and Ying<sup>2</sup> have performed a spin-fluctuation calculation for the electronic surface states of vanadium. Their results predict fluctuation-induced, localized surface moments which obey Curie's law. An antiferromagnetic exchange-interaction strength was also calculated, indicative of a magnetic transition temperature of 3.8 K. It is my conjecture that the experimental results and theoretical interpretation described above also allow for a more physical explanation than has heretofore been advanced for the finite value of the extrapolation length  $b$  extracted from sheathstate  $H_{c3}/H_{c2}$  data, reported for clean, freshly etched, vanadium bulk specimens.<sup>3,</sup>

The extrapolation length  $b$  is a measure of mixed boundary conditions at a superconducting interface:  $b = \Psi_0/$  $(d\Psi/dx)_0$ , where  $\Psi_0$  and  $(d\Psi/dx)_0$  are, respectively, the Ginsburg-Landau (GL) order parameter and its normal derivative at the surface of the superconductor.<sup>5</sup> Finite values for  $b$  are required in the linear GL theory for  $H_{c3}/H_{c2} \neq 1.69$  as  $T \rightarrow T_c$ . It possibly needs to be reiterated that the  $H_{c3}/H_{c2}$  data of Ref. 4 are "linearized" according to the prescription

$$
\left(\frac{H_{c3}}{H_{c2}}\right)_l = \left(\frac{H_{c3}}{H_{c2}}\right)_e \times \frac{1}{C(T)}
$$

where T is the temperature,  $(H_{c3}/H_{c2})_e$  the experimental value of the critical field ratios,  $C(T)$  a polynomial in T that is the result of a microscopic calculation by Hu and Korenman<sup>6</sup> which effectively gives the nonlinear Ginsberg-Landau contributions to the  $H_{c3}/H_{c2}$  ratio, and  $(H_{c3}/H_{c2})_l$  the resultant "linearized" values of  $H_{c3}/H_{c2}$ . The success of the procedure is evidenced by the excellent fit of a calculation of  $H_{c3}/H_{c2}$  using the linear GL theory weighted by  $C(T)$  to the data of Ref. 4 for  $T \ge 3$  K.

The theoretical treatment of choice that has previously

been employed to explain  $H_{c3}/H_{c2} \neq 1.69$  near  $T_c$  for clean, nomogeneous specimens is one proposed by  $Hu^7$  In this model *b* is expressed in terms of the enhancement or depression of the electron-electron interaction strength  $\Delta V$ over a surface region of depth d. Specific sources of the modified electronic states of the surface region have thus far not been identified, especially for the pure, well annealed specimens of Nb and V to which Hu's phenomenology has been applied.<sup>3,4</sup> Furthermore, the suggestion has been previously advanced in Ref. 4 that the abrupt departure of the good fit at  $T \approx 3$  K may be interpreted in the context of the Hu model as evidence of a very deep "A" region; i.e., the depth of the perturbed surface region  $d$  is of the order of  $\xi$ , the superconducting coherence length at that temperature 675 A. This leaves one with the puzzle of explaining an intrinsically modified surface region of such great depth.

I now suggest that the  $H_{c3}/H_{c2}$  data near  $T_c$  of Ref. 4 can be understood in the context of a model in which the pairbreaking perturbation is localized spins at the free surface of the vanadium  $(d = 3-6$  Å). It is further suggested that the abrupt decrease in b at  $T \cong 3$  K required to fit the data is associated with the onset of magnetic ordering of the surface moments and a concomitant increase in their pairbreaking strength. Such ordering was inferred by Akoh and Tasaki<sup>1</sup> to occur at  $T<4$  K for their vanadium micropowder specimens; and, as already indicated, Grempel and Ying<sup>2</sup> estimated an antiferromagnetic transition temperature of  $T \approx 3.8$  K in their fluctuation-induced surface moment calculation. Thus, the thrust of this paper is to provide a plausible explanation for the finite value of the extrapolation length  $b$  and its temperature dependence numerically calculated from the integral form of the linear GL equations with  $(H_{c3}/H_{c2})_l$  as the input parameter.

### II. MODEL

The simple model that I wish to present is essentially a synthesis of two related calculations. The first is that of de

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Gennes and Sarma for the extrapolation length b of a superconducting surface coated with a normal metal. $5$  The second is Werthamer's calculation of the proximity effect of localized moments on the surface of a superconducting film, i.e., the reduction of the transition temperature of the film.<sup>8</sup>

de Gennes' expression for  $b$  is given in terms of the normal layer-superconducting substrate electronic parameters:

$$
\frac{1}{b} = \frac{\sigma_n}{\sigma_s} K \tanh(d_n K) \quad , \tag{1}
$$

where  $\sigma_n$  and  $\sigma_s$  are the electrical conductivity of the normal and superconducting materials, respectively,  $d_n$  the thickness of the normal layer, and  $K^{-1}$  the diffusion length of the Cooper pairs into the normal region. In this model I assume that the "normal" region consists only of fluctuation-induced, localized moments that extend to a depth of 1-2 atomic layers-in effect, a clean strain-free surface.

I next assume that Werthamer's proximity effect calculation may be applied in order to develop a simple relationship between the diffusion length  $K^{-1}$  and the mean free path  $l_n$  of the "normal" layer.<sup>8,9</sup> For the "normal" surface layer assumed, Eq. (9) of Ref. 8 reduces to

$$
K^{-1} = (D_n \tau_n)^{1/2} = \frac{\nu_F \tau_n}{\sqrt{3}} = \frac{l_n}{\sqrt{3}} \quad , \tag{2}
$$

where  $D_n$  is the "normal" region diffusion coefficient,  $1/\tau_n$ the rate of pair destruction by the localized moments via the exchange interaction, and  $v_F$  is the Fermi velocity. The resulting expression<sup>10</sup> for the extrapolation length  $\overrightarrow{b}$  from the combination of Eqs. (1) and (2), and from  $\sigma_n/$  $\sigma_s = (N_n/N_s)^2 l_n/l_s$ , is

$$
\frac{1}{b} = \frac{\sqrt{3}}{l_s} \left( \frac{N_n}{N_s} \right)^2 \tanh\left( \frac{\sqrt{3}d_n}{l_n} \right) , \qquad (3)
$$

where now  $l_s$  and  $l_n$  are, respectively, the interior and surface electronic mean free paths and  $N_n/N_s$  the ratio of the surface and interior density of states, a quantity calculated by Grempel and Ying.

I now assume an additional'pair breaking mechanism associated with the onset of ordering of the localized moments at a temperature  $T_m$  and, further, that the strength of the "ordered" pair-breaking component is proportional to the magnetic order parameter  $m(t)$ . Therefore, the effective pair-breaking rate  $1/\tau'_n$ , for  $T < T_m$  is now given by

$$
\frac{1}{\tau'_n} = \frac{1}{\tau_n} + \frac{m(t)}{\tau_0} \quad , \tag{4}
$$

where  $m(t)/\tau_0$  is the increment in the rate of pair breaking associated with magnetic ordering and  $t$  is the reduced magnetic temperature  $(T/T_m)$ . For reasons of clarity and ease of calculation, I take  $m(t)$  to be the magnitude of the normalized, mean-field theory magnetization for spin- $\frac{1}{2}$  moments:  $m(t) = \tanh[m(t)/t]$ . Finally,

$$
\frac{1}{b(t)} = \frac{\sqrt{3}}{l_s} \left( \frac{N_n}{N_s} \right)^2 \tanh\left[ \frac{\sqrt{3}d_n}{l_n} \left( 1 + \frac{l_n m(t)}{l_0} \right) \right] , \qquad (5)
$$

where  $l_0 = v_F \tau_0$  and may be interpreted as the mean free path associated with magnetic ordering in the "normal" surface layer.

TABLE I. Values of parameters used in fit of Eq. (5) to  $\xi_0/b$ data.

$l_s = 9030 \text{ Å}$	(See Ref. 4)	$d_n / l_n = 0.31$
$\xi_0 = 450 \text{ Å}$	(See Ref. $4$ )	$l_n / l_0 = 0.52$
$N_s/N_n \cong 0.7$	(See Ref. 2)	$T_m = 3$ K
$d_n = 3 - 6$ Å	(See Refs. 1 and 2)	

# III. RESULTS AND DISCUSSION

The adjustable parameters of this model are the ratios of lengths characteristic of the "normal" surface region,  $d_n / l_n$ and  $l_n / l_0$ , and the magnetic transition temperature  $T_m$  of this region. Values of these parameters as determined by a least-square fit of Eq. (5) to the published values of  $\xi_0/b$ are also given in Table I. [The extrapolation length calculations of Ref. 4 were checked resulting in slight corrections  $(< 5\%)$  to the previously published values. The resulting reasonably good fit is shown in Fig. 1. The value chosen for the magnetic transition temperature  $T_m = 3$  K is comparable to the approximate calculation by Grempel and  $\text{Ying}^2$ of a Néel temperature of 3.8 K and consistent with the magnetic ordering temperature that may be inferred from Akoh and Tasaki's<sup>1</sup> data.  $K^{-1} \cong 5.7 - 11.4$  Å, is calculated from the assumed value for  $d_n = 3-6$  Å chosen to agree with the results of Ref. <sup>1</sup> and the calculation of Ref. 2, the fitted value of the ratio  $d_n / l_n$ , and Eq. (2). This value is consistent with the calculation of Hauser and co-workers of  $K^{-1}$  from proximity effect measurements on magnetic material-superconducting film sandwiches.<sup>8</sup> For an Fe-Pb film it was estimated that  $K^{-1} \cong 6$  Å. On the other hand, the increased degradation of the sheath-state superconductivity that accompanies the onset of magnetic ordering, as indicated by the fitted value of  $l_n/l_0 \approx 0.5$ , is not so easily understood. Both the measurements<sup>1</sup> and calculations<sup>2</sup> of localized moments on the free surface of vanadium are indicative of antiferromagnetic ordering and it has been argued that such ordering in the bulk should reduce the exchange scattering by localized spins of Cooper pairs.<sup>5</sup> Nevertheless,



FIG. 1. Inverse extrapolation length vs temperature and reduced magnetic temperature. The solid line is a plot of Eq. (5) and the points are calculated values of  $1/b$  from Ref. 4.

the increase in pair scattering associated with surface magnetic ordering that is implied by the fit of this model to  $b(t)$  from  $H_{c3}/H_{c2}$  data is certainly consistent with the results of Hauser and co-workers for superconducting antiferromagnetic composite films which showed decreases in the transition temperature comparable to that achieved by ferromagnetic coatings or localized moments imbedded in a normal metal coating.

The results of this calculation are further testable<sup>10</sup> through the linear dependence of the extrapolation length b on the interior or bulk mean free path  $l_s$  given by Eq. (5). The terms in the coefficient of  $l_s$  are relatively insensitive to the purity level of the specimen provided that  $l_s \gg l_n$  or  $l_0$ and that the free surface is clean and unstrained. Thus, measurements of  $H_{c3}/H_{c2}$  as a function of temperature on a series of vanadium specimens of varying residual resistance<br>ratio  $R_R$  (25 <  $R_R$  < 500) would allow for a test of the model. The only other published  $H_{c3}/H_{c2}$  data for vanadium of which I have knowledge are for an annealed,

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polycrystalline specimen whose  $R_R = 720$  ( $l_s = 1.3 \times 10^4$  Å).<sup>3</sup> The implied [through Eq.  $(5)$ ] values for b of this specimen result in an  $H_{c3}/H_{c2}$  temperature dependence that is not experimentally distinguishable from the data analyzed here or presented in Ref. 3.

Finally, it is an interesting speculation that the observed temperature dependence of  $H_{c3}/H_{c2}$  in well annealed, freshemperature dependence of  $H_{c3}/H_{c2}$  in well annealed, fresh-<br>y etched niobium specimens<sup>3,11</sup> may have a similar physical origin to that proposed herein for vanadium —localized, fluctuation-induced surface moments that depress sheathstate superconductivity and result in a finite value for the extrapolation length or the mixed boundary condition parameter b. This conjecture can of course be tested directly by a search for surface moments in niobium specimens with reasonably large surface to volume ratios, and indirectly by measurements of the temperature dependence of  $H_{c3}/H_{c2}$  for a series of niobium specimens of varying purity levels.

R. D. Parks (Dekker, New York, 1969), pp. 1005-1034. <sup>10</sup>The first test of this model presents itself at this point since,

$$
\frac{l_s}{\sqrt{3}b(t)} \left(\frac{N_s}{N_n}\right)^2 = \tanh\left(\frac{\sqrt{3}d_n}{l_n}\right) \le 1
$$

The data listed in Table I and the experimentally derived maximum value of  $1/b(t)$  shown in Fig. 1 satisfy this constraint, with the left-hand side of the equality = 0.68, for  $(1/b(t))_{max} \approx 1/b(0)$ K).

11J. E. Ostenson and K. K. Finnemore, Phys. Rev. Lett. 22, 188 (1969).