Amorphization of a quasi-one-dimensional Ising ferromagnet

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The amorphization of a quasi-one-dimensional Ising ferromagnet is investigated with the use of a new type of effective-field theory with correlations. The exchange parameters of interchain and intrachain interactions are assumed to have δ-function distributions. We find some characteristic behavior for the amorphization.

I. INTRODUCTION

The magnetism of structurally amorphous alloys has become the subject of both experimental and theoretical interest in solid-state physics. A number of experimental and theoretical investigations have led to the results that magnetic long-range order may exist in amorphous systems, although the atomic states in these solids are inequivalent both structurally and magnetically. At the same time, because of this disordered structure, many interesting physical properties not observed in the corresponding crystalline magnets are now becoming apparent.[1]

Theoretically, there exists a great amount of sophisticated techniques. With regard to the difficulties of the theoretical description of such complicated magnetic systems, it is sometimes necessary to do some simplifications. Therefore, for studying such systems, the lattice model of amorphous magnets has often been applied, in which the structural disorder is replaced by the random distribution of the exchange integral. In fact, experiments of the Mössbauer effect and the magnetization of amorphous ferromagnets indicate that, at least in some materials, there are large fluctuations in the exchange interaction. It is also found that the magnetization of an amorphous ferromagnet is in general lower than that of its crystalline counterpart. That fluctuation may be that underlying cause for amorphous magnets has been proposed by Gubanov,[2] Kaneyoshi,[3] and Handrich.[4]

On the other hand, a number of crystalline quasi-one-dimensional magnets have been found and studied in detail. The type of magnetic order and the temperature of the magnetic phase transition in quasi-one-dimensional magnets are determined by the weak exchange interactions which couple the magnetic chains. In the crystalline state, such substances are characterized by only two intrachain and interchain exchange-interaction parameters, the magnitudes of which are fixed over the whole crystal. For the amorphization (for instance, due to the fast-neutron irradiation) of a quasi-one-dimensional system, however, the exchange interactions do not distribute uniformly, and the magnetic properties may change essentially from those of the crystalline state.

Recently, Kaneyoshi and co-workers[5] have developed for the spin-$\frac{1}{2}$ pure Ising model a new type of effective-field theory with correlations (based on the use of a convenient differential operator in the Callen's spin-correlation identity[6]). The theory, within a mathematically simple framework, substantially improves the standard molecular-field approximation (MFA) results. This approach shares with the MFA a great versatility and has already been applied to a variety of interesting situations, such as pure systems,[7] site-random and bond-random magnets,[8] amorphous magnets,[9] the transverse Ising model,[10] the Potts model,[11] and surface problems.[12] Especially, in the previous papers,[13,14] we have investigated the amorphization of a diluted crystalline Ising ferromagnet in a square lattice by using the theory. Some interesting effects of amorphization came up in the thermal behavior of the physical properties (in particular, the magnetization, the susceptibility, and the high-field magnetization).

In this paper, the physical properties of a quasi-one-dimensional Ising spin-$\frac{1}{2}$ system with randomly distributed interchain and intrachain exchange parameters are investigated by using the effective-field theory, in order to clarify the effect of amorphization on the crystalline quasi-one-dimensional system. We calculate the most relevant thermodynamical quantities (the transition temperature, the phase diagram, the magnetization, and the initial susceptibility).

The outline of this paper is as follows. In Sec. II, we briefly review the basic points of the simple effective-field theory with correlations. In Sec. III, it is applied to the problem for the amorphization of a quasi-one-dimensional ferromagnet. The analytical forms of the relevant thermodynamical properties are obtained. In Sec. IV, the numerical results of the quantities are studied and discussed. We find some interesting behavior characteristic of the amorphization of a quasi-one-dimensional ferromagnet.

II. THEORY

The system consists of N identical spins, $\mu_i = \pm 1$, arranged on a square lattice. The Hamiltonian is given by

$$H = -\tfrac{1}{2}\sum_{i,j} J_{ij}\mu_i\mu_j - H\sum_i \mu_i \ , \tag{1}$$

where J_{ij} are the exchange interactions with $J_{ii}=0$, and H is the applied magnetic field. In order to describe the amorphization of a quasi-one-dimensional system, the stochastic lattice model is used; as shown in Fig. 1, the

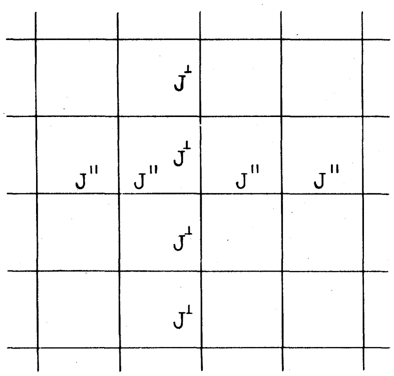

FIG. 1. Exchange interactions in a square lattice.

nearest-neighbor exchange interactions are given by two independent random variables,

$$P(J_{ij}^{\|})=\tfrac{1}{2}[\delta(J_{ij}^{\|}-J-\Delta J)+\delta(J_{ij}^{\|}-J+\Delta J)]\ , \tag{2}$$

and

$$P(J_{ij}^{\perp})=\tfrac{1}{2}[\delta(J_{ij}^{\perp}-J_{\perp}-\Delta J_{\perp})+\delta(J_{ij}^{\perp}-J_{\perp}+\Delta J_{\perp})]\ , \tag{3}$$

where the coupling constant $J_{\perp}$ is assumed to be weaker than J.

Formal identities for the correlation functions of the Ising model have appeared in the literature for the same time. The starting point for the statistics of our spin system is the exact relation due to Callen[6]

$$\langle \mu_i \rangle = \left\langle \tanh\left[h+\beta\sum_j J_{ij}\mu_j\right]\right\rangle\ , \tag{4}$$

where $h=\beta H$ and $\beta=(k_B T)^{-1}$. The angular brackets indicate the usual ensemble average

$$\langle\langle\cdots\rangle\rangle=\mathrm{Tr}[\exp(-\beta H)(\cdots)]/\mathrm{Tr}\exp(-\beta H)\ .$$

Here, in order to write the identity (4) in a form which is particularly amenable to approximation, let us introduce the differential-operator technique proposed by Honmura and Kaneyoshi[5] as follows:

$$\begin{aligned}\sigma_i=\langle\mu_i\rangle&=\left\langle \exp\left[D\sum_j J_{ij}\mu_j\right]\right\rangle \tanh(h+x)\,|_{x=0}\\ &=\left\langle \prod_j [\cosh(Dt_{ij})+\mu_j\sinh(Dt_{ij})]\right\rangle \tanh(h+x)\,|_{x=0}\ ,\end{aligned} \tag{5}$$

where $D=\partial/\partial x$ is a differential operator and $t_{ij}=\beta J_{ij}$.

By assuming the statistical independence of lattice sites, namely,

$$\langle \mu_i\mu_j\cdots\mu_l\rangle \cong \langle\mu_i\rangle\langle\mu_j\rangle\cdots\langle\mu_l\rangle\ ,$$

Eq. (5) may be rewritten as

$$\begin{aligned}\sigma_i=\prod_\delta &[\cosh(Dt_{i,i+\delta})+\delta_{i+\delta}\sinh(Dt_{i,i+\delta})]\\ &\times\tanh(h+x)\,|_{x=0}\ ,\end{aligned} \tag{6}$$

where δ only takes the nearest neighbors of a central site i. That is to say, in a disordered system, spin-spin correlation should be more reduced than that of its corresponding nonrandom system. The approximation has led, in spite of its simplicity, to quite satisfactory results. In fact, the approximation essentially corresponds to the Zernike approximation in the nonrandom problem.[15,16] The formalism has been applied to a number of disordered magnetic systems.[8–15]

For a disordered system with random bonds, we must perform the random configurational average to Eq. (6). In the case that the exchange interactions are given by independent random variables, as discussed in Ref. 15, Eq. (6) reduces to, upon performing the random average,

$$\begin{aligned}m&=\langle\sigma_i\rangle_r\\ &=\prod_\delta[\langle\cosh(Dt_{i,i+\delta})\rangle_r+m\langle\sinh(Dt_{i,i+\delta})\rangle_r]\\ &\qquad\times\tanh(h+x)\,|_{x=0}\ ,\end{aligned} \tag{7}$$

where $\langle\cdots\rangle_r$ expresses the random-bond average. By means of (2) and (3), the random-bond averages are then given by

$$\begin{aligned}&\langle\cosh(Dt_{i,i+\delta}^{\|})\rangle_r=\cosh(2Dt\delta)\cosh(Dt)\\ &\langle\sinh(Dt_{i,i+\delta}^{\|})\rangle_r=\cosh(2Dt\delta)\sinh(Dt)\\ &\langle\cosh(Dt_{i,i+\delta}^{\perp})\rangle_r=\cosh(2Dt\alpha\delta_{\perp})\cosh(D\alpha t)\\ &\langle\sinh(Dt_{i,i+\delta}^{\perp})\rangle_r=\cosh(2Dt\alpha\delta_{\perp})\sinh(D\alpha t)\ ,\end{aligned} \tag{8}$$

where $t=\beta J$, $\delta=\Delta J/2J$, $\alpha=J_{\perp}/J$, and $\delta_{\perp}=\Delta J_{\perp}/2J_{\perp}$. As mentioned above, the parameter α is assumed to be $\alpha\le 1$. The factors δ and $\delta_{\perp}$ are dimensionless parameters which measure the amount of fluctuation of exchange interactions. As noted in Ref. 13, the result (8) can also be obtained by using the so-called "lattice model" of amorphous magnets discussed by Handrich.[4]

In this section, we have briefly reviewed the effective-field theory with correlations in a Ising ferromagnet with random bonds. We are in a position to examine the effects of amorphization in a quasi-one-dimensional ferromagnet on the most relevant thermodynamical quantities, such as the phase diagram, the magnetization, the transition temperature, and the initial susceptibility. In the following sections, we shall study the physical properties within this framework.

III. AMORPHIZATION OF A QUASI-ONE-DIMENSIONAL FERROMAGNET

In this section, let us study the amorphization of a quasi-one-dimensional ferromagnet, as depicted in Fig. 1. By the use of (8), Eq. (7) can be expanded, i.e.,

$$m=2Am+2Bm^3+h[C+Dm^2+Em^4]+O(h^2)\ , \tag{9}$$

with

$$A=\cosh^2(2Dt\delta)\cosh^2(2Dt\delta_\perp\alpha)[\cosh(Dt)\sinh(Dt)\cosh^2(D\alpha t)+\cosh^2(Dt)\cosh(D\alpha t)\sinh(D\alpha t)]\tanh(x)\,|_{x=0}\,, \quad (10)$$

$$B=\cosh^2(2Dt\delta)\cosh^2(2Dt\delta_\perp\alpha)[\sinh^2(Dt)\cosh(D\alpha t)\sinh(D\alpha t)+\cosh^2(Dt)\sinh(Dt)\sinh^2(D\alpha t)]\tanh(x)\,|_{x=0}\,, \quad (11)$$

$$C=\cosh^2(2Dt\delta)\cosh^2(2Dt\delta_\perp\alpha)\cosh^2(Dt)\cosh^2(D\alpha t)\mathrm{sech}^2(x)\,|_{x=0}\,, \quad (12)$$

$$D=\cosh^2(2Dt\delta)\cosh^2(2Dt\delta_\perp\alpha)[\sinh^2(Dt)\cosh^2(D\alpha t)+\cosh^2(Dt)\sinh^2(D\alpha t)$$
$$+4\cosh(Dt)\sinh(Dt)\cosh(D\alpha t)\sinh(D\alpha t)]\mathrm{sech}^2(x)\,|_{x=0}\,, \quad (13)$$

and

$$E=\cosh^2(2Dt\delta)\cosh^2(2Dt\delta_\perp\alpha)\sinh^2(Dt)\sinh^2(D\alpha t)\mathrm{sech}^2(x)\,|_{x=0}\,. \quad (14)$$

Equation (9) was then derived by expanding $\tanh(h+x)$ in Eq. (7) with h and retaining the terms linear to h.

For $h=0$, the averaged magnetization m is given by

$$m=\left[\frac{1-2A}{2B}\right]^{1/2}. \quad (15)$$

The critical ferromagnetic frontier can be derived from the condition

$$2A-1=0\,, \quad (16)$$

by which the transition temperature and the phase diagram can be determined as functions of the three parameters, α, δ, and $\delta_\perp$. Then, by applying a mathematical relation $e^{\alpha D}f(x)=f(x+\alpha)$, all the coefficients (10)–(14) can be expressed as a sum of transcendental functions $\tanh X$ (or $\mathrm{sech}^2 X$) with an appropriate argument X. That is to say, for the special case of $\alpha=1$ and $\delta=\delta_\perp=0$, the coefficient A is given by

$$A_0=\tfrac{1}{4}[\tanh(4t)+2\tanh(2t)]\,.$$

From Eq. (16), the critical temperature associated with $\alpha=1$ and $\delta=\delta_\perp=0$ (which, within the present description, corresponds to the pure square lattice) is given by $T_c=3.0898J/k_B$, which is to be compared with the result Bethe-Peierls $T_c=2.8854J/k_B$ (MFA leads to $T_c=4J/k_B$). For the pure case, on the other hand, the parameter B is given by

$$B_0=\tfrac{1}{4}[\tanh(4t)-2\tanh(2t)]\,.$$

At $T=0$, therefore, the parameters A_0 and B_0 reduce to the values $A_0=\frac{3}{4}$ and $B_0=\frac{1}{4}$, by which the magnetization is given by, upon using Eq. (15), $m=1$. Thus the magnetization is well defined at $T=0$.

The initial susceptibility is defined by

$$\chi=\lim_{H\to 0}\frac{\partial m}{\partial H}=\frac{t}{J}\left.\frac{\partial m}{\partial h}\right|_{h=0}, \quad (17)$$

from which the inverse susceptibility is given by, on using Eq. (9),

$$(J\chi)^{-1}=\frac{1-2A-6Bm^2}{t[C+Dm^2+Em^4]}\,. \quad (18)$$

Thus, the effects of the amorphization on the thermodynamical quantities in the quasi-one-dimensional ferromagnet can be obtained by solving Eqs. (15), (16), and (18) numerically. The results are shown in the next section.

IV. NUMERICAL RESULTS AND DISCUSSION

A. Transition temperature and phase diagram

By solving Eq. (16), the critical frontiers in the (T,δ) space are plotted in Figs. 2–4 for typical values of α and $\delta_\perp$. For the case of $\delta_\perp=\delta$, the critical frontiers are shown in Fig. 2, in which the curve (a) is the same as that (the curve labeled as $P=1.0$ in Fig. 4 of Ref. 13) for the amorphization of a diluted crystalline Ising ferromagnet in a square lattice. Increasing the fluctuation of exchange couplings, i.e., δ, the transition temperature, as is well known, decreases. For the curve (a) with $\alpha=1$, the change of the transition temperature for small δ is given by

$$\frac{\Delta T_c}{T_c^0}=-0.3437\delta^2\,,$$

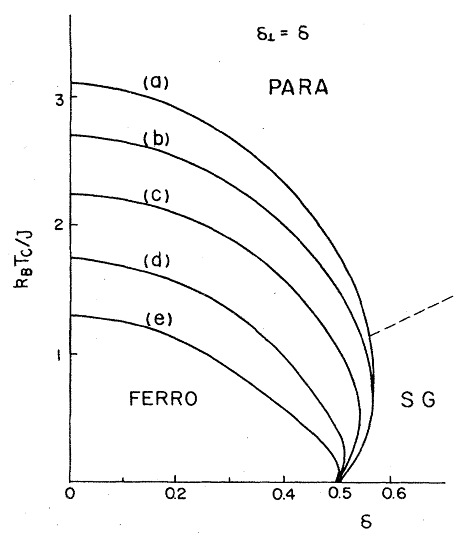

FIG. 2. Phase diagrams in the (T,δ) space for the systems with $\delta_\perp=\delta$. Here the interface between the spin-glass (SG) and the paramagnetic phases (dashed line) is that predicted by the usual MFA (Ref. 18). (a)–(d) denote the magnitudes of the parameter α; (a) $\alpha=1$, (b) $\alpha=0.75$, (c) $\alpha=0.5$, (d) $\alpha=0.25$, and (e) $\alpha=0.1$.

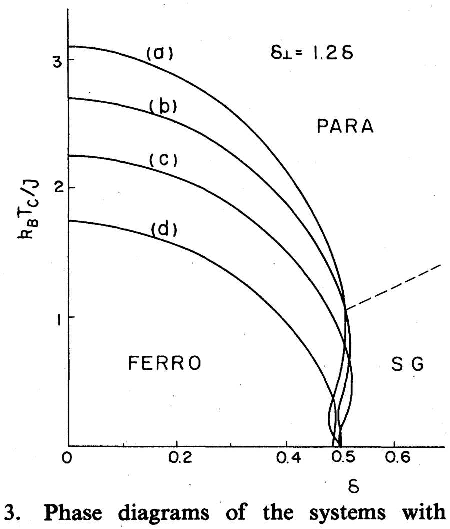

FIG. 3. Phase diagrams of the systems with $\delta_\perp = 1.2\delta$. (a)—(d) denote the magnitudes of the parameter, as indicated in Fig. 2.

where T_c^0 is the critical temperature of the ordered system with $\delta=0$, i.e., $T_c^0 = 3.0898J/k_B$ and $\Delta T_c = T_c - T_c^0$. Quantitatively similar results have found in other theories for the lattice model of amorphous ferromagnets.[17] Thus the effect of treating the exchange interaction as a random variable is commonly to lower the transition temperature by an amount proportional to the second moment of the distribution of the bond strengths.

In curve (a) of Fig. 2, on the other hand, there exist two possible different values of T_c for a given value of δ in the range $0.5 < \delta < 0.565$. As discussed in Refs. 13 and 14, if we admit the existence of a spin-glass phase below a small value of the two T_c's, the result may support the reentrant phenomenon that the transition from the spin-glass phase to the paramagnetic phase passing through the ferromagnetic phase is possible (see also Fig. 5). For the isotropic fluctuation of exchange couplings, i.e., $\delta_\perp = \delta$, as seen from the figure, both the ferromagnetic region and the region of δ showing the reentrant phenomenon become narrower than those of $\alpha = 1$, when the value of α decreases (the system becomes a quasi-one-dimensional one).

In Figs. 3 and 4, the critical frontiers in the (T,δ) space are depicted for the two typical cases of the anisotropic fluctuations ($\delta_\perp \neq \delta$). For the systems with $\delta_\perp = 1.2\delta$, as shown in Fig. 3, the transition temperatures of small values of δ take nearly the same values as the corresponding ones of Fig. 2. However, the behavior of the curves near $\delta = 0.5$ in Fig. 3 are very different from those of Fig. 2. In curve (a) of Fig. 3, for instance, there exist three possible different values of T_c for a given value of δ in the range $0.48 < \delta < 0.50$, although the region becomes narrower, on decreasing α. That is to say, in contrast with the cases of Fig. 2 expressing the reentrant phenomenon discussed above, for the selected value (δ,α) in the region the ferromagnetic phase (in which the system may be inhomogeneous) exists below the smallest transition temperature of the three T_c, although the system exhibits the reentrant phenomenon at the other two T_c (see also the magnetization curve with $\delta = 0.49$ in Fig. 6). Thus, owing to the different fluctuations between intrachain and interchain exchange interactions, it seems that the effect of frustration may be suppressed partially and the peculiar behavior appears at very low temperatures. For the case (d) with $\alpha = 0.25$ in Fig. 3, on the other hand, the curve has the form similar to the corresponding curve [curve (d)] in Fig. 2; since the strength of $J_\perp$ is weaker than that of J, the effect from the interchain fluctuation $\delta_\perp$ on the critical frontier is reduced there.

In Fig. 4, the critical frontiers in the (T,δ) space are depicted for $\delta_\perp = 0.5\delta$. The results are very different from those of Fig. 2 and 3. That is to say, for the weak interchain fluctuation, the effect of frustration is more suppressed in comparison with those of Figs. 2 and 3. The reentrant phenomenon seems to be impossible, since the interchain interactions $J_{ij}^\perp$ are always positive even for large δ. Instead of it, the critical frontiers of the systems with a given value of α in the range $0.75 < \alpha \le 1$ reveal the tails for large values of δ. As will be shown later, the behavior of the physical properties (the magnetization and the susceptibility) for large values of δ are very different from those for small values of δ.

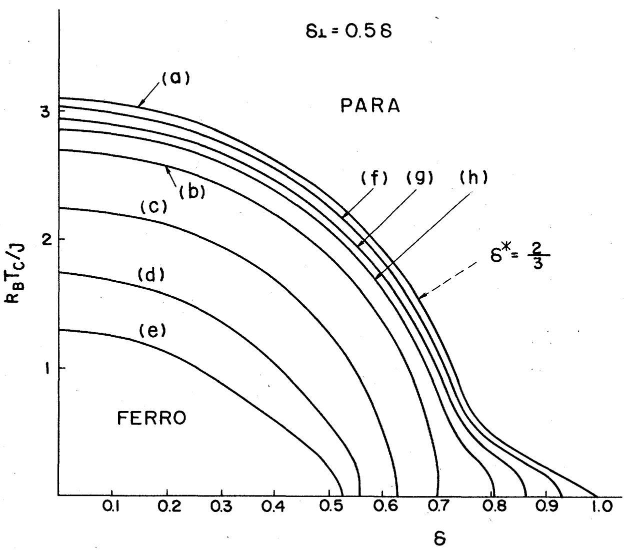

FIG. 4. Phase diagrams of the systems with $\delta_\perp = 0.5\delta$. The meaning of (a)—(e) is the same as that of Fig. 2. (f), (g), and (h) denote (f) $\alpha = 0.95$, (g) $\alpha = 0.9$, and (h) $\alpha = 0.85$.

B. Magnetization

By solving Eq. (15), the behavior of the averaged magnetization versus temperature for fixed pair of values (δ,α) are presented in Figs. 5—8 under the given restrictions of δ. From Fig. 5 depicted for $\delta_\perp = \delta$, we can see that for selected values (δ,α) [see, for instance, the solid curve labeled $(0.505,a)$] the magnetization, which does not exist until a certain temperature, starts to increase, passes through a maximum value, and decreases to zero with increasing temperature. The behavior of the averaged magnetization is similar to the experimental results of amorphous ferromagnets, showing the reentrant phenomenon.

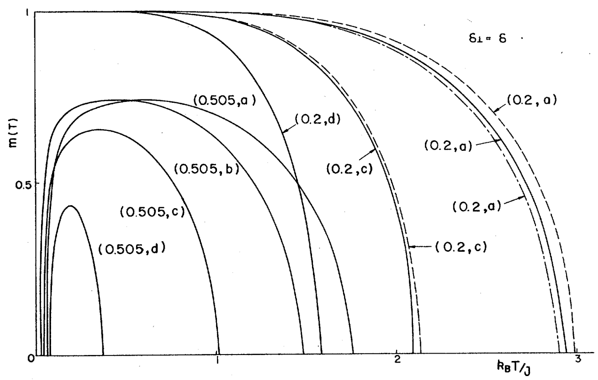

FIG. 5. Averaged magnetizations vs temperature for the systems with $\delta_\perp = \delta$. The numbers and letters associated with each curve denote the fixed pair of values (δ,α). The a–d indicate the magnitudes of α, corresponding to (a)—(d) in Fig. 2. The dash and dashed-dotted lines are for the systems with $\delta_\perp = 1.2\delta$ and for the system with $\delta_\perp = 0.5\delta$, respectively.

In relation to the result, it is worth noting the curves in Fig. 2; the value $\delta = 0.505$ is in the range $0.5 < \delta < 0.565$. The magnetization curves showing the reentrant phenomenon become gradually symmetric, on decreasing α. In the figure, the magnetization curves of the fixed value of δ ($\delta = 0.2$) are also plotted for selected values of α; it is clarified how the magnetization curve changes, when the system becomes quasi-one-dimensional. For comparison, we show in the figure the magnetization curves of the anisotropic exchange fluctuations ($\delta_\perp = 1.2\delta$ and $\delta_\perp = 0.5\delta$), especially for the system with $\delta = 0.2$ and corresponding values of α. For the small value of δ the magnetization curves take the same form as that of $\delta_\perp = \delta$, when the value of α decreases nearly to $\alpha = 0.25$. The result can be also seen from the phase diagrams of Figs. 2—4.

For the case of $\delta_\perp = 1.2\delta$, the magnetization curves of selected values (δ,α) are presented in Fig. 6. As mentioned in Sec. IV A, for the curve labeled $(0.49,a)$ the magnetization starts to decrease from the value ($m = 0.577$) at $T = 0$, and disappears at the smallest value of the three T_c's, although it shows the reentrant phenomenon at the other two values of T_c. In the figure, the curves of the fixed value of δ, i.e., $\delta = 0.4$, are also plotted, on varying α. It is seen that the magnetization for a large value of α [for instance the curve labeled $(0.4,a)$] initially decreases linearly from the saturation value at $T = 0$ with increasing temperature. The linear region, however, disappears, when the system becomes to be quasi-one-dimensional [see the curve labeled $(0.4,d)$].

In Fig. 7, the magnetization curves of $\alpha = 1$ are shown for the case of $\delta_\perp = 0.5\delta$, on changing the value of δ. An interesting feature comes up in the figure, in contrast with the results of Figs. 5 and 6; the behavior of magnetization curves [see the curves labeled $(0.6,a)$ and $(0.7,a)$] are very different, on increasing the value of δ from $\delta = 0.6$ to 0.7. In Fig. 8, therefore, the magnetization curves in the region are studied in detail. Then we find that the saturation magnetization at $T = 0$ varies discontinuously from the value of unity to the value given by $m = 0.577$ at a value of δ [δ^* shown in Fig. 4], and the system, at $T = 0$, may

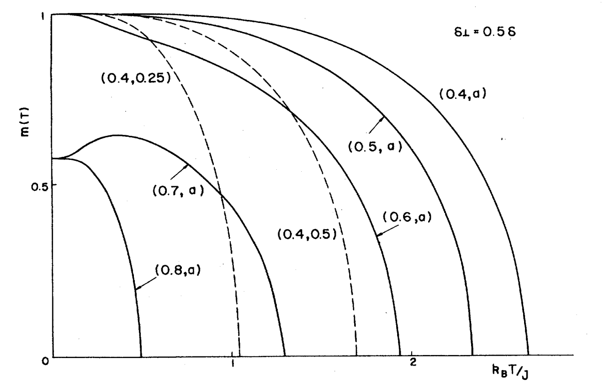

FIG. 7. Averaged magnetization vs temperature for the systems with $\delta_\perp = 0.5\delta$. Lines and labels have the same meaning as those of Fig. 5.

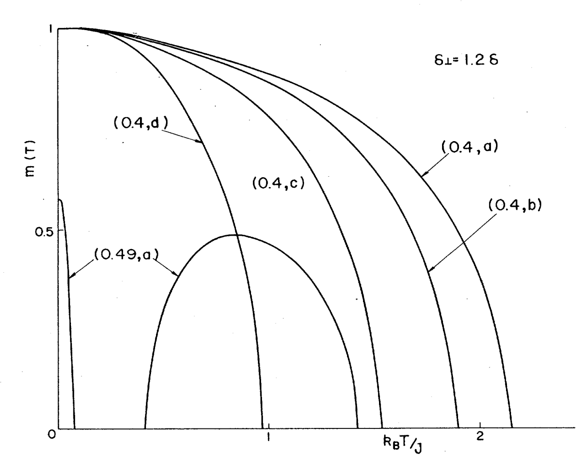

FIG. 6. Averaged magnetizations vs temperature for the systems with $\delta_\perp = 1.2\delta$. The numbers and the letters have the same meaning as those of Fig. 5.

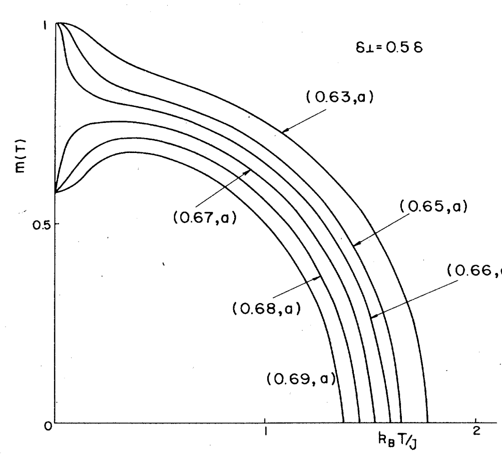

FIG. 8. Averaged magnetizations vs temperature for the systems ($\alpha = 1$ and $\delta_\perp = 0.5\delta$) with δ in the region $0.6 < \delta < 0.7$.

exhibit a first-order phase transition. The result comes from the fact that the transcendental functions $\tanh[at(1-b\delta)]$ with positive a and b are included in the coefficients A and B of Eq. (15), and the functions, at $T=0$, can take values

$$\tanh\{at(1-b\delta)\}=\begin{cases}1 & \text{for } \delta<\dfrac{1}{b}\,,\\[2mm] 0 & \text{for } \delta=\dfrac{1}{b}\,,\\[2mm] -1 & \text{for } \delta>\dfrac{1}{b}\,,\end{cases}$$

depending on the value of δ. The characteristic feature was found in the amorphization problem of a diluted crystalline ferromagnet,[13,14] when we use the amorphization given by Eq. (2). As the value of δ approaches the value of $\delta=\delta^*$, the averaged magnetization starts to decrease abruptly from the saturation value at $T=0$, and then changes in the usual way with increasing temperature. When the value of δ becomes larger than the critical value δ^*, on the other hand, the magnetization initially starts to increase, passes through a maximum value, and then decreases to zero with increasing temperature. For the larger value of δ, however, the magnetization curve decreases monotonically, as seen from the curve labeled $(0.8,a)$ in Fig. 7. Thus, at the point $\delta=\delta^*$ in Fig. 4, the peculiar behavior of magnetization may be observed.

Finally, a great number of experimental and theoretical works have reported that the temperature dependence of the reduced magnetization in amorphous and dilute ferromagnets has a characteristic feature—it consistently falls below that of the corresponding crystalline ferromagnets. Therefore, the temperature dependences of reduced magnetization for some values of δ and α are shown in Fig. 9, in comparison with the pure crystalline system given by $\delta=0$ and $\alpha=1$. Thus, we can understand that the effect of increasing the disorder in the pure system (except the special cases mentioned above) is generally an increase of the depression of the reduced magnetization curve over the entire temperature range for $T\leq T_c$, as observed in dilute and amorphous ferromagnets.[1]

C. Initial susceptibility

By solving Eq. (18), the behavior of the inverse initial susceptibility versus temperature is presented in Figs. 10—13. Except the special cases exhibiting the reentrant phenomenon in Figs. 5 and 6 and the anomalous behavior of magnetization in Fig. 8, the susceptibility diverges only at the transition temperature, as usually observed in ferromagnets, even for the amorphization. In the following, therefore, the inverse susceptibilities are shown in Figs. 10—12 only for the special cases.

An interesting behavior is the case expressing the reentrant phenomenon, namely, each case labeled $(0.505,a)$—$(0.505,d)$ in Fig. 5, for which the susceptibility diverges three times—twice at the critical points where the averaged magnetization disappears and once at $T=0$, because of the existence of finite clusters, as depicted in Fig. 10. The behavior of the inverse susceptibilities at very low temperatures is plotted in the inset of the figure. For the case labeled $(0.49,a)$ in Fig. 6, on the other hand, the susceptibility diverges four times, one at $T=0$ (finite-cluster contribution, as understood from the saturation magnetization given by $m=0.577$ at $T=0$) and three times at the critical points where the magnetization disappears. The result is shown in Fig. 11. Thus, for the cases exhibiting the reentrant phenomenon, we observe the coexistence of a Curie-Weiss—type law with a Curie-type one within one formalism. This was also discussed in Ref. 13 and 14.

In Fig. 12 the inverse susceptibilities of the systems with $\delta_\perp=0.5\delta$ and $\alpha=1$ are depicted, for which the magnetizations exhibit some peculiar behavior, as plotted in Fig. 8. Notice that for approaching the value of δ to δ^* the low-temperature region expresses an extremely interesting behavior; for the system with δ slightly smaller

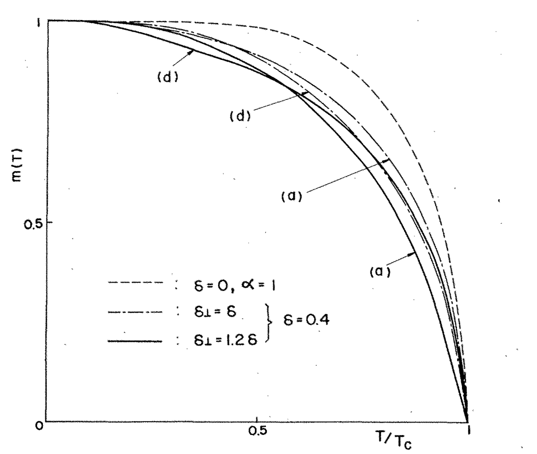

FIG. 9. Reduced magnetization curves for selected values of (δ,α). For comparison, the reduced magnetization curve of the pure system $(\delta=0,\alpha=1)$ in a square lattice is depicted. (a) and (d) denote the values of α; (a) $\alpha=1$ and (d) $\delta=0.25$.

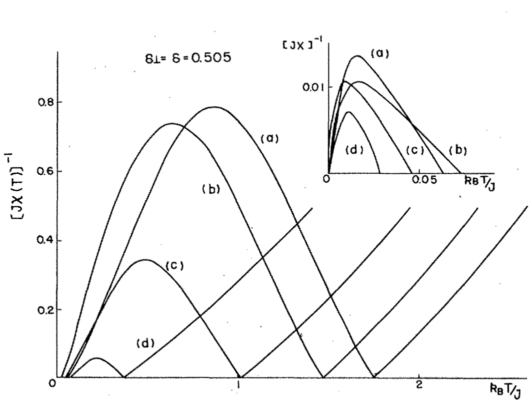

FIG. 10. Thermal dependence of the inverse susceptibility for the systems $(\delta_\perp=\delta=0.505)$ expressing the reentrant phenomenon. The (a)—(d) correspond to those of Fig. 2. The parts of its very low temperatures are depicted in the inset.

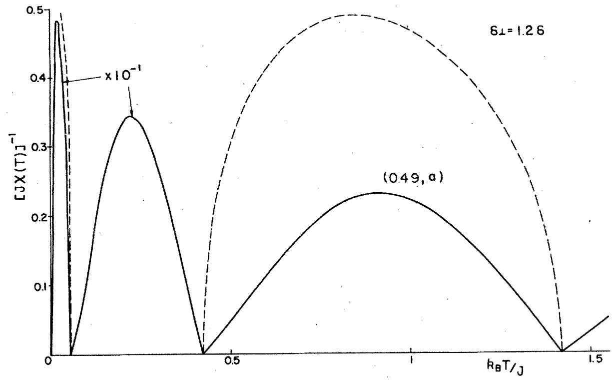

FIG. 11. Thermal dependence of the inverse susceptibility for the system with $\delta_\perp = 1.2\delta$ and $(0.49,a)$ in Fig. 6. The values of the first two maxima must be reduced by 10^{-1}. For reference, the averaged magnetization curve (dashed line) is also plotted.

than δ^* the inverse susceptibility shows a maximum and a minimum below its critical temperature. Below the minimum point, the inverse susceptibility diverges with decreasing temperature, since there are no finite clusters in the system. For the system with δ larger than δ^*, however, the susceptibility diverges twice, once at the critical point and again at $T=0$ (finite-cluster contribution, as seen from Fig. 8). Thus, above and below the critical value δ^* where the system exhibits the first-order phase transition at $T=0$, its physical properties exhibit very peculiar behavior, although we cannot understand the facts only from the phase diagram of Fig. 4.

Finally, in order to observe the behavior of the susceptibility above the transition tempertaure clearly, the inverse paramagnetic susceptibilities χ_{para}^{-1} are shown in Fig. 13 for selected values of δ and α. A characteristic behavior is that the deviation from the Curie-Weiss law is observed more remarkably than that of the pure (0,1) system in a square lattice, on increasing disorder, which phenomenon

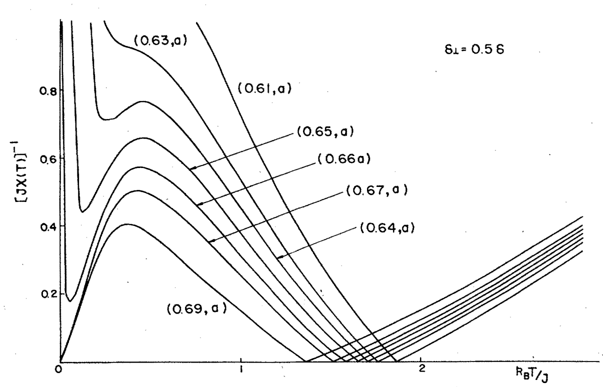

FIG. 12. Thermal dependence of the inverse susceptibility for the systems corresponding to Fig. 8.

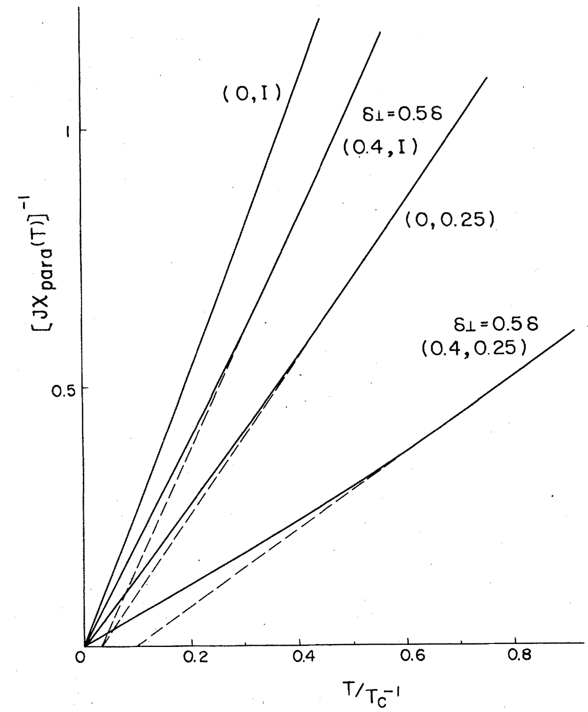

FIG. 13. Thermal dependence of the inverse paramagnetic susceptibility for selected values of (δ,α).

is observed in amorphous ferromagnets. As seen from the curve labeled (0,0.25), the deviation from the Curie-Weiss law is also observed, when the pure system becomes quasi-one-dimensional.

V. CONCLUSION

We have discussed the amorphization of a quasi-one-dimensional Ising ferromagnet. Within the effective-field theory with correlations, we evaluated the most relevant thermodynamical quantities, namely, the magnetization, the critical temperature, the phase diagram, and the susceptibility. Some interesting effects of amorphization are revealed in the thermal behavior, especially for the magnetizations and the susceptibilities of the systems with anisotropic exchange fluctuations. The reentrant phenomenon is observed in the magnetization curves of some systems, as shown in Figs. 5 and 6. The susceptibilities of the systems show the effect of the eventual coexistence of an infinite cluster with finite ones.

Especially for the system with $\delta_\perp = 0.5\delta$, we find that there exists a critical value of δ (namely, δ^* in Fig. 4) at which the magnetization, at $T=0$, shows the discontinuous change. Near the δ^*, the magnetization and the susceptibility exhibit some peculiar behavior. Thus, it will be an interesting problem to study whether or not the results obtained in this paper are general by using more elaborate theories.

In these calculations, we have applied a decoupling in the effective-field framework introduced by Honmura and

Kaneyashi and co-workers.[5] As discussed in Ref. 15, the approximation essentially corresponds to the Zernike approximation.[16] This formalism is, from the analytical standpoint, almost as simple as the standard MFA, and because of negligence of multispin correlations, shares with it the fact that topology of the system is only partially taken into account, essentially through the coordination number. However, it is worth noting the following fact; if one applies the standard MFA to study the critical frontiers, it is well known in the lattice model of amorphous magnets that the critical frontiers are independent of the exchange fluctuations, in contrast with the result of Figs. 2—4.[19]. Thus, we verify that its results are quite superior to the other effective-field theories and exhibit some characteristic behavior for the amorphization of a quasi-one-dimensional ferromagnet.

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