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Determination of the Hall coefficient by direct generation of ultrasound

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A theoretical study is made of the generation of ultrasound as a function of an applied, static magnetic field for propagation along a [110] direction in a potassium crystal. For certain orientations of the incident radio-frequency field the variation of acoustic amplitude with field presents a cusp. The position of this minimum can be used to determine the departure of the Hall coefficient from its free-electron value.

I. INTRODUCTION

In a recent paper, Zhu and Overhauser[1] showed that one can explain the induced torque anomalies in potassium by assuming an anisotropic Hall coefficient. Direct evidence of such an anisotropy had been obtained by Chimenti and Maxfield,[2] who measured the Hall coefficient R_H at high fields by a helicon transmission method. They found that R_H is larger than the free-electron value, $R_0 = -1/nec$, and that the deviation from R_0 depends on the crystal orientation. In this paper we propose a different way of obtaining the Hall coefficient, one that makes use of the direct generation of ultrasound, and which is sensitive to a deviation of R_H from R_0.

It is well known[3-7] that a sound wave can be generated in a crystal by application of an alternating electromagnetic field. The field penetrates the metal (within a skin depth) and accelerates the electrons, which transfer their excess momentum to the positive ions through collisions (with impurities or phonons). The resulting force on the positive ions is called the collision-drag force. The combined effect of this force and the direct electric force acting on the positive ions will accelerate them away from their equilibrium positions and generate a sound wave that propagates through the crystal. The presence of a static magnetic field H_0 (perpendicular to the sample surface and parallel to the propagation direction) accentuates this effect since it causes the collision-drag force to be nonaligned with the direct electric force. Since the angle between the forces depends on the magnitude of the field, the amplitude of the ultrasonic wave will likewise depend on H_0.

Puskorius and Trivisonno[8] have measured the variation of the ultrasonic amplitude as a function of H_0 for the slow shear mode generated in potassium and propagating along a [110] direction. Their result, Fig. 1, is not the monotonic curve predicted by using a free-electron model and neglecting the difference in velocities of the two shear modes.

We present a calculation which takes into account this difference in the velocities. The results obtained depend very strongly on the direction of polarization of the incident rf field with respect to the crystal axis. Fig. 2 shows the geometry of the experiment and Fig. 3 shows the force components acting on the positive ions along the slow-shear-mode axis. This figure enables one to understand the variation of the effect with polarization angle.

The collision drag force rotates with increasing H_0 since the Lorentz force is proportional to H_0. For a specific value of H_0, the electric and collision-drag force components parallel to the shear-mode polarization axis will cancel, thereby creating a minimum in the generation

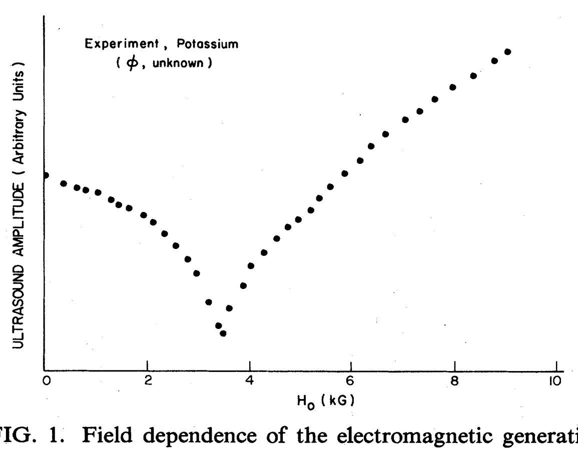

FIG. 1. Field dependence of the electromagnetic generation amplitude for the slow shear mode in K at 4.2 K (Puskorius and Trivisonno, Ref. 8). The orientation of the 10-MHz rf field relative to the crystal axes was unknown.

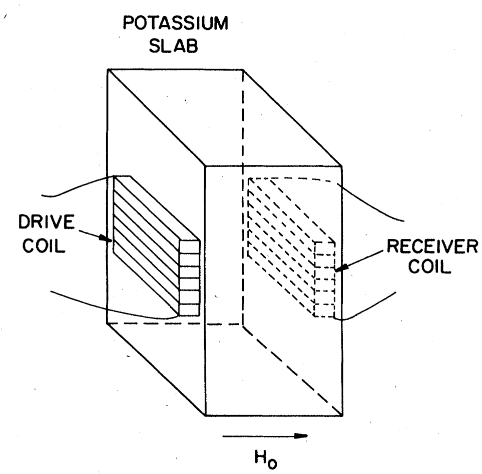

FIG. 2. Schematic illustration of the direct generation experiment described in Ref. 8.

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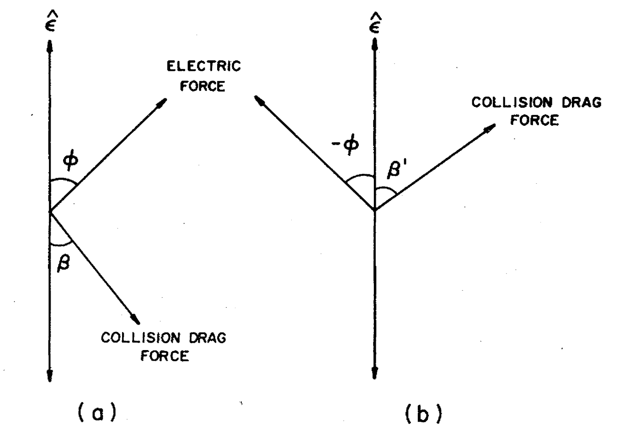

FIG. 3. Electric and collision-drag forces relative to the polarization vector $\hat{\epsilon}$ of the shear mode. (a) For positive ϕ the components along $\hat{\epsilon}$ tend to cancel. (b) For negative ϕ the components along $\hat{\epsilon}$ add. (If $H_0=0$, the forces shown would be antiparallel.)

amplitude. Since the value of H_0 for which this occurs depends on the polarization angle ϕ of the rf electric field, it follows that the position of the minimum (versus H_0) will depend dramatically on ϕ. In particular, if ϕ is changed into $-\phi$ (which is equivalent to reversing H_0), the minimum will disappear since, now, the components of the forces along the shear-mode polarization axis add instead of cancel. See Fig. 3(b).

This study of ultrasonic generation assumes specular reflection of the electrons at the surface of the metal. Inclusion of a surface force that would result from diffuse scattering[9] does not change the results significantly. This insensitivity to boundary conditions has been noted previously.[10]

The emphasis of this work is on the location of the cusp described above, and on how that location can be used to measure the deviation of R_H from R_0. Zhu and Overhauser[1] found that some experiments indicate deviations as large as 30%, which could arise only from a highly preferred orientation of the charge-density-wave wave vector. It seems important to confirm the existence of such anomalous Hall effects by alternative methods. Single-crystal samples having the required geometry for direct generation experiments have been employed by Penz and Kushida[11] for helicon resonance studies. Their results (on samples with very shiny surfaces) indicated both specular reflection at the surface and a highly oriented charge-density-wave structure.

II. THEORY

Consider a radio-frequency electromagnetic wave incident on a metal surface which is perpendicular to the [110] direction of propagation. A static magnetic field $\vec{\mathrm{H}}_0$ is applied parallel to the [110] direction. For a cubic metal, two shear acoustic modes are generated: a slow mode with velocity proportional to $(c_{11}-c_{12})^{1/2}$ and polarization parallel to the $[1\bar{1}0]$ direction, and a fast mode with velocity proportional to $c_{44}^{1/2}$ and polarized along the [001] axis. As expected, from Maxwell's equations, the Boltzmann transport equation and the elastic wave equation, one can calculate the amplitudes of the generated acoustic waves. The calculation is simplified by the fact that the ionic current is small compared to the electronic current because the electromechanical coupling is weak.

We have, for the two transverse modes, the following wave equations:

$$\rho\frac{\partial^2\xi_y}{\partial t^2}=F_y+C_{44}\frac{\partial^2\xi_y}{\partial z^2}\ , \tag{1}$$

$$\rho\frac{\partial^2\xi_x}{\partial t^2}=F_x+\frac{c_{11}-c_{12}}{2}\frac{\partial^2\xi_x}{\partial z^2}\ , \tag{2}$$

where the z axis is along [110]. $\vec{\mathrm{F}}$ is the force on the ions (per unit volume):

$$\vec{\mathrm{F}}=n\left[e\vec{\mathrm{E}}-\frac{e\,\vec{\mathrm{j}}}{\sigma_0}\right]\ , \tag{3}$$

where σ_0 is the zero-field conductivity

$$\sigma_0=\frac{ne^2\tau}{m}\ , \tag{4}$$

τ is the relaxation time, and n is the number of electrons (and ions) per unit volume.

Assuming $\vec{\mathrm{E}}(z,t)=\vec{\mathrm{E}}(z)\exp(-i\omega t)$, etc., we find that Eqs. (1) and (2) reduce to

$$\frac{d^2\xi_i(z)}{dz^2}+\left[\frac{\omega}{S_i}\right]^2\xi_i(z)=-\frac{F_i(z)}{\rho S_i^2}\ , \tag{5}$$

with

$$S_x=[(c_{11}-c_{12})/2\rho]^{1/2} \tag{6}$$

and

$$S_y=(c_{44}/\rho)^{1/2}\ . \tag{7}$$

The solution of Eq. (5) is[12]

$$\xi_i(z)=\frac{i}{\rho\omega S_i}\left[-\cos(q_iz)\int_z^\infty e^{iq_iz'}F_i(z')dz' + e^{iq_iz}\int_0^z\cos(q_iz')z'dz'\right]\ , \tag{8}$$

where

$$q_i=\omega/S_i\ . \tag{9}$$

Then, the amplitude of the sound at the observation point is (upon neglect of any ultrasonic attenuation)

$$|\,\xi_i(\infty)\,|=\frac{1}{\rho\omega S_i}\left|\int_0^\infty\cos(q_iz)F_i(z)dz\right|\ . \tag{10}$$

For specular reflection $F_i(z)$ is even in z, so that

$$|\,\xi_i(\infty)\,|=\frac{\pi}{\rho\omega S_i}\,|\,\tilde{F}_i(q_i)\,|\ , \tag{11}$$

where $\tilde{F}_i(q_i)$ is the Fourier transform of $F_i(z)$.

We need only determine $F(z)$ in Eq. (3). For this we employ the relation between $\vec{\mathrm{j}}$ and $\vec{\mathrm{E}}$ obtained from the Boltzmann transport equation[13] (we shall assume that σ_0

has cylindrical symmetry about the z axis, since the crystal is cubic),

$$\tilde{j}^{\pm}(q)=\sigma_0\tilde{G}^{\pm}(q)\tilde{E}^{\pm}(q) , \tag{12}$$

where $\tilde{j}^{\pm}\equiv\tilde{j}_x\pm i\tilde{j}_y$, etc., and

$$\tilde{G}^{\pm}(q)=\frac{e^2}{4\pi^2\hbar^2\sigma_0}\int_{-k_F}^{k_F}dk_z\frac{m\tau v_\perp^2}{1+i(\omega\tau\mp\omega_c\tau-qv_z\tau)} . \tag{13}$$

$\vec{v}$ is the electron velocity, $\omega_c\equiv eH_0/mc$ is the cyclotron frequency, and k_F is the radius of the Fermi sphere. Then, we have

$$\tilde{F}^{\pm}(q)=ne\tilde{E}^{\pm}(q)[1-\tilde{G}^{\pm}(q)] . \tag{14}$$

$\tilde{E}(q)$ can be obtained from Maxwell's equations (one may neglect the displacement current)

$$\frac{d^2E_y(z)}{dz^2}=-\frac{i\omega}{c}\left[\frac{4\pi}{c}j_y(z)\Theta(z)+2H_x(0)\delta(z)\right] , \tag{15}$$

$$\frac{d^2E_x(z)}{dz^2}=-\frac{i\omega}{c}\left[\frac{4\pi}{c}j_x(z)\Theta(z)-2H_y(0)\delta(z)\right] , \tag{16}$$

where $\Theta(z)$ is the step function. From these equations we find, for $z>0$,

$$\frac{\partial^2E^{\pm}(z)}{dz^2}=-\frac{i\omega4\pi}{c^2}j^{\pm}(z)\pm\frac{2\omega}{c}H^{\pm}(0)\delta(z) . \tag{17}$$

After using Eq. (12) and integrating, we obtain

$$\tilde{E}^{\pm}(q)=\frac{\mp\omega}{c\pi q^2}H^{\pm}(0)\Big/\left[1-\frac{4\pi i\omega}{c^2q^2}\sigma_0\tilde{G}^{\pm}\right] . \tag{18}$$

Substitution of this result into Eq. (14) leads to

$$\tilde{F}^{\pm}(q)=\frac{\mp ne\omega}{c\pi q^2}\left[\frac{[1-\tilde{G}^{\pm}(q)]H^{\pm}(0)}{1-\frac{4\pi i\omega}{c^2q^2}\sigma_0\tilde{G}^{\pm}(q)}\right] . \tag{19}$$

We are now prepared to find the amplitude of the acoustic wave for the slow shear mode, for which Eq. (11) becomes

$$|\xi_x|=\frac{\pi}{\rho\omega S_x}\left|\frac{\tilde{F}^{+}(q)+\tilde{F}^{-}(q)}{2}\right| . \tag{20}$$

Equation (19) involves $\tilde{G}^{\pm}(q)$, Eq. (13), which can be evaluated by a tedious integration. $H^{\pm}(0)$ is, of course, the amplitude of the driving rf field at the surface of the sample.

III. RESULTS AND CONCLUSIONS

In order to evaluate $G^{\pm}$ from Eq. (13), we must specify the electronic energy spectrum. For simplicity we use the free-electron model. We have also performed calculations based on a modified $E(\vec{k})$ associated with a charge-density-wave structure. The results obtained did not differ significantly from the ones presented here using $E(\vec{k})=\hbar^2k^2/2m$. Equation (13) can be analytically integrated and the result substituted in Eqs. (19) and (20). Numerical results were obtained for $ql=6$ and $\nu=10$ MHz, values appropriate to the experiments of Puskorius and Trivisonno.[8] (For ql between 4 and 10 the results do not change significantly.)

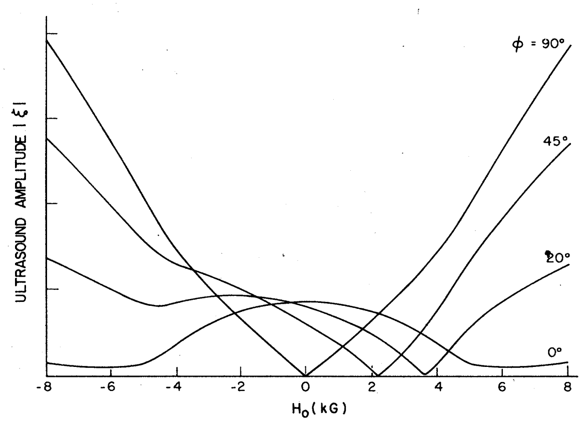

FIG. 4. Amplitude of the ultrasonic slow shear wave as a function of the static magnetic field H_0. ($\nu=10$ MHz.)

Figure 4 shows the amplitude of the slow shear wave as a function of the static magnetic field for several orientations, ϕ, of the rf field. Note the striking variation with ϕ, and the asymmetry about $H_0=0$, whenever $0<|\phi|<90°$. (ϕ is the angle between the rf electric field $\vec{E}$ and the $[1\bar{1}0]$ axis; positive ϕ corresponds to $\vec{E}$ rotated away from $[1\bar{1}0]$ in a direction opposite to the cyclotron rotation.) Figure 5 juxtaposes theory and experiment. Since the experimental orientation of the rf field was unknown, a truly meaningful comparison cannot be made. We present, however, the theoretical curve for $\phi=20°$, which optimizes the qualitative similarity.

It is the sharp, cusplike minimum in the ultrasonic amplitude (versus H_0) which is the focus of this paper. Its location depends strongly on ϕ, as shown in Fig. 6. Also shown are the corresponding curves that result if the Hall coefficient exceeds its classical value by a factor t. An anomalous (and anisotropic) Hall coefficient in K is a manifestation of the charge-density-wave broken symme-

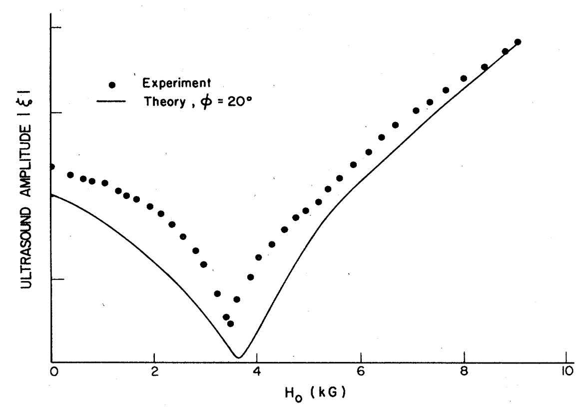

FIG. 5. Comparison between free-electron theory and experiment in K at 4.2 K. The orientation angle of the rf field ($\phi=20°$ for the theoretical curve) was not measured. ($\nu=10$ MHz.)

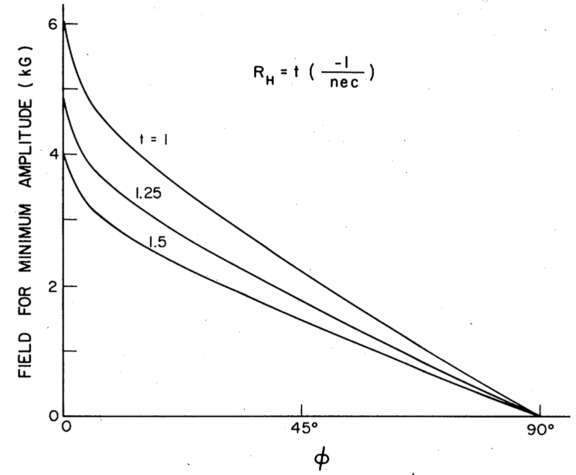

FIG. 6. Variation of the field for minimum amplitude, e.g., Fig. 4, as a function of the orientation ϕ of the rf field and the Hall coefficient factor t. ($\nu = 10$ MHz.)

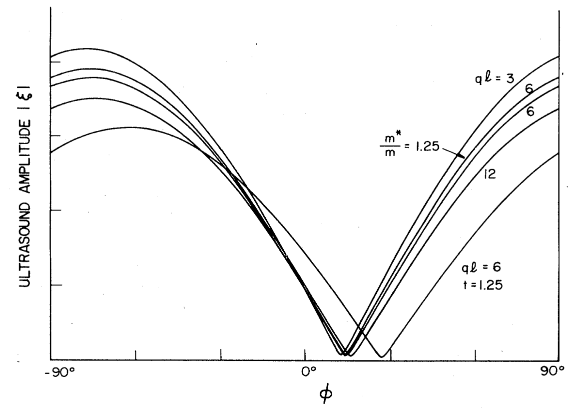

FIG. 8. Generation amplitude vs ϕ for $H_0 = 4$ kG. The variation with ql for $m^* = m$, $t = 1$ is shown. The small change caused by $m^* = 1.25m$ contrasts with a (eightfold) larger change arising from a Hall coefficient $= 1.25R_0$. ($\nu = 10$ MHz.) All curves are for $m^* = m$ and $t = 1$ except where indicated.

try.[1] It is caused by the multiply connected Fermi surface arising from the minigaps and heterodyne gaps, which come into existence from the presence of two incommensurate periodic potentials in the Schrödinger equation.[14] The effect of an altered Hall coefficient can be incorporated into the theory given above by replacing ω_c with $t\omega_c$ in Eq. (13). The net effect is that the value of H_0 for the cusplike minimum is reduced by the factor t. This scaling is illustrated in Fig. 6.

An equivalent determination of the Hall coefficient can also be made by the location of the cusp in the ultrasonic amplitude versus ϕ (for fixed H_0). Typical theoretical curves (for $t = 1$) are shown in Fig. 7. If $t \neq 1$, the general scaling relation is $\xi(\phi, H) = \xi_0(\phi, H/t)$.

It is important to note that changing m to m^*, which also alters ω_c, does *not* lead to a similar change. The effective mass also enters σ_0 and $\vec{v}$ in Eqs. (13) and (19) as well as ω_c. But these effects tend to cancel. For the free-electron model $\tilde{G}^{\pm}(q)$, Eq. (13) can be written as

$$G^{\pm}(q) = \frac{3}{4}\int_0^{\pi} \frac{\sin^3\alpha\, d\alpha}{1 + i[\omega\tau \mp \omega_c\tau - (q\tau k\hbar\cos\alpha/m)]}, \tag{21}$$

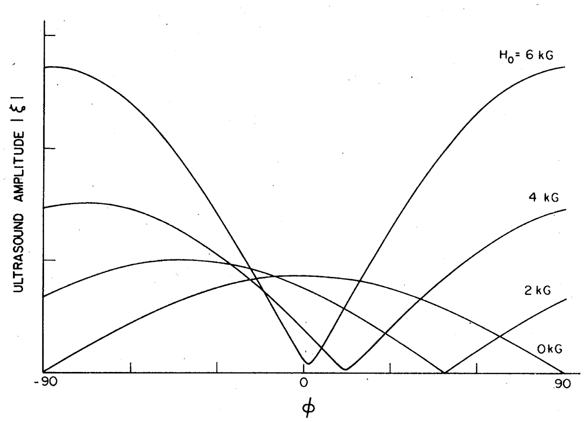

FIG. 7. Amplitude of the slow shear ultrasonic wave as a function of the rf orientation angle ϕ for fixed values of H_0. ($\nu = 10$ MHz.)

where $v_z \equiv v\cos\alpha$. If we change m to m^*, we have

$$G^{\pm *}(q) = \frac{3}{4}\int_0^{\pi} \frac{\sin^3\alpha\, d\alpha}{1 + i[\omega\tau \mp (eH_0\tau/m^*c) - (q\tau k\hbar\cos\alpha/m^*)]}. \tag{22}$$

Now, in the expression for $\tilde{F}^{\pm}(q)$, Eq. (19), we note that in the denominator $\tilde{G}^{\pm}(q)$ is multiplied by $\sigma_0 = ne^2\tau/m^*$. Therefore, for the dominant terms in Eq. (19), the masses cancel, i.e.,

$$\sigma_0^* \tilde{G}^{\pm *}(q) = \frac{3ne^2\tau}{4}\int_0^{\pi} \frac{\sin^3\alpha\, d\alpha}{m^* + i[\omega\tau m^* \mp (eH_0\tau/c) - q\tau k\hbar\cos\alpha]}. \tag{23}$$

The residual effect of an effective mass is relatively small, since the last two terms of the denominator are the dominant ones.

In Fig. 8 we illustrate how an increase in effective mass of 25% leaves the position of the minimum almost unchanged. In contrast, a 25% change in the Hall coefficient has a large effect. Also shown is the slight dependence of the minimum on ql.

Since ql can generally be determined from an independent experiment, we believe that the cusplike feature found in direct generation experiments for a "free-electron" metal (such as K) can be used to probe the Hall coefficient. We anticipate that results will differ from specimen to specimen (on account of varying domain texture of the charge-density wave). Accordingly such studies can possibly be used as a probe of domain orientation and its dependence on sample preparation.

ACKNOWLEDGMENT

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