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Domain-wall renormalization-group study of the three-dimensional random Ising model

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The three-dimensional random Ising model with a Gaussian distribution of nearest-neighbor interactions is studied for the pure spin-glass case where the average interaction vanishes. The distribution of domain-wall energies at zero temperature is calculated using a Monte Carlo–quench algorithm to find the ground-state energy for finite lattices. A renormalization-group transformation is set up which preserves the domain-wall energy distribution when the lattice parameter is changed. In the strong-coupling regime (zero temperature) the model iterates toward strong coupling and therefore exhibits a spin-glass phase transition at nonzero temperature. The thermal exponent is $\nu = 3.0 \pm 1.0$ and the heat capacity exponent is $\alpha = -7 \pm 3$.

I. INTRODUCTION

The two-dimensional random Ising model has been studied by a variety of techniques including Monte Carlo simulation,[1-3] transfer matrix simulation,[1,4] and a domain-wall renormalization-group (DWRG) method.[5] All studies agree that there is no spin-glass phase transition at nonzero temperature but that the correlation length diverges algebraically at zero temperature; one says that there is a "phase transition at zero temperature" and that the dimensionality is less than the lower critical dimensionality for a phase transition at nonzero temperature. There have been several suggestions[6-9] that the lower critical dimension for the random Ising model is four. Binder and Young[10] have analyzed Young's Monte Carlo data[11] for the three-dimensional ($\pm J$) random Ising model and find that the data are consistent with the transition temperature being zero.

The DWRG[5] method has proved to be the most powerful method in two dimensions. For the two-dimensional random Ising model a transfer matrix approach provides a numerically feasible way to calculate the domain-wall free energy; in three dimensions the transfer matrix method is not feasible and we use instead a Monte Carlo–quench algorithm to find the ground-state energies for periodic and antiperiodic boundary conditions; the energy difference is the domain-wall energy at zero temperature.

II. DOMAIN-WALL RENORMALIZATION GROUP

We use the DWRG method described in Ref. 5. We consider a cubic lattice with n^3 sites with lattice spacing a and lattice size $L = na$. The Hamiltonian is

$$H = -\sum_{ij} J_{ij} S_i S_j \ , \tag{1}$$

with nearest-neighbor interactions J_{ij} chosen from a Gaussian distribution with zero mean and variance $\tilde{J}$. We choose either periodic boundary conditions in all three directions or periodic boundary conditions in two directions and antiperiodic boundary conditions in the third; antiperiodic boundary conditions introduce a domain wall. For a given configuration of interactions let E^p and E^a be the ground-state energy for periodic and antiperiodic boundary conditions, respectively. The domain-wall energy at zero temperature is then $W = E^a - E^p$. For each configuration of interactions one obtains a different domain-wall energy and one is interested in the distribution function of the domain-wall energies. We characterize that distribution by its mean $\overline{W}$ and variance $\tilde{W}$; $\overline{W}$ is zero for the model considered here.

We set up a renormalization-group transformation by considering two systems with the same physical size but different lattice spacings. The first system has lattice spacing a, n^3 lattice sites with $L = na$, and Hamiltonian parameter $\tilde{J}$. The second system has lattice spacing $a' > a$, n'^3 lattice sites with $L = n'a'$, and Hamiltonian parameter $\tilde{J}'$. We require that the two systems have the same macroscopic properties and that the variances of the distribution of domain-wall energies be equal.

$$\tilde{W}_{n'}(\tilde{J}') = \tilde{W}_n(\tilde{J}) \ . \tag{2}$$

We choose $\tilde{J}'$ so that (2) is satisfied and (2) is the implicit recursion relation for $\tilde{J}$.

III. MONTE CARLO–QUENCH ALGORITHM

In order to implement the DWRG method we need a numerical algorithm to generate the ground-state energy for a particular configuration of interactions. One could use the Metropolis Monte Carlo algorithm to move the system through its spin-configuration space and record the lowest-energy state visited; eventually, the system would visit its ground state. An algorithm consisting of a Monte Carlo update followed by a spin quench finds the ground state somewhat more quickly and we use that procedure here. First the Metropolis[12] Monte Carlo algorithm with Glauber[13] dynamics is used to update each spin once. Then we examine each spin in turn and reverse it if the total energy is reduced by doing so. We then calculate the total energy of this spin quenched spin configuration. The Monte Carlo spin configuration, not the spin quenched spin configuration, is used for the next Monte Carlo update cycle. Let E_M be the energy of the lowest state visited after M iterations of the Monte Carlo update–spin quench procedure. Then for M sufficiently large $E^a_M - E^p_M$ is the domain-wall energy, where a and p indicate antiperiodic or periodic boundary

conditions. We repeat the entire calculation N times (N typically 10^4 to 4×10^4) with different interaction configurations to obtain N samples of the domain-wall energy; the variance is then estimated from these N samples. We find in this way the variance in $E_M^e - E_M^p$ after M cycles of update-quench; this variance $\tilde{W}_M$ approaches $\tilde{W}$ as M goes to infinity. We find that the variance converges as $\exp[-A\ln^2(M)]$ and we fit this form to $\tilde{W}_M$ for finite M and extrapolate M to infinity. We choose the maximum M to minimize the total statistical error for fixed computation time. We also vary the temperature used in the Monte Carlo procedure to optimize the convergence. If the temperature is too low the relaxation time over the largest energy barriers is too slow and the algorithm does not rapidly find the ground state. If the temperature is too high the Monte Carlo spin configuration is a highly excited state and the spin quench rarely finds the ground state. The temperature referred to here is the temperature used in the Monte Carlo-update dynamics; the physical temperature is zero since we only deal with the ground state.

IV. RESULTS

The Monte Carlo–quench results for the variance of the domain-wall energy distribution are given in Table I for two and three dimensions. N is the number of interaction configurations sampled to estimate $\tilde{W}$; M is the total number of Monte Carlo–quench cycles for each interaction configuration; T is the temperature of the Monte Carlo–update algorithm. The errors quoted are three estimated standard deviations. The scaling form for $\tilde{W}$ is

$$\tilde{W}_n = An^{\lambda}\tilde{J} \ . \tag{3}$$

In two dimensions the scaling form provides a good fit to the data with eigenvalue $\lambda = -0.306 \pm 0.015$. The interaction iterates toward weak coupling and there is a phase transition at zero temperature with thermal exponent $\nu = 1/\lambda = 3.27 \pm 0.16$ in agreement with the previous DWRG[5] estimate $\nu = 3.56 \pm 0.06$. In three dimensions the scaling form provides an acceptable fit (using the χ-squared test at the 98% confidence level) with $\lambda = +0.17 \pm 0.05$. The interaction iterates toward strong coupling showing that there is a spin-glass phase transition at finite temperature. If we assume a T^2 correction term to the recursion relation at finite temperature[5] we find for the thermal exponent $\nu = 1/2\lambda = 3.0 \pm 1.0$. The heat-capacity exponent is $\alpha = 2 - d\nu = -7 \pm 3$ indicating an undetectable heat capacity anomaly. Interpolating the eigenvalue λ linearly between two and three dimensions we find a lower critical dimensionality (at which λ vanishes) of $d_l = 2.64 \pm 0.10$.

We therefore find that the three-dimensional random Ising model exhibits a spin-glass phase transition at finite temperature and that the lower critical dimension is approximately 2.6. The computations required 700 h on the University of Illinois (Urbana) Monte Carlo computer.

TABLE I. The Monte Carlo–quench results for the random Ising model in two and three dimensions. d is the dimensionality, n^d is the number of the lattice sites; N is the number of interaction configurations; M is the total number of update cycles per configuration; T is the Monte Carlo temperature; $\tilde{J}$ is the variance of the interaction distribution; $\tilde{W}$ is the variance of the domain-wall energy distribution. The errors quoted are three estimated standard deviations.

d	n	N	M	$T/\tilde{J}$	$\tilde{W}/\tilde{J}$
2	3	40 000	2^9	1.7	1.527 ± 0.018
2	4	40 000	2^9	1.7	1.394 ± 0.018
2	5	40 000	2^9	1.7	1.307 ± 0.016
2	6	40 000	2^{10}	1.7	1.231 ± 0.015
2	8	10 000	2^{13}	1.2	1.140 ± 0.030
3	3	40 000	2^{10}	1.7	2.554 ± 0.036
3	4	40 000	2^{11}	1.55	2.650 ± 0.039
3	5	10 000	2^{13}	1.4	2.780 ± 0.099
3	6	10 000	2^{14}	1.3	2.987 ± 0.159

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