Linear conductance of short semiconductor structures

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(Received 26 September 1983; revised manuscript received 13 February 1984)

We study the length dependence of the linear conductance in semiconductor samples sandwiched between two metallic contacts. In very short samples the conductance is given by the Landauer formula which accounts for the quantum-mechanical reflection in the semiconducting region. In long samples, where semiclassical transport concepts are applicable, the conductance is derived by solving the Boltzmann equation with the appropriate boundary conditions imposed by the metallic contacts. Depending on the relative magnitudes of the sample length L and the carrier mean free path l we can distinguish between three specific modes of the electrical transport: the ordinary collision-controlled conductance for $L \gg l$, thermionic emission for $L \sim l$, and tunneling at the Fermi level for $L \ll l$ (below $L = 100$ Å, typically).

Progress in semiconductor technology is pushing device size ever smaller. At present, commercially available devices have a feature size of about 1 μm and research laboratories are processing prototypes with dimensions well in the submicron range. From the technological point of view it has been suggested that the minimum device size can be in the future as small as a few hundred angstroms.[1]

The study of the transport properties of semiconductor devices is conventionally based on solutions of the Boltzmann equation, which are valid in infinitely large samples only. In practice this implies that the device dimensions must be much larger than the carrier mean free path l. At room temperature $l \simeq 500$ Å in n-type Si and 1500 Å in n-type GaAs. Thus, the sample length L is in the submicron range only a few mean free paths and consequently some changes from the long sample behavior can be expected. One type of modification arises from the boundary conditions imposed by the environment. The other type of changes stem from the space and time constrained kinetic effects such as finite collision duration, size quantization, breakdown of effective-mass approximation, etc.[1,2] Some of these phenomena can be treated as corrections to the semiclassical Boltzmann theory, but, finally, in very small dimensions a full quantum treatment of electrical transport is needed. There are various formulations of quantum transport theory but owing to the complexity of these theories only very few results exist for short devices. This paper is intended to describe within a simple model the behavior of the linear conductance in short semiconductor structures. We study a one-dimensional metal-semiconductor-metal (M-S-M) structure shown in Fig. 1. The metallic regions are described by the free-electron model and the semiconductor by the Kronig-Penney chain of variable length. The dashed lines stand for the band edges obtained by allowing L to approach infinity.

In ultrashort samples ($L \ll l$) an electron traverses through the semiconductor without being scattered. However, the semiconducting region introduces a perturbation in potential energy that causes the electron wave function to be partially reflected. This gives rise to a finite conductance given by the Landauer formula[3,4]

$$G = \frac{e^2}{\pi\hbar} \frac{\int dE\,\mathscr{T}(E)(-\partial f/\partial E)}{1-\langle \mathscr{T} \rangle} . \tag{1}$$

Here, f is the Fermi-Dirac distribution function and $\mathscr{T}(E)$ the energy-dependent transmission coefficient for the M-S-M structure. The average $\langle \mathscr{T} \rangle$ is defined by

$$\langle \mathscr{T} \rangle = \frac{\int dE\, v(E)\,\mathscr{T}(E)\left[-\frac{\partial f(E)}{\partial E}\right]}{\int dE\, v(E)\left[-\frac{df(E)}{dE}\right]} , \tag{2}$$

where $v(E)$ is the electron velocity. The numerator of the Landauer formula is familiar from the tunneling theories. In ordinary formulation the distribution function is taken as the thermal equilibrium distribution. By correcting the dis-

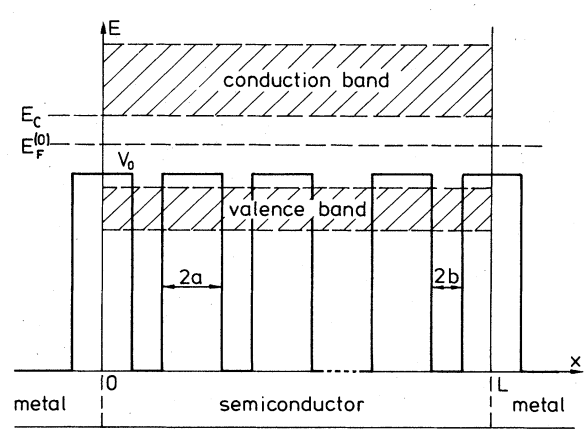

FIG. 1. One-dimensional metal-semiconductor-metal (M-S-M) structure. The conduction and valence bands refer to an infinitely long semiconductor. Parameter values used in the calculations are $V_0 = 7$ eV, $E_F = 7.75$ eV, $a = 2.26$ Å, and $b = 0.24$ Å.

tribution function for the current carrying state the renormalization factor $1-\langle \mathcal{T} \rangle$ is obtained.

The transmission coefficient for the M-S-M structure can be easily calculated by the usual transfer-matrix method.[5] The results for the conductance show that above $L \sim 100$ Å the conduction takes place by the ordinary band conduction via the semiconductor bands. Thus, for $L > 100$ Å we can study the conductance with the aid of the Boltzmann equation.

In the relaxation time approximation the general solution of the Boltzmann equation can be expressed in the path-integral form[6]

$$\delta\Phi(\vec{p}) = -\int_{-\infty}^{t} dt' \frac{df^{(0)}(\vec{p}(t'))}{dt'} \exp\left[-\int_{t'}^{t} \frac{dt''}{\tau(\vec{p}(t''))}\right] . \quad (3)$$

Here, $f^{(0)}$ stands for the local equilibrium distribution, $\vec{p}(t)=(k(t),x(t))$ for the electron trajectory and τ for the relaxation time. Since $f^{(0)}$ depends on time through the position-dependent Fermi energy $E_F(x)$ only, we see that $df^{(0)}/dt' \sim dE_F/dx'$. In the metallic regions dE_F/dx is orders of magnitude smaller than in the semiconducting part. Therefore, we can limit the t' integration in Eq. (3) to the semiconductor side only. Physically this means that the metallic regions supply thermalized electrons into the semiconductor side, thus giving rise to the boundary condition $\delta\Phi=0$ at the incoming contact.

In the linearized treatment of the nonequilibrium distribution the t' integration is taken along the free-electron trajectory. Furthermore, by changing the integration variable from t' to $x'=x-v(t-t')$ and assuming a constant mean free path $l=v\tau$ we obtain from Eq. (3) for the current density

$$J=\frac{ev_0 n}{2k_B T}\int_0^L dx' \frac{dE_F(x')}{dx'} \exp(-|x-x'|/l); \quad 0 \leq x \leq L . \quad (4)$$

Here, v_0 is the thermal velocity and n the carrier density. Equation (4) is an integral equation for the gradient of the quasi-Fermi level, having the solution

$$\frac{dE_F}{dx}=\frac{Jk_B T}{ev_0 n}[\delta(x)+\delta(L-x)+1/l] . \quad (5)$$

In Eq. (5) the δ functions are understood to be within the region of integration. The term proportional to $1/l$ represents the normal long sample behavior, whereas the sudden changes of the Fermi energy at the contacts account for the specific contact resistance of the metal-semiconductor interface.

The integration of Eq. (5) gives the total voltage and thus we obtain for the conductance of the M-S-M structure

$$G=G_{\mathrm{TE}}/(1+L/2l) , \quad (6)$$

where

$$G_{\mathrm{TE}}=\frac{e^2 v_0 n}{2k_B T}=\frac{e^2}{\pi\hbar}\exp\left[-\frac{E_C-E_F^{(0)}}{k_B T}\right] \quad (7)$$

is the linear conductance of the classical thermionic emission theory. In Eq. (7) E_C stands for the energy of the conduction-band edge and $E_F^{(0)}$ for the equilibrium Fermi energy.

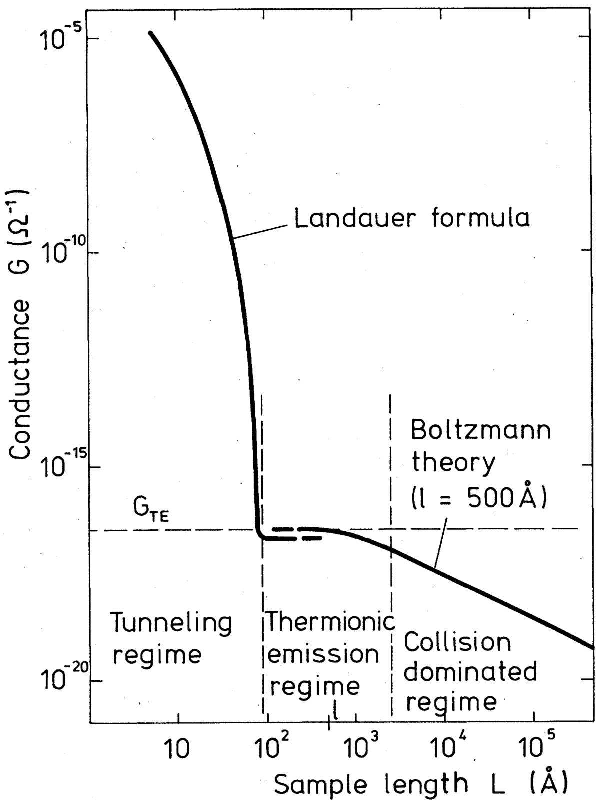

FIG. 2. Conductance of the one-dimensional metal-semiconductor-metal structure at $T=300$ K as a function of the length of the semiconducting region.

Figure 2 shows the length dependence of the conductance of the M-S-M structure as derived from Eqs. (1) and (6) for an n-type semiconductor sample at $T=300$ K. In Eq. (6) $l=500$ Å is assumed. For long semiconductor samples the classical behavior $G \sim 1/L$ is clearly seen down to $L \approx 2l$. Below this, G saturates to the classical thermionic emission value G_{TE}, which thus represents the contact resistance of the structure. For the sample thicknesses of a few atomic layers, only, the Landauer formula gives an exponentially decreasing conductance, which indicates that the current flows by tunneling through the semiconducting region. After $L \approx 100$ Å the conductance rapidly saturates to a constant value, which is roughly a factor of 2 smaller than G_{TE}. The saturation is clearly due to the fact that now the current flows through the regions of high transmission coefficient associated with the semiconductor conduction band. Furthermore, in the Landauer formula the exact transmission coefficient has been used, which explains the slight deviation of the saturation value from G_{TE}.

To summarize, the electrical transport in the M-S-M structures can be characterized by three regimes, as shown in Fig. 2. The classical length dependence can be seen in the collision dominated regime ($L \gg l$), where the results of the standard Boltzmann theory are applicable. In the opposite limit ($L \ll l$) the tunneling at the contact Fermi level dominates, and the conductance is given by the Landauer formula. Between these two extremes ($L \approx l$) the thermionic emission current is prevailing. Some refinement of the theory is still needed in the intermediate regime, but, as seen from Fig. 2, the classical and quantum treatments already agree within a factor of 2.

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