Mean-field simulation of field-induced domains and hysteretic behavior in dilute Ising antiferromagnets

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A mean-field free-energy surface of a dilute Ising antiferromagnet on two-, three-, and four-dimensional lattices is studied numerically as a function of temperature and an applied magnetic field. In the presence of the field parallel to spins, the dilute Ising antiferromagnet is believed to map onto the random-field Ising model providing a convenient experimental realization. From the hysteretic behavior we identify three temperature regions: (1) a high-temperature equilibrium region above the hysteresis boundary T_H where an accessed state is independent of the sample history; (2) a low-temperature region below T_c where the antiferromagnetic state has the lowest free energy; (3) an intermediate-temperature region $T_c < T < T_H$ where the accessed state depends on the sample history, but the domain state has a lower free energy than the antiferromagnetic state. When the sample is cooled in a finite field, the domains are formed and stabilized below T_H. Thus the antiferromagnetic state is inaccessible in our simulation. In contrast, once the antiferromagnetic order is prepared at low temperatures and the temperature is raised in the finite field, the sample remains in the antiferromagnetic state until T_c and then the domains nucleate above T_c, and finally the sample recovers the high-temperature equilibrium state at T_H. In recent neutron scattering experiments on three-dimensional (3D) Ising antiferromagnets, the analogous behavior is observed. Using the simulation results, we argue that strong domain-wall pinning prevents long-range order from occurring in 3D samples when cooled with $H \neq 0$, despite the theoretical consensus that the lower critical dimension in equilibrium is 2 for the random-field Ising model.

I. INTRODUCTION

What effects a random ordering field has on a three-dimensional (3D) Ising system has persisted as a difficult question both experimentally and theoretically. Imry and Ma[1] first demonstrated theoretically the dramatic influence such random fields would have on the ordering transition. For $d \simeq 6$, a system is expected to behave as if its dimensionality were decreased by 2 ($\tilde{d} = d - 2$). For the 3D Ising model, however, the situation is not as clear. If, for this case, $\tilde{d} = d - 2$, no phase transition will occur for finite temperature since such is the case for the 1D Ising model with no random field.[2] All of the most recent theoretical work indicates that the $\tilde{d} = d - 2$ mapping is not valid for the 3D Ising system and long-range order would be expected at $T = 0$ in equilibrium.[3]

Fishman and Aharony[4] first proposed that an experimentally realizable case of the random-field Ising model is the anisotropic, uniaxial antiferromagnet with random bond dilution with a uniform field H applied parallel to the spin direction. In practice, most suitable systems are site diluted and the results have been recently extended by Cardy to this case.[5] Several experimental studies[6] of random-field effects have followed the pioneering work in which Rohrer[7] investigated the susceptibility of $GdAlO_3$:La near its bicritical point. We would like, however, to focus mainly on a set of experiments in the 3D *Ising* systems $Fe_cZn_{1-c}F_2$ and $Co_cZn_{1-c}F_2$ and the 2D *Ising* system $Rb_2Co_cMg_{1-c}F_4$.

The neutron scattering experiments of Yoshizawa *et al.*[8] first showed that large domains form in both the 2D and 3D Co samples when cooled in an applied field. Similar behavior was later observed in the 3D Fe system by Cowley *et al.*[9] The domain state has been cited as evidence that the phase transition to long-range antiferromagnetic order is destroyed by the random field for both $d = 2$ and $d = 3$. The inverse domain structure correlation length κ is observed as the width of the neutron scattering line shape at low T. Theory[1-3] has predicted that κ will have an exponential dependence on the average of the square of the random field for $d = d_1$, the lower critical dimension below which no long-range order occurs. For $d < d_1$ a power-law dependence has been predicted. When the samples are cooled in a field, a power law is in fact observed, with $\kappa \propto H^{\nu_H}$, with $\nu_H \sim 2$ for both $d = 2$ and 3. However, when the samples are cooled to low T in zero field, long-range antiferromagnetic orders persist after applying the field. Hence, freezing of the spin configuration at low T prevents a clear comparison with the low-T theory.

Simultaneously with, and independently of the neutron scattering experiments, the behavior of the magnetic specific heat was investigated using the linear birefringence technique by Belanger *et al.*[10] in the systems $Fe_cZn_{1-c}F_2$ and $Mn_cZn_{1-c}F_2$. The linear birefringence measurements are apparently not directly sensitive to the existence of large domains. No hysteresis is observed and the transitions appear sharp, although significantly altered when the field is applied. Subsequent results[11] in $Fe_{0.6}Zn_{0.4}F_2$ reveal new apparent random-field critical behavior with the specific heat exponent $\alpha \simeq 0$ and the amplitude ratio $A/A' \simeq 1$. This behavior is consistent with

an effective dimensionality $\tilde{d} \simeq 2$. Ferreira *et al.*[12] have recently shown that the phase transition is clearly destroyed in the 2D Ising system $Rb_2Co_{0.85}Mg_{0.15}F_4$ by very small applied fields in a way described by scaling theory. The changes in the critical behavior for both $d=2$ and 3 are consistent with the prediction that $d_1=2$.

Most recently the neutron scattering in $Fe_{0.6}Zn_{0.4}F_2$ by Belanger *et al.*[13] has revealed critical-like behavior in the inverse correlation length above a hysteresis boundary. In this sample the boundary for observed hysteresis coincides with the apparent phase boundary within experimental accuracy. Stable domain structure is observed below the hysteresis boundary when the sample is cooled with $H \neq 0$.

The results of the various experiments must eventually be brought together into one coherent physical picture of the behavior of randomly dilute anisotropic antiferromagnets in applied fields. Hysteretic behavior plays an integral part of such a picture. One must understand, for example, why domains form in samples when cooled in a field, but not when cooled in zero field. To obtain insight into this matter, we have performed mean-field numerical studies in site-dilute Ising antiferromagnets with uniform applied fields in dimensions $d=2$, 3, and 4.

II. THE MEAN-FIELD SIMULATION TECHNIQUE

The magnetically dilute fluoride crystals used in the random-field experiments have short-range antiferromagnetic interactions of uniform strength with no frustration in zero field. The randomness of the systems enters only in the distribution of magnetic ions on the lattice. The magnetic behavior should not, therefore, be equated with that associated with spin glasses. However, the randomization in the spin configuration and the irreversibility observed experimentally in a field strongly suggest complicated free-energy surfaces. We were thus motivated to use a mean-field simulation technique similar to that developed by Soukoulis *et al.*,[14] which has proven to be quite successful in the study of spin-glass behavior, which also involves complicated free-energy surfaces. The chaotic pinning of solitons in nearly commensurate modulated systems has recently been studied in a similar way.[15]

For the mean-field model of a dilute antiferromagnet in a uniform applied field H, the local free energy may be expressed as

$$F = J \sum_{i<j} \epsilon(i)\epsilon(j) m_i m_j - H \sum_i \epsilon(i) m_i + \tfrac{1}{2} T \sum_i \epsilon(i)[(1+m_i)\ln(1+m_i) + (1-m_i)\ln(1-m_i)] , \quad (1)$$

where i and j are nearest-neighbor lattice-site labels, J is the exchange energy strength, m_i is the thermal average of the magnetization at site i, and $\epsilon(i)$ is unity if site i is occupied and zero otherwise. The values m_i are determined by the self-consistent equations

$$m_i = \tanh\left[\beta\left[H - J\sum_j \epsilon(j) m_j\right]\right] , \quad (2)$$

where $\beta = 1/kT$, which are derived from the minimization of the free-energy $\partial F/\partial m_i = 0$. The random-vacancy configuration $\{\epsilon(i)\}$ is quenched, and the moments are then all updated in a *random* sequence using the expression

$$m_i^{(n)} = \tanh\left[\beta\left[H - J\sum_i \epsilon(j) m_i^{(n-1)}\right]\right] , \quad (3)$$

where n is the number of iterations of the moment configuration. We use the convergence criterion

$$\frac{\sum_i (m_i^{(n)} - m_i^{(n-1)})}{\sum_i (m_i^{(n)})^2} < 10^{-10} . \quad (4)$$

Various sample sizes with periodic boundary conditions have been used, ranging up to 80^2 sites for $d=2$, 18^3 for $d=3$, and 8^4 for $d=4$. In all simulations each spin has 2D possible nearest neighbors. In studying temperature-dependent effects, small enough temperature increments were taken such that the observed behavior was independent of the increment size. The moment was allowed to converge [Eq. (4)] at each temperature. Although most results presented were obtained using one 60×60, 2D lattice with $c=0.85$, other sizes, concentrations, and impurity configurations were examined to assure ourselves that the results are not anomalous to that particular sample.

The limited sample sizes used in this study preclude *quantitative* statements regarding, for example, the dependence of the domain sizes on H, c, or d. Furthermore, the effects of thermal fluctuations are greatly suppressed by the mean-field approximation. This allows, for example, the ordering of just two isolated spins at $T=J$. Hence, the method is not useful for determining the lower critical dimension of real systems. We note, however, that several *qualitative* features of domain formation observed in the experiments are reproduced in the simulations.

This mean-field procedure differs significantly from the most simple mean-field calculation in which each spin sees the average of all other spins on the opposite sublattice and hence is insensitive to the dimensionality of the spin configuration. In our simulation each spin sees the average only of its nearest neighbors. Clearly, m_i is not equal to the configurational average of moments on the same sublattice. Dimensionality does, therefore, play a vital role in the geometric correlation of moments through the sample. This point is important with respect to the domain-wall-energy arguments to be discussed in Sec. III.

III. ZERO-FIELD, FIELD COOLED (FC), AND ZERO-FIELD-COOLED (ZFC) STATES

A nondiluted ($c=1.00$) sample always forms a single domain when the phase boundary is crossed from the paramagnetic side by reducing either the temperature or field. We observed that a 2D, 80×80, $c=1.00$ sample relaxes into a single domain for $T \gtrsim 3J$ ($T_N = 4J$) even if the simulation is started with a random configuration.

Independently of sample size and dimensionality, the simulated dilute antiferromagnet always forms a single domain when cooled from high temperature in zero field.

This behavior is entirely consistent with that observed experimentally. Both linear birefringence and neutron scattering data show random-exchange critical behavior[16] in $Fe_cZn_{1-c}F_2$ with zero field and resolution-limited long-range order is observed at low T in the scattering experiments. It is to be noted, however, that if one begins with a random configuration below the transition temperature T_N, the dilute mean-field Ising lattice does not necessarily relax into the single domain state. The simulated behavior for a dilute lattice in an applied field H is much more complex, in a manner very similar to that of the neutron scattering experiments. One very apparent result is that an extremely small field ($H=0.001J$) is sufficient to produce a multidomain structure in a 60×60, $c=0.85$ lattice when cooled slowly in the field (FC) at low T. In contrast, if the sample is cooled at low T in zero field, resulting in a single domain, the antiferromagnetic configuration is stable (except for very small clusters) when large fields are applied ($H<3J$). All spins will align with the field at $T=0$ when $H>H_c=4J$, the maximum exchange any spin sees in the 2D lattice.

Figure 1 shows the lattice in a FC configuration at low T but with a field $H=J$. The domain structure is evident, with isolated islands of each register. Note, however, that one cluster is actually of infinite extent in this periodic lattice. A very large field $H\approx3J$ is required to make all clusters finite in size at low T in this sample.

An example of 3D lattices was also examined. A FC 12^3, $c=0.70$ sample with $H=3J$ ($H_c=6J$) shows the formation of two domains in a manner similar to the 2D sample. However, for a similar number of spins, the minimum field H_m required to form a multidomain configuration for $d=3$ is conspicuously larger than for $d=2$. For example, a FC 18^3, $c=0.60$ sample (3500 spins) forms a domain state for $H\gtrsim J$. This contrasts with the behavior in a FC 60^2, $c=0.85$ sample (3060 spins) where a field of only $H=0.001J$ is sufficient to create two domains. For a given dimensionality, H_m increases with increasing concentration and decreasing lattice size.

One would expect the original low-temperature wall-energy arguments of Imry and Ma[1] to apply to this mean-field calculation. The Zeeman energy decrease obtained by forming domains of size L is proportional to the statistical imbalance of spins on the two sublattices within the domains, which in turn is proportional to $L^{d/2}$. The domain-wall-energy increase when such a domain is formed is proportional to L^{d-1}. Domains should form readily for $d\leq2$ in agreement with the simulation results, but not for $d=3$. Consideration of the effects of domain-wall roughening has been used to argue that domains will also form for $d=3$, although the results have been questioned.[2-3] We therefore investigated a $d=4$ lattice for which there is little ambiguity about the prediction that domains should not form in equilibrium. However, we found that a field $H=J$ ($H_c=8J$) can produce two domains in a 8^4, $c=0.40$ FC sample. We see then, that the simple energy arguments do not explain the existence of domains formed under FC conditions. We see a similar situation in the experiments in which FC domains form for both $d=2$ and $d=3$, despite the theoretical prediction that $d_1=2$.

One feature which is evident upon inspection of Figs. 1—4 is that domain walls are not randomly situated in the sample, but rather follow paths passing through large

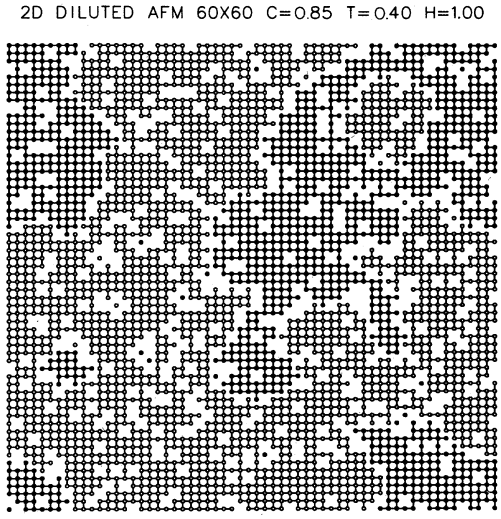

FIG. 1. 60×60, $c=0.85$ lattice of Ising spins cooled in a field (FC) of $H=J$ to a temperature well below the zero-field-ordering temperature. Connected solid circles represent spins whose mean-field moments are antiferromagnetically aligned. Open connected circles likewise represent antiferromagnetically aligned spins which, however, are out of register with the spins in the solid circle domain. Periodic boundary conditions are imposed, so this FC sample consists of two domains which are infinite in extent in the horizontal direction.

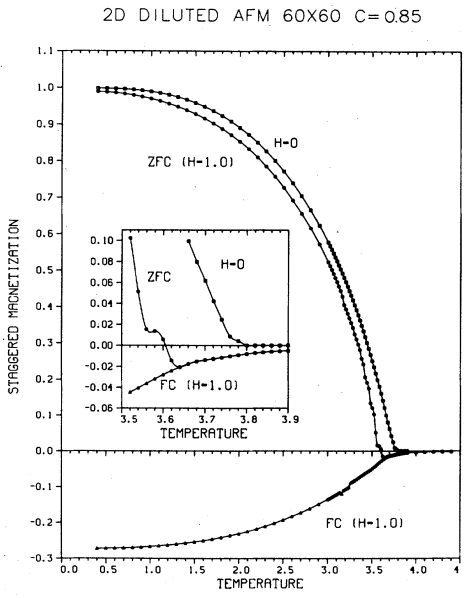

FIG. 2. Average staggered moment per spin as a function of temperature observed for the lattice shown in Fig. 1 under zero field, field-cooled (FC) and zero-field-cooled (ZFC) conditions. The kinks are a result of clusters flipping.

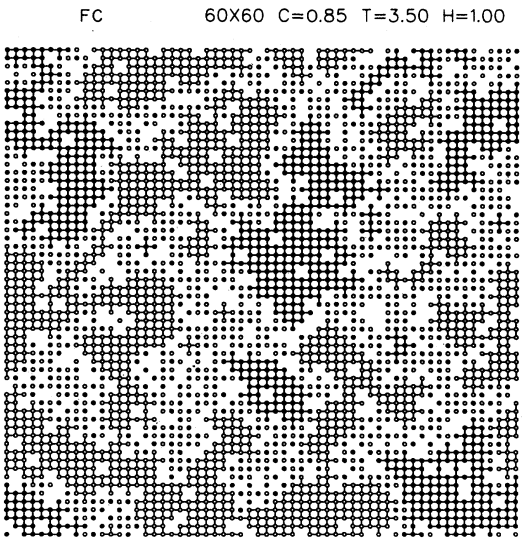

FIG. 3. 60×60, $c=0.85$ lattice cooled in a field $H=J$ to a temperature $T=3.5J$, which is just below the hysteresis boundary. Further cooling results in the configuration shown in Fig. 1.

numbers of vacancies. In this way, the number of broken bonds is kept to a minimum and the domains make less attempt at minimizing their surface than one might expect in a random-field Ising ferromagnet which has no vacancies. The significant role played by vacancies is also demonstrated by simulations in which a 2D, 80×80 uniform ferromagnet is subjected to a static random field with a strength ranging between $-H$ and H at each site assigned with a uniform probability. In this case, the domain walls cannot take advantage of vacancies and we find that fields typically two orders of magnitude larger ($H\approx J$) are required to form multidomain FC configurations.

We have now demonstrated that in the mean-field model domains form readily in the dilute samples when cooled in a field. It will be established, however, that the FC state is not the low-T ground state in this model since for low T, the long-range antiferromagnetic state has a lower local free energy. To show why the FC metastable domain state occurs, we concentrate primarily on the 60×60, $c=0.85$ sample shown in Fig. 1, with either zero field or a field $H=J$ applied.

The staggered magnetization per spin (M_s) observed as a function of temperature under three distinct physical procedures is shown in Fig. 2. For $H=0$, M_s is zero above $T_N=3.8J$ (for $c=100$, $T_n=4J$). Below T_N, M_s rises abruptly, as expected for the order parameter, and saturates at $M_s=J$ at low T. The $H=0$ results exhibit no hysteresis. In an applied field $H=J$, however, hysteresis is observed below $T=3.62J$. We define the temperature below which hysteresis occurs to be T_H. If the sample is initially prepared in a single domain by cooling to low T in zero field, and a field $H=J$ is subsequently applied (ZFC), M_s behaves on heating very much like it did for $H=0$. The significant differences are that the spontaneous contribution vanishes at a temperature T_H lower than T_N, M_s is nonzero at higher T, and kinks are observed in M_s versus T close to T_H. When the sample is cooled from high temperature with $H=J$ (FC), the hysteresis below T_H is evident in that M_s remains negative and saturates at a smaller magnitude $M_s\lesssim-0.3$. The negative finite M_s at low T in the FC domain state is a finite size effect because M_s tends to decrease rapidly as the lattice size is increased since M_s is then averaged over more domains. The same holds for the disordered state for $T>T_H$. M_s, as shown in Fig. 2, does, however, reflect the temperature dependence of the *local* antiferromagnetic order for $H\neq0$. Small kinks in M_s vs T are again observed near T_H.

Hysteresis at low T is very obvious when one compares the FC lattice which exhibits domain structure with the single domain obtained for the same lattice under ZFC conditions. At higher temperatures the difference is more subtle. Figure 3 shows the FC lattice for $H=J$ and $T=3.5J$. The domains are not as well formed as they are at low T, and many spins are not antiferromagnetically aligned with neighboring spins but rather are aligned with H. As T is lowered, the domains grow in size until all spins are antiferromagnetically aligned except those which are isolated and those as the well-defined domain walls shown in Fig. 1. The ZFC sample with $H=J$ retains its antiferromagnetic order when heated until $T\simeq3J$, when clusters of opposite register begin to appear. This is also the region in which kinks are observed in M_s vs T. Long-range antiferromagnetic order is completely lost when $T\simeq3.5J<T_H$ as shown in Fig. 4. Comparing the FC and ZFC states in Figs. 3 and 4, respectively, one observes domains of similar size and about the same degree of ordering in the lattice. Only the orientation of some

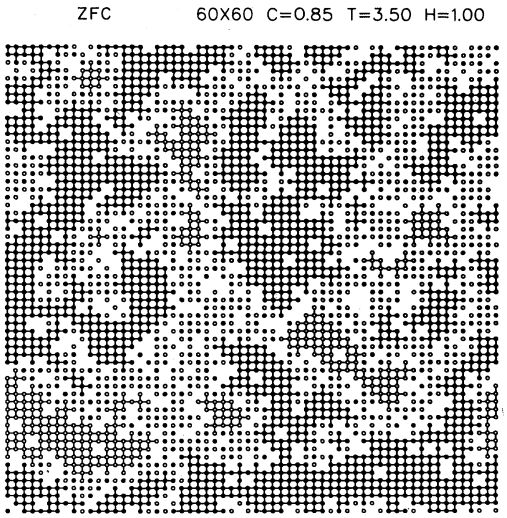

FIG. 4. Same lattice at the same temperature and field as that shown in Fig. 3. However, the sample was first cooled in zero field to low temperature, resulting in a single domain. The field $H=J$ was then applied and the sample was finally heated to $T=3.5J$. This procedure (ZFC) results in a configuration similar to the FC one shown in Fig. 3 except that some clusters are oriented in the opposite direction.

domains differs. Above T_H the configuration is identical whether obtained by the FC or ZFC procedure. Just above T_H, most spins are aligned with H. However, well-separated clusters with field-induced antiferromagnetic correlations do form in the more concentrated regions. These clusters are the nucleation sites for the formation of the FC domains observed in Figs. 1 and 3.

Thermal fluctuations are suppressed in the mean-field approximation. Nevertheless, in real systems, for $T < T_N$ of the corresponding pure system, magnetically concentrated regions will begin developing average local antiferromagnetic correlations, with the sublattice with statistically more spins oriented with the field H. These regions surely act as the nucleation sites for the observed domain formation as the real system is cooled in a field.

The difference in energy between the FC and ZFC states is shown versus T in Fig. 5. Above T_H the states are identical and no energy difference is observed. Just below T_H the FC domain state has a slightly lower energy, indicating that the Zeeman energy decrease obtained by having oppositely ordered domains is larger than the domain-wall-energy increase. At lower temperature the ZFC state has lower energy. The change in sign of the difference in energy of the FC and ZFC states as T decreases may be understood by considering the local average moments of spins at the edges of clusters. As previously noted, if domain walls form, they follow paths along as many vacancy sites as possible. Clusters have only weak links across these paths which are not well ordered except at low T. This is demonstrated in Fig. 6, in

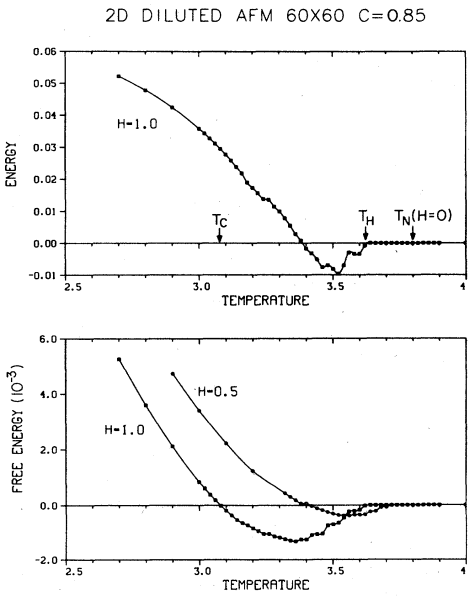

FIG. 5. Energy of the FC state minus the energy of the ZFC state versus temperature for $H=J$ and the analogous difference in the local free energy for $H=J$ and $H=0.5J$. T_H is the temperature above which there is no difference between the FC and ZFC states. T_c is the temperature below which the ZFC long-range antiferromagnetic state has the lowest free energy. The kinks observed between T_c and T_H are the result of cluster flips in the FC and ZFC configurations.

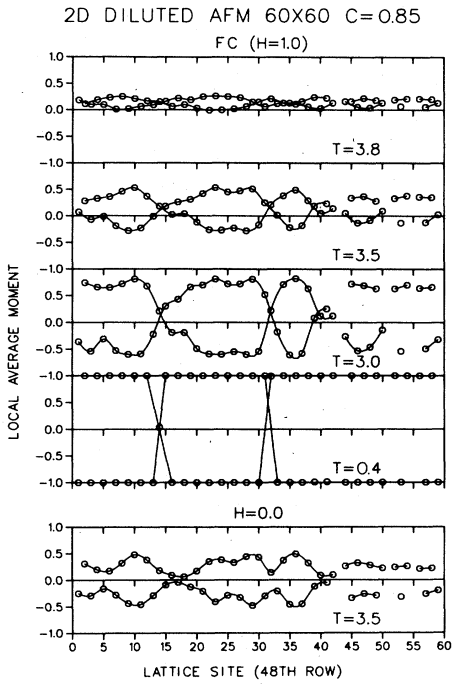

FIG. 6. Mean-field moments at each spin site along the 48th row from the bottom of the lattice in Fig. 1 for $T=3.8$, 3.5, 3.0, and $0.4J$ when the lattice is field cooled with $H=J$ and for $T=3.5J$ for $H=0$. The solid lines connect spins on the same sublattice. Breaks in the solid lines occur at vacancies.

which the local average moment is plotted along a typical row (48th row from the bottom) of our sample. For $T=3.8J$, which is well above $T_H=3.62J$, a staggered moment is appearing in higher-density clusters. The spins between clusters show essentially no staggered moment. At $T=3.5J$, below T_H, the FC antiferromagnetic domains are well ordered, with $|m_i| \sim 0.5$, whereas the spins at the interfaces still show little staggered moment. The same sample is shown cooled in zero field at the bottom of Fig. 6 for the same temperature, $T=3.5J$. In this case all domains are oriented in the same direction. One should note how little the degree of spin ordering is affected by the relative orientation of the domains. The Zeeman energy decrease, by having domains favorably oriented with H, more than compensates for the energy increase by introducing domain walls. This temperature, $T=3.5J$, is where the minimum in the energy difference between the FC and ZFC state occurs. As T is lowered, the spins near the domain walls become more and more ordered and the wall-energy increase then makes the domain configuration unfavorable.

Considering entropy, we calculate the difference in the local minima of the free energy for the FC and ZFC states using Eq. (2) as shown in Fig. 5. The FC state has the lower local free energy just below T_H, whereas the ZFC state has the lower local free energy at low temperatures ($T \lesssim 3.08$). Also shown in Fig. 5 is the difference in free energy for $H=0.5J$. The temperature T_c below which the ZFC state has the lower free energy is closer to T_H in this case.

In the mean-field model we see that, for $H \neq 0$, the magnetization at each site within clusters has a spontaneous contribution below a boundary $T_H(H)$. The long-range correlation of spins, on the other hand, depends upon the sample history. The hysteresis boundary lies just above the inaccessible boundary $T_c(H)$ below which the single domain state is the ground state. At $H=0$, $T_H=T_C$ and no hysteresis is observed since the antiferromagnetic ground state is immediately accessed from the high-temperature paramagnetic state.

IV. FIELD HYSTERESIS

Hysteresis may be observed in the 60×60, $c=0.85$ sample (Fig. 1) when the field is varied at fixed temperature, as exemplified in Fig. 7 for $T=3.2J$. The sample is first cooled in zero field, resulting in a single domain with $M_s=0.47$. As the field is then increased to $4J$, M_s decreases. Above $H=3J$ all spins point along H. When the field is subsequently reduced to zero, domain structure is present. Hence, M_s reaches a value of only 0.13, slightly less than $\frac{1}{3}$ of the ZFC value. The domain structure for $H=0$ at $T=3.2J$ is stabilized by the vacancies. The field is further reduced to $H=-4J$ and raised again to $H=4J$ to complete the hysteresis cycle. As T approaches T_N, the hysteresis boundary occurs at smaller H and M_s at $H=0$ is always about $\frac{1}{3}$ of the ZFC value after cycling H. The field-induced uniform magnetization, also shown in Fig. 7, is nearly linear in H, although hysteresis of less than 0.006 is present.

Similar hysteretic effects are observed in neutron scattering experiments on $Fe_{0.6}Zn_{0.4}F_2$.[13] When the field is reduced at a constant temperature just below T_N, the hysteresis boundary is crossed from the paramagnetic side and domain structure is present. Even if H is then reduced to zero, the antiferromagnetic state is not readily recovered. Hysteresis in the $Fe_cZn_{1-c}F_2$ system is also observed in pulsed high-field magnetization measurements by King, *et al.*[17] when samples cooled in zero field are pulsed above a hysteresis boundary. Long-range antiferromagnetic order persists until the boundary is crossed. As the field then decreases, the samples enter a domain state. From zero-temperature mean-field simulations[17] it was concluded that the long-range antiferromagnetic state is the ground state but that this state is inaccessible when approaching the phase boundary from the high-field disordered state. A neutron scattering study has been made by Wong and Cable[18] of the hysteretic behavior of the dilute antiferromagnetic system $Fe_cMg_{1-c}Cl_2$ when the field is cycled.

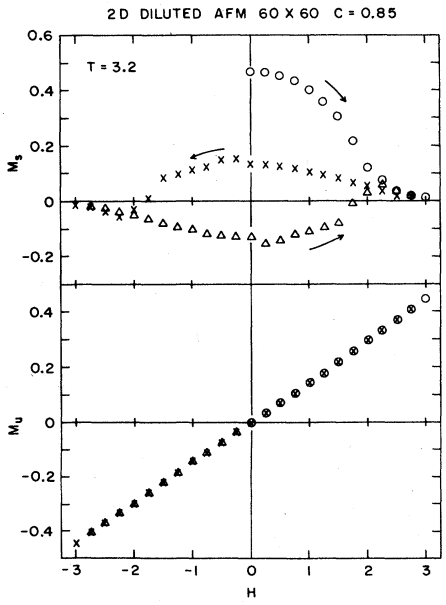

FIG. 7. Staggered (M_s) and uniform (M_u) average moment per spin for the lattice in Fig. 1 at $T=3.2J$ as the field H is cycled.

V. DISCUSSION

In the mean-field model, a well-defined hysteresis boundary $T_H(H)$ occurs in the phase diagram above the phase boundary $T_c(H)$ for $H\neq0$. In a FC sample the field-induced domain structure is locked in below the hysteresis boundary and the low-T antiferromagnetic ground state is not accessed. A ZFC sample, on the other hand, does not immediately disorder as it reaches the phase boundary upon heating. In our model, the inaccessibility of the antiferromagnetic ground state under FC conditions is rather straightforward to understand. Since spins are updated singly and in a way governed by the mean-field expression in Eq. (3), it is essentially impossible for a domain wall to pass through a locally well-ordered domain. Hence, the domain walls which form below T_H are often pinned and the domains cannot necessarily reorient themselves into an antiferromagnetic configuration at T_c. Analogously, domains do not form readily when the ZFC system is heated in a field above T_c. Some domains do flip for $T_c<T<T_H$, as evidenced by the kinks in M_s versus T in Fig. 2.

A domain configuration does not occur when the sample is cooled below T_N with $H=0$ simply because there are no clusters with field-induced staggered moments above T_N. However, as previously noted, if domains are artificially introduced below T_N, the system does not necessarily relax to a long-range antiferromagnetic state. The nonzero staggered moment below T_N is largest inside clusters and prevents domain walls from passing through them. The phenomena of domain wall pinning is not simply a product of the mean-field nature of the model. The same behavior has been observed and interpreted in a similar manner by McMillan[19] in Monte Carlo simulations of the 2D frustrated Ising model.

Equilibrium theories predict that the lower critical dimension, d_1, of the random-field Ising model is 2.[1-3] It is implicit in such theories that the low-temperature ground state is accessible to the system. We have presented, on the other hand, a mean-field Ising system which, when cooled in a field, does not achieve its low-temperature antiferromagnetic ground state. Instead, for $H\neq0$, the antiferromagnetic order parameter M_s is not well behaved in two respects: first, M_s is dependent upon the sample's history for $T<T_H$; second, M_s is not a

smooth function of temperature for $T_c < T < T_H$, showing kinks resulting from clusters flipping in both FC and ZFC procedures. If thermal fluctuations were included in a more sophisticated model of the dilute antiferromagnet, the pinning of domain walls might still occur as a dynamical effect, again disrupting the phase transition in a manner analogous to that observed in our simple model. We propose that the experimental situation lies between that of the equilibrium theories, which assume complete thermal relaxation, and the mean-field simulation, in which thermal fluctuations are suppressed.

Somewhat similar behavior is proposed for the random-field Ising model by Binder.[20] As T is decreased for $H \neq 0$, a boundary is encountered below which the correlation length ceases to grow much, but the moment increases spontaneously within the resulting domains. This is analogous to the behavior observed near the hysteresis boundary in the mean-field simulation. Bruinsma[21] has obtained a phase diagram for the random-field Ising model on a Bethe lattice which includes cluster flips just above the ferromagnetic phase boundary for $H \neq 0$.

In real systems, for $d > d_1$, the FC domain state is metastable and should relax, perhaps very slowly, toward the long-range antiferromagnetic state just below the hysteresis boundary. In the experiments on the very anisotropic 3D system $Fe_{0.6}Zn_{0.4}F_2$,[13] the phase boundary determined from the critical behavior is indistinguishable from the hysteresis boundary below which stable domain structure is observed in FC samples. In the context of the model, it is natural to suggest that the hysteresis boundary is slightly above the phase boundary, thereby preventing the antiferromagnetic long-range order which is otherwise expected since d_1 is believed to be 2.

One would expect the size of the domain structure observed in real systems at low T to reflect the degree to which relaxation takes place. Specifically, in a FC sample with smaller anisotropy, transverse spin fluctuations will provide additional thermal relaxation and larger low-T domain structure should be observed. Although the random fields generated in the related system $Mn_cZn_{1-c}F_2$ are known to have effects on the critical behavior comparable to those in $Fe_cZn_{1-c}F_2$,[10] the anisotropy in MnF_2 is dipolar in origin and is an order of magnitude smaller than the single ion anisotropy in FeF_2.[22] Greater relaxation toward long-range order would therefore be expected and the observed domain structure at low T in $Mn_cZn_{1-c}F_2$ is indeed much larger than in $Fe_cZn_{1-c}F_2$.[23]

Combining the birefringence, neutron scattering, and mean-field simulation results, we conclude that the critical behavior observed in dilute Ising antiferromagnets is consistent with the theoretical expectation that $d_1 = 2$, whereas domain walls are pinned for $d = 3$ under field-cooled conditions and the system does not achieve complete equilibrium. We hope that the nonequilibrium aspects of domain formation for $d > d_1$ will be more thoroughly addressed theoretically as well as experimentally. After completing this manuscript we received two reports, one from Villain[24] and one from Bruinsma and Aeppli,[25] in which it is argued that quenched domain walls should be permanently pinned in $d = 3$ Ising systems. These studies begin with quenched domains below T_c as contrasted with the present mean-field study in which we obtain domains by field cooling, the procedure by which they are obtained in the experiments. It is still a key issue to go beyond mean field to include thermal fluctuations in order to better answer why and how domains form when the system is cooled slowly through the critical region.

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