Spin-orbit and paramagnon effects on magnetoconductance and tunneling

A. J. Millis and P. A. Lee
Department of Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139

(Received 16 July 1984)

Spin-splitting effects on the density of states and magnetoconductivity in disordered conductors with spin-orbit scattering are calculated. Particular attention is given to "paramagnon-enhanced" materials, in which spin-splitting effects are largest.

In this Brief Report we extend a previous calculation[1] of spin-splitting effects on the single-particle density of states $N(E)$ and magnetoconductivity $\delta\sigma(H)$ in disordered conductors to include spin-orbit scattering. We present explicit formulas for $N(E)$ and $\delta\sigma(H)$. Our results are valid in the magnetic field range defined by

$$\hbar c/2eD < H \quad , \tag{1}$$

$$eH/mc < 1/\tau \quad . \tag{2}$$

H is the magnetic field and τ the elastic scattering time. Condition (1) ensures that particle-particle channel effects[2] are suppressed; condition (2) that Landau-level effects are not important.

Before presenting the results of our calculation, we will outline the physics involved. In the usual "interaction theory" of electrons in disordered conductors,[3] it is argued that electrons nearby in energy have functions strongly correlated in space. This correlation leads[4] to singular corrections to such physical quantities as the density of states, given in two (2D) and three dimensions (3D),

$$\delta N(E) \sim \begin{cases} \sqrt{E} & (3\mathrm{D}) \quad , \\ \ln E & (2\mathrm{D}) \quad , \end{cases}$$

where E is the energy measured from E_F. These singularities are cut off by the thermal effects at energies $E \sim T$.

When a disordered conductor is placed in a magnetic field H, the two spin subbands split; hence two electrons in opposite spin bands with energy separated by ω_s ($\omega_s = g\mu_B H$ is the spin-splitting energy, μ_B is the Bohr magneton, and g is the electronic g factor) have strongly correlated wave functions. Mathematically, this leads to the magnetoresistance first discussed in (1),

$$\begin{aligned} \delta\sigma(H) &= \sigma(H) - \sigma(0) \\ &\sim \sqrt{H} \quad (3\mathrm{D}) \\ &\sim \ln H \quad (2\mathrm{D}) \quad . \end{aligned}$$

As Altshuler and Aronov have shown,[6] spin splitting also leads to new singularities in the density of states $E = \pm\omega_s$,

$$\delta N(E) \sim \begin{cases} |E \pm \omega_s|^{1/2} & (3\mathrm{D}) \quad , \\ \ln|E \pm \omega_s| & (2\mathrm{D}) \quad . \end{cases}$$

Spin-orbit scattering mixes the spin bands. One expects the singularity in the magnetoconductivity and the new contributions to the density of states to be cut off by spin-orbit scattering at energies small compared to the spin-orbit rate $1/\tau_s$. The singularities are also cut off by thermal fluctuations at energies small compared to T. Thus, there are three energies in the problem, ω_s, $1/\tau_s$, and T. To simplify matters, we consider only the limit $T < 1/\tau_s$ and $T < \omega_s$. In this limit, thermal effects are irrelevant and T will not appear in our formulas. $1/\tau_s$ sets the energy scale, and the new density-of-states singularities and the magnetoconductivity are functions only of $x = E\tau_s$ and $y = \omega_s\tau_s$. Spin-splitting effects will, in general, be observable only for fields large enough that $y > 1$.

One further parameter is needed to describe spin-splitting phenomena. F is a dimensionless number that sets the strength of the relevant part of the electron-electron interaction. The magnitude of the spin-splitting effects is set by F and is large when F is large, and small when F is small. In almost ferromagnetic materials (such as Pd, Pt, and their alloys), we shall show $F >> 1$. We shall also show that the condition for the observability of spin-splitting effects may be relaxed to $y > 2/F$ in these materials.

We turn now to the details of our calculation. We use the Altshuler-Aronov interaction theory,[3] which has recently been extended[5] to handle both spin-dependent phenomena and Coulomb interactions correctly. We outline some of the relevant aspects here.

The essential point of the theory is that in order to describe disordered conductors, one must take account of the diffusion of both particle density and spin density. The important quantities are the diffusion propagator $\mathscr{D}(q,w)$ and the particle-hole interaction amplitude $A(q,\omega)$. We will discuss $\mathscr{D}$ first, which is shown in Fig. 1. The dashed lines represent scattering (both potential and spin orbit) from impurities. $\mathscr{D}$ is an impurity-averaged propagator describing motion of a particle-hole pair. It is important to note that after impurity averaging both the total spin j and the projection along the magnetic field M of the particle-hole pair are conserved in collisions with impurities.[6,7] Thus, in the (j,M) representation, $\mathscr{D}$ is a simple function of its spin variables. For $j=0$, $\mathscr{D}$ is the density-density

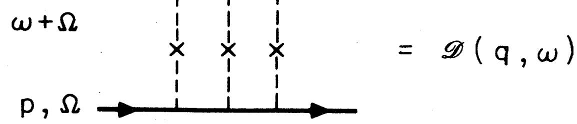

FIG. 1. The diffusion propagator.

correlation function. It has the familiar diffusion pole form

$$\mathscr{D}^{j=0}(q,\omega)=Dq^2/(-i\omega+Dq^2)\tau \ , \tag{3}$$

where $D=E_F\tau$ is the diffusion constant. Because it has quantum number $j=0$ it is rotationally invariant in spin space and hence is unaffected by spin-splitting or spin-orbit scattering. For $j=1$, the situation is more involved. $\mathscr{D}$ is the spin-density–spin-density correlation function, thus is cut off by spin-flip scattering, and (if the particle and hole are in different subbands) is shifted by spin splitting. Straightforward calculation shows[6,8,9]

$$\mathscr{D}^{j=1,M}(q,\omega)=\frac{Dq^2+1/\tau_s}{[-i(\omega+M\omega_s)+Dq^2+1/\tau_s]\tau} \ . \tag{4}$$

Now consider A. A is the screened, impurity-averaged particle-hole interaction amplitude. It also conserves j and M. A satisfies the Dyson equation shown in Fig. 2. A_0 is the unscreened, two-particle irreducible interaction amplitude. $\mathscr{D}$ is the polarizaton operator for disordered systems. For $j=0$, A_0 is dominated by the isolated Coulomb propagator, $A_0 \sim 1/q^2$. By solving the Dyson equation, one easily sees that A is the random-phase approximation (RPA) screened Coulomb interaction

$$A^{j=0}=(1/N_0)[(-i\omega+Dq^2)/Dq^2] \ , \tag{5}$$

where N_0 is the unperturbed density of states. Note that because of the long range of the Coulomb interaction, A is independent of the details of band structure, etc.

For $j=1$, the situation is more involved. The interaction is affected by spin splitting and spin scattering. Further, A_0 cannot contain an isolated Coulomb line (which would carry spin $j=0$) and hence must go to a constant as $q,\omega \rightarrow 0$. The solution of the Dyson equation can be written[5]

$$A^{j=1,M}=\frac{-F}{2N_0}\frac{-i(\omega+M\omega_s)Dq^2+1/\tau_s}{-i(\omega+M\omega_s)(1+F/2)+Dq^2+1/\tau_s} \ . \tag{6}$$

F is a parameter characterizing the strength of the $j=1$ interaction. It is nonuniversal: it may be positive or negative and its magnitude varies from material to material depending on details of band structure, coupling to phonons, etc. These details affect long-time, large distance diffusive behavior only in the $j=1$ channel, where the Coulomb interaction does not operate. It is easy to see, however, that in "almost ferromagnetic" materials (such as Pd, Pt, and their alloys) $F >> 1$. These materials have a very large static spin susceptibility, caused by long-lived spin fluctuations. They are well described by the paramagnon model.[9] In this model, it is assumed that the relevant electron-electron interaction, denoted by I, is of zero range and acts only between electrons of opposite spin. This assumption implies a specific form for $A_0^{j=1}$, which yields an interaction A given by our Eq. (6) with $F/2=(1-I)^{-1} >> 1$. $(1-I)^{-1}$ is the Stoner enhancement factor, and is shown in the paramagnon literature to give the enhancement of the spin susceptibility over that of a noninteracting electron gas. As spin susceptibility is proportional to the spin-splitting energy divided by the field, one may write the effective g factor as

$$g_{\text{eff}}=g_0(F/2) \ . \tag{7}$$

$g_0=2$ is the independent-electron g factor.

Given A and $\mathscr{D}$, we may compute the self-energy Σ as shown in Fig. 3. From this one may obtain the density-of-states correction

$$\delta N^{j,M}(E)=-(1/\pi)\,\text{Im}\int[d^dp/(2\pi)^d]\Sigma^{jM}(p,E) \ . \tag{8}$$

The conductivity is obtained in the standard way from diagrams generated from Σ in a conserving approximation. The density of states and conductivity results are as follows:

$$\delta N(E)/N_0=\delta N_d^{j=0}(|E|)+\sum_{m=-1,0,1}\delta N_d^{j=1}(|E-M\omega_s|) \ , \tag{9}$$

$$\delta N_3^{j=0}=[1/4\pi^2\sqrt{2}(\hbar D)^{3/2}]\sqrt{E}, \quad d=3 \ , \tag{10a}$$

$$\delta N_3^{j=1}=\frac{-1}{16\pi^2\sqrt{2}(\hbar D)^{3/2}\sqrt{\tau_s}}[g((1+F/2)x)-g(x)] \ , \quad d=3 \ , \tag{10b}$$

$$\delta N_2^{j=0}=(1/8\pi^2\hbar D)\ln(E/D^2K^4\hbar\tau)\ln E\tau, \quad d=2 \ , \tag{10c}$$

$$\delta N_2^{j=1}=(-1/32\pi^2\hbar D)[h((1+F/2)x)-h(x)], \quad d=2 \ , \tag{10d}$$

$$g(x)=2[(1+x^2)^{1/2}+1]^{1/2}+\frac{1}{\sqrt{2}}\ln\left|\frac{[(1+x^2)^{1/2}+1]^{1/2}-\sqrt{2}}{[(1+x^2)^{1/2}+1]^{1/2}+\sqrt{2}}\right| \ , \tag{11}$$

$$h(x)=\int_0^x dy(1/y)\ln(1+y) \ , \tag{12a}$$

$$h(x)=\begin{cases}\frac{1}{2}\ln^2x-(1/x)+(1/2^2x^2)-(1/3^2x^3)+\cdots & x>1 \ , \quad (12b)\\ x-(x^2/2^2)+(x^3/3^2)+\cdots & x<1 \ . \quad (12c)\end{cases}$$

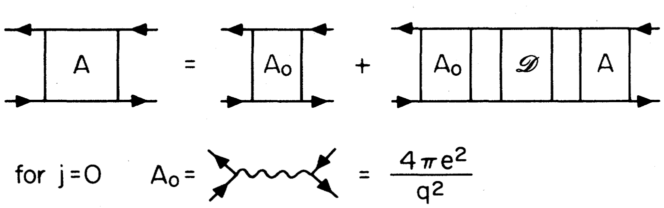

FIG. 2. Diagrammatic representation of the particle-hole interaction A. The diffusion propagator is represented by $\mathscr{D}$.

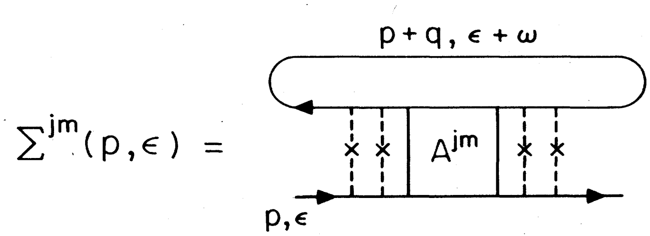

FIG. 3. Diagram representing the self-energy correction due to the interaction A.

K is the screening length. The factor $\ln(E/D^2k^4\hbar\tau)$ enters, because in 2D certain integrals fall off too slowly for the usual asymptotic approximations of the interaction theory to be valid.[10]

The magnetoconductivity results are as follows:

$$\delta\sigma(H)=\sigma(H)-\sigma(0)\ ; \tag{13}$$

$$\begin{aligned}\frac{\delta\sigma(H)}{\sigma_0}=&\ \frac{-1}{4\pi\hbar D}\left[\frac{3}{2}\frac{1+F/2}{F}[g((1+F/2)y)-g(y)]\right]\\ &+\frac{1+F/2}{F}\left[\tfrac{1}{2}\{[1+(1+F/2)^2y^2]^{1/2}+1\}^{1/2}-\frac{1}{\{[1+(1+F/2)^2y^2]^{1/2}+1\}^{1/2}}\right]\\ &+\frac{1}{F}\left[[(1+y^2)^{1/2}+1]^{1/2}-\frac{1}{[(1+y^2)^{1/2}+1]^{1/2}}\right],\quad d=3\ ;\end{aligned} \tag{14a}$$

$$\begin{aligned}\frac{\delta\sigma(H)}{\sigma}=&\ \frac{-1}{4\pi^2N_0D}\frac{2}{Fy}\{\tan^{-1}[(1+F/2)y]-\tan^{-1}(y)\}+\frac{1+F/2}{F}\ln[1+(1+F/2)^2y^2]-\frac{1}{F}\ln(1+y^2)-1\\ &+\frac{1+F/2}{2F}h((1+F/2)^2y^2)-h(y^2),\quad d=2\ .\end{aligned} \tag{14b}$$

We will now discuss the magnetoconductivity. Note that it is a function of y only. For $y<(1+F/2)^{-1}$ $F(>0)$ or $y<1(F<0)$ we have

$$\delta\sigma(H)\sim y^2\ ,$$

and for large y,

$$\delta\sigma\sim\begin{cases}\sqrt{|y|} & (3\text{D})\ ,\\ \ln|y| & (2\text{D})\ .\end{cases}$$

The crossover is relatively sharp in 2D and slower in 3D.

In the previous calculation,[1] a similar behavior was found, with $\delta\sigma(H)\sim H^2$ for small fields and $\sim\sqrt{H}$ or $\ln H$ for large fields. But in that calculation, the singularities were cut off by thermal effects, hence "small" meant ω_s small relative to T. Here the spin-orbit rate provides the cutoff, and the thermal effects are not included.

Now consider the density-of-state corrections. Note all three $j=1$ terms are proportional to

$$|x+My|^2\ \text{ for } |x+My|<(1+F/2)^{-1}\ ,$$

while for large y

$$\delta N^{j=1,M}\sim\begin{cases}|x+My|^{1/2} & (3\text{D})\ ,\\ \ln|x+My| & (2\text{D})\ .\end{cases}$$

As before the crossover is slow in 3D, faster in 2D. Thus to observe the singularities at all, one must have $y>>(1+F/2)^{-1}$. This condition must be enforced much more stringently in 3D than in 2D.

The sign of the $j=1$ terms is opposite of the $j=0$ terms for $F>0$. The magnitude of the $j=1$ terms relative to the $j=0$ terms is determined by F, and is small when F is small, and large when F is large. As can be seen from Fig. 4 (which shows the 3D density-of-states correction for $y=\omega_s\tau_s=16$ and $F=2$), the new singularities in density of

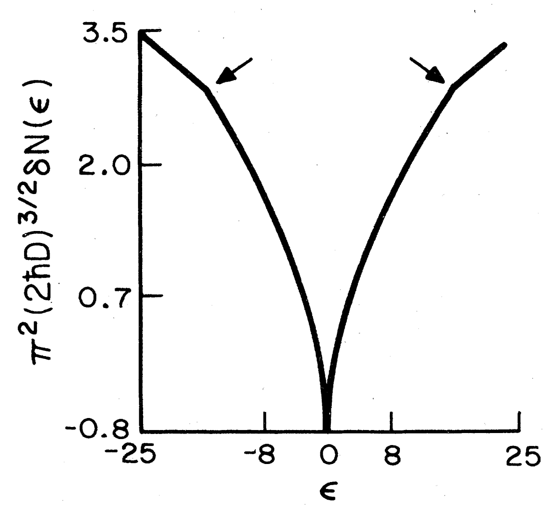

FIG. 4. Dimensionless 3D density-of-states correction for $F=2$ and $y=16$. The arrows indicate singularities at $E=\omega_s$. E is in units such that $1/\tau_s=1$.

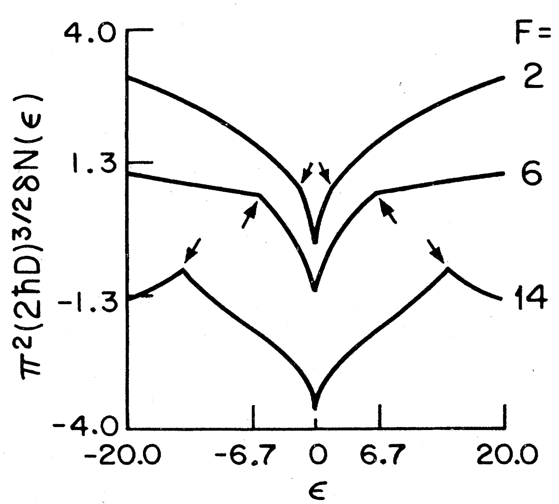

FIG. 5. F dependence of 3D density-of-states correction. The arrows indicate singularities at $E=\omega_s$. The paramagnon enhancement of ω_s with increasing F is included. The energy scale is such that the $F=0$ spin splitting $g_0\mu_BH=2$; $1/\tau_s=1$.

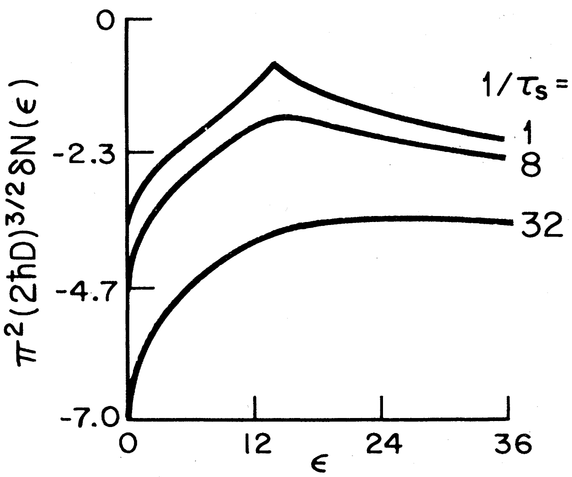

FIG. 6. Spin-orbit rate dependence of 3D density-of-states correction $F=14$; $\omega_s=14$. The energy scale is the same as in Fig. 5.

states (indicated by arrows in the figure) are barely noticeable for $F \cong 1$. For large F, there are several qualitative changes in the density-of-states curve. For $E >> \omega_s$, the density-of-states correction goes as $\sqrt{E}$ (3D) or $\ln E$ (2D), but with a prefactor that is positive if F is small, and negative if F is large. Further, for $F >> 1$, $\delta N^{j=1}$ and $\delta\sigma$ become functions of $\frac{1}{2}Fx$ and $\frac{1}{2}Fy$ only. Finally, the effective g factor is enhanced [Eq. (7)]. Thus, the condition for the observability of spin effects becomes

$$(\tfrac{1}{2}F)^2 g_0 \mu_B H \tau_s >> 1 \ . \tag{15}$$

This is shown in Fig. 5, in which the 3D density-of-states correction is plotted for several F values at fixed τ_s, H. For $F=2$, the new singularities are invisible. As F (and thus the effective g factor) is increased, the new singularities move out from the origin and become more sharply defined. Note that the density-of-states correction is defined to be zero at $E=0$ in zero magnetic field. In finite field,

$$\delta N(0) = 2\delta N^{j=1}(|\omega_s|) < 0 \quad \text{for } F > 0 \ . \tag{16}$$

Note finally that the sharpness of the central singularity is independent of F. This is due to the $j=0$ density-of-states correction, which is cut off only by thermal effects.

In Fig. 6 we show the effects of increasing spin-orbit scattering. F and ω_s are fixed at the values used in the $F=14$ curve in Fig. 5. Note the smearing of the $j=1$ singularity as $1/\tau_s$ is increased.

We thank W. F. Brinkman for an enlightening discussion on the paramagnon effects. We acknowledge NSF Grant No. DMR-84-41030 for support.

[1]P. A. Lee and T. V. Ramakrishnan, Phys. Rev. B **26**, 4009 (1982).

[2]B. L. Altshuler, D. Khmel'nitzkii, A. I. Larkin, and P. A. Lee, Phys. Rev. B **22**, 5142 (1980).

[3]B. L. Altshuler and A. G. Aronov, Zh. Eksp. Teor. Fiz. **77**, 2028 (1979) [Sov. Phys. JETP **50**, 968 (1979)].

[4]E. Abrahams, P. W. Anderson, P. A. Lee, and T. V. Ramakrishnan, Phys. Rev. B **24**, 6783 (1981).

[5]B. L. Altshuler and A. G. Aronov, Solid State Commun. **44**, 137 (1982).

[6]B. L. Altshuler and A. G. Aronov, Pis'ma Zh. Eksp. Teor. Fiz. **37**, 145 [JETP Lett. **37**, 175 (1983)].

[7]Thus the fact that spin-orbit scattering mixes the spin and space parts of the electronic wave function is irrelevant here. If spin-flip scattering (as from magnetic impurities) is present, with rate $1/\tau_{sf}$, then one should replace $1/\tau_s$ in our formulas by $(1/\tau_s + 1/\tau_{sf})$.

[8]We have defined the spin-orbit scattering rate to be the low-energy cutoff in the diffusion pole, i.e., to be the spin-fluctuation decay rate. One may alternatively define it as the reciprocal of the mean time t between spin-orbit scatterings for a single electron. It can be shown that $1/\tau_s = \frac{4}{3}(1/t)$.

[9]W. F. Brinkman and S. Engelsberg, Phys. Rev. Lett. **21**, 1187 (1968).

[10]B. L. Altshuler, A. G. Aronov, and P. A. Lee, Phys. Rev. Lett. **44**, 1288 (1980).