Cyclotron resonance of polarons confined to a surface

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Corrections to the cyclotron resonance energy induced by electron—LO-phonon interaction for electrons confined to a heterojunction with magnetic field perpendicular to the surface are calculated. A weak Frölich interaction with bulk LO phonons is assumed, and correlation due to the Coulomb repulsion of the electrons is neglected. Various approximations are compared for the two-dimensional polaron in the zero-density limit. The transition from two-dimensional to bulk behavior is studied, and an approximate method of calculation for an arbitrary confining potential is proposed and tested. Corrections to the cyclotron-resonance energy due to nonzero electron density are calculated by summing the "most divergent" terms in perturbation theory to all orders. It is suggested that correlation may be important for electrons in full or nearly full Landau levels.

INTRODUCTION

Recent cyclotron-resonance experiments on electrons at the GaAs/$Ga_xAl_{1-x}As$ interface show that sharp cyclotron absorption lines can be observed in heterojunctions which have high electron mobilities.[1] In addition, magnetotransport measurements on the same type of heterostructures indicate that the electrons interact with the bulk LO phonons of the GaAs.[2] These results suggest that it should be possible to observe in such systems "polaron corrections" to the cyclotron-resonance frequency, corrections which arise from the interaction of electrons at the interface with bulk LO phonons of the GaAs. Such effects have been observed in InSb inversion layers.[3] Electron—LO-phonon interactions have long been known to shift cyclotron-resonance and impurity transition frequencies in bulk semiconductors,[4] but, because this interaction tends to be weak, sharp transition lines are required for its observation.

Of particular interest is the situation in which a pair of unperturbed electron energy levels has energy separation close to $\hbar\omega_{LO}$, the LO-phonon energy. Then, if these levels are coupled by the electron—LO-phonon interaction, a "polaron-resonance" splitting of the upper level occurs, giving rise to a splitting of optical transition lines which involve the upper level as a final or initial state. In cyclotron-resonance experiments it is possible to produce such a pair of levels from the $n=0$ and $n=1$ Landau levels by adjusting the unperturbed cyclotron frequency ω_c in such a way that $\omega_c \cong \omega_{LO}$ where, if the unperturbed electron band mass is m, then

$$\omega_c = eB/mc \ .$$

For the case of semiconductor heterojunctions with magnetic field B perpendicular to the interface, the upper level ($n=1$) interacts via electron—LO-phonon interaction with every unoccupied lower level ($n=0$), all of which are, to a first approximation, degenerate. Under these conditions a polaron-resonance splitting of the $n=1$ level occurs, giving rise to relatively large shifts in the observed cyclotron-resonance energies.

Das Sarma and Madhukar[5] (DM) have already discussed this resonance splitting in semiconductor heterojunctions. In their work formal calculations are presented for the limit of zero electron density and weak electron-phonon coupling, and off-resonance terms in perturbation theory are neglected.

The work described here is an attempt to improve the DM approach for polaron cyclotron resonance in the zero-density limit by including all significant nonresonant terms for the two-dimensional polaron, thus giving an accurate description both near the polaron resonance ($\omega_c=\omega_{LO}$) and away from it. In addition, the transition between two-dimensional and three-dimensional polaron cyclotron resonance is studied in a simple, soluble model, and a practicable method is proposed and tested for evaluating approximately the off-resonance perturbation contributions for electrons confined to a surface by an arbitrary one-dimensional potential. The Fröhlich model, in which electrons interact with bulk LO phonons, is employed throughout. For cyclotron resonance in GaAs in the two-dimensional limit, off-resonance terms are not experimentally negligible; they contribute corrections which are typically ~ 0.5 meV.

Another important difference between the present work and that of DM is that here, for simplicity, the polaron levels are taken to be perfectly sharp; no phenomenological damping parameters are introduced into the calculations.

Finally, the effect of nonzero electron density is investigated for an ideal gas of two-dimensional electrons. It is found necessary, in principle, to sum terms for all orders in perturbation theory to describe exactly the polaron resonance to lowest nonvanishing order in the dimensionless coupling constant α when the density is not zero. This summation gives rise, in the cyclotron-resonance calculations, to a discontinuity in the polaron correction as a function of the filling factor ν of the lower level at $\nu=1$. Here the filling factor is defined as usual by

$$\nu = 2\pi\rho(\hbar/m\omega_c) \ ,$$

where ρ is the number of electrons per unit area. In gen-

eral, it is found that there is an interplay between the Pauli principle and the polaron-polaron interaction; it is this interplay which determines the size of the density-dependent part of the cyclotron frequency.

One interesting consequence of density-dependent polaron corrections is that, in a given magnetic field, spin-up and spin-down cyclotron frequencies are slightly different when these spin states are unequally populated. Thus, for example, when $1>\nu>2$ so that both spin states associated with the $n=0$ Landau levels are occupied, two distinct cyclotron-resonance frequencies may be expected.

I. LOW-DENSITY LIMIT

Let us assume, as is usual in polaron problems, that at sufficiently low densities of electrons the cyclotron-resonance corrections can be found from a one-electron Fröhlich Hamiltonian, which can be written in polaron units in the form

$$H_{0\perp}=p_y^2+(p_x-\tfrac{1}{2}\lambda^2 y)^2,\quad H_{0z}=p_z^2+V(z)\ ,$$
$$H_0=H_{0\perp}+H_{0z}+\sum_{\vec{k}} b^\dagger_{\vec{k}} b_{\vec{k}}\ , \tag{1}$$
$$H=H_0+\sum_{\vec{k}} \nu_k (e^{-i\vec{k}\cdot\vec{r}} b^\dagger_{\vec{k}}+e^{i\vec{k}\cdot\vec{r}} b_{\vec{k}})\ ,$$

where $b^\dagger_{\vec{k}}$ creates a bulk LO phonon of wave vector $\vec{k}$, $\vec{k}=(k_x,k_y,k_z)$, $\lambda^2=\omega_c/\omega_{\rm LO}$,

$$\nu_k=\left[\frac{4\pi\alpha}{\Omega}\right]^{1/2}\Big/k,\quad \alpha=\frac{1}{2}\frac{1}{\hbar\omega_{\rm LO}}\frac{e^2}{r_0}(1/\epsilon_\infty-1/\epsilon_0)$$

($\alpha\cong 0.06$ in GaAs), ϵ_∞ and ϵ_0 denote the high-frequency and static dielectric constant, respectively, of the semiconductor in which the electrons reside, and r_0 is the polaron radius, equal to $(\hbar/2m\omega_{\rm LO})^{1/2}$.

The Landau gauge has been employed in deriving Eq. (1). In "polaron units" all energies are in units of $\hbar\omega_{\rm LO}$ and lengths are in units of r_0. What is meant by "sufficiently low density" in the context of the present model, in which the Coulomb repulsion between electrons is ignored,[6] will be clarified in Sec. II.

If $V(z)$ is sufficiently strong to confine electron motion to a sufficiently small region in the z direction, the low-lying eigenstates of Eq. (1) are those of a two-dimensional polaron in a magnetic field. If, on the other hand, $V(z)\to 0$, H becomes the Hamiltonian of a bulk polaron in a magnetic field. Corrections to the cyclotron frequency for the transition between the lowest $n=0$ Landau level ($k_z=0$) and the $n=1$ Landau level have already been found by second-order Rayleigh-Schrödinger perturbation theory (RSPT) (exact to order α in the limit $\alpha\to 0$) for $\lambda^2<1$ in the latter case.[7] In Sec. IA, RSPT is evaluated for the two-dimensional polaron, and a more general and accurate solution giving the split cyclotron frequencies exact to order α at $\lambda^2=1$ is described. Various approximate solutions will also be described. In Sec. IB, methods are proposed for calculating the cyclotron-resonance corrections when the polaron has an intermediate form between the two-dimensional and three-dimensional limits.

It is convenient to work in the representation of eigenstates of $H_{0\perp}$. These are the well-known Landau wave functions

$$\psi_{n,q_x}=[\lambda/(\sqrt{2\pi}\,2^n nL)]^{1/2}H_n((\lambda/\sqrt{2})(y-y_0))e^{-\lambda^2(y-y_0)^2/4}e^{iq_x x}\ , \tag{2}$$

where we assume that the interface is a square with side L, $y_0=2q_x/\lambda^2$, and the H_n are Hermite polynomials. The associated eigenvalues are $(n+\frac{1}{2})\lambda^2$. Matrix elements and related quantities involving the wave functions of Eq. (2) and useful in developments to follow are listed below for future reference:

$$M_{n0}(-\vec{k},q_x)\equiv\langle\psi_{n,q_x-k_x}|e^{-i\vec{k}_\perp\cdot\vec{r}}|\psi_{0,q_x}\rangle=(n!)^{-1/2}[(k_x-ik_y)/\lambda]^n\exp[-ik_y(2q_x-k_x)/\lambda^2]\exp(-k_\perp^2/2\lambda^2)\ ,$$

$$M_{0n}(\vec{k},q_x)\equiv\langle\psi_{0,q_x+k_x}|e^{i\vec{k}_\perp\cdot\vec{r}}|\psi_{n,q_x}\rangle=(n!)^{-1/2}[(k_x+ik_y)/\lambda]^n\exp[ik_y(2q_x+k_x)/\lambda^2]\exp(-k_\perp^2/2\lambda^2)\ , \tag{3}$$

$$|M_{1n}(\vec{k},q_x)|^2\equiv|\langle\psi_{1,q_x+k_x}|e^{i\vec{k}_\perp\cdot\vec{r}}|\psi_{n,q_x}\rangle|^2=(n!)^{-1}(k_\perp^2/\lambda^2)^{n-1}(n-k_\perp^2/\lambda^2)^2\exp(-k_\perp^2/\lambda^2)\equiv|M_{1n}(\vec{k})|^2\ ,$$

where $\vec{k}_\perp=(k_x,k_y,0)$, which being the component of $\vec{k}$ in the interface plane is therefore perpendicular to $\vec{B}$.

The eigenfunctions of H_{0z} are denoted ϕ_s with corresponding eigenvalues E_s. The ground-state wave function and energy are ϕ_0 and E_0, respectively. It is convenient to adjust the zero of energy so that $E_0=0$. Useful matrix elements are defined by

$$\mathcal{M}_{ls}(k_z)\equiv\langle\phi_l|e^{ik_z z}|\phi_s\rangle\ .$$

Perhaps the cyclotron-resonance transition of greatest experimental interest at higher magnetic fields is the transition from an $n=0$ Landau level to an $n=1$ Landau level within the lowest subband state ($s=0$). The initial unperturbed electronic state is $\psi_{0,q_x}\phi_0$ and the final unperturbed state is $\psi_{1,q_x}\phi_0$. With H_0 in Eq. (1) as unperturbed Hamiltonian and $H-H_0$ as the perturbation, the intermediate states in RSPT are of the form $\psi_{n,q_x-k_x}\phi_s b^\dagger_{\vec{k}}|0\rangle$ where $|0\rangle$ is the LO-phonon vacuum. The correction to the cyclotron-resonance frequency of interest is denoted $\Delta E_{\rm CR}$, which is the difference between the correction to the upper unperturbed level and that of the lower. In RSPT,

$$\Delta E_{\rm CR}=-\sum_{s'=0}^{\infty}\sum_{\vec{k}}v_k^2\,|\mathcal{M}_{0s'}(k_z)|^2\sum_{n=0}^{\infty}\left[\frac{(n-x^2)^2}{(n-1)\lambda^2+1+E_{s'}}-\frac{x^2}{n\lambda^2+1+E_{s'}}\right]\frac{x^{2n-2}e^{-x^2}}{n!}$$

$$=-\sum_{s'=0}^{\infty}\sum_{\vec{k}}v_k^2\,|\mathcal{M}_{0s'}(k_z)|^2\sum_{n=0}^{\infty}\frac{g_n(x)}{(n-1)\lambda^2+1+E_{s'}}\,, \tag{4}$$

where

$$x=k_\perp/\lambda \text{ and } g_n(x)=[n(n-1)-2nx^2+x^4]x^{2n-2}e^{-x^2}/n!\,.$$

Equation (4) is the fundamental equation for the RSPT polaron shift of the cyclotron-resonance transition energy in the low-density limit; it is most useful when the perturbed cyclotron transition energy is not too close to 1 (the LO-phonon energy). Major limitations of Eq. (4) are that it does not predict two coexisting branches of the cyclotron-resonance spectrum and that it diverges at $\lambda^2=1$, due to vanishing of the $n=0$, $s'=0$ energy denominator. Nevertheless, RSPT gives an accurate description of the ground-state energy and well represents off-resonance contributions (terms for which $n>0$ and/or $s'>0$) to the perturbed $n=1$ level.

A. Cyclotron resonance in the two-dimensional limit

Under conditions in which $V(z)$ confines the electron to a region of z which is small compared to both r_0 and to the cyclotron radius, the subband excitation energies obey

$$E_{s'}\gg 1 \text{ and } E_{s'}\gg\lambda^2 \text{ for } s'\geq 1 \tag{5}$$

and[8]

$$|\mathcal{M}_{00}(k_z)|^2\simeq 1 \text{ for } k_z^2\sim 1 \text{ and } k_z^2\sim\lambda^2\,. \tag{6}$$

We regard Eqs. (5) and (6) as defining the two-dimensional limit. In this limit we need consider only the $s'=0$ term in Eq. (4), which becomes

$$\Delta E_{\rm CR}=-\sum_{\vec{k}}v_k^2\sum_{n=0}^{\infty}\frac{g_n(x)}{(n-1)\lambda^2+1}\,. \tag{7}$$

In the bulk-polaron problem one can always convert the sum on $\vec{k}$ to an integral according to

$$\sum_{\vec{k}}\rightarrow\frac{\Omega}{(2\pi)^3}\int d^3k\,. \tag{8}$$

The same conversion can be made in the heterojunction and quantum-well problems, provided that the thickness of the material supplying the LO phonon [GaAs in the case of the conventional GaAs/(GaAl)As interface] is much greater than r_0, the cyclotron radius, and the confinement distance. (In GaAs, $r_0\simeq 40$ Å.)

With the replacement indicated in Eq. (8) one obtains the two-dimensional limit,

$$\sum_{\vec{k}}v_k^2\rightarrow\frac{\Omega}{(2\pi)^3}\frac{8\pi^2\alpha}{\Omega}\int_0^\infty dk_\perp k_\perp\int_{-\infty}^{\infty}dk_z\frac{1}{k_\perp^2+k_z^2}$$

$$=\alpha\int_0^\infty dk_\perp\,,$$

so that Eq. (7) becomes

$$\Delta E_{\rm CR}=-\alpha\lambda\int_0^\infty dx\sum_{n=0}^{\infty}\frac{g_n(x)}{(n-1)\lambda^2+1}\,. \tag{9}$$

The summation over n in Eq. (9) can be converted into an integral by the method of Ref. 7. There, for performing a sum such as that in Eq. (9), use is made of the identities

$$\frac{1}{a}=\frac{a}{|a|}\int_0^\infty e^{-|a|t}dt\,,$$

$$\sum_{n=0}^{\infty}\frac{n(n-1)}{n!}q^{n-1}=q\,e^q\,,$$

$$\sum_{n=0}^{\infty}\frac{n}{n!}q^{n-1}=e^q\,,$$

and

$$\sum_{n=0}^{\infty}\frac{q^{n-1}}{n!}=\frac{1}{q}e^q\,, \tag{10}$$

taking $a=(n-1)\lambda^2+1$ and $q=x^2e^{-\lambda^2t}$. The result, for $\lambda^2<1$ (hence $a>0$ for all n), is

$$\Delta E_{\rm CR}=-4\alpha\lambda\int_0^\infty dt\,e^{-t}\int_0^\infty dx\,x^2e^{-[x^2(1-e^{-\lambda^2t})]}\times\sinh^2\left[\frac{\lambda^2t}{2}\right]$$

$$=-\sqrt{\pi}\,\lambda\alpha\int_0^\infty dt\,e^{-t}\sinh^2\left[\frac{\lambda^2t}{2}\right](1-e^{-\lambda^2t})^{-3/2}\,. \tag{11}$$

This can also be written in the closed form,

$$\Delta E_{\rm CR}=-\frac{\pi\alpha}{8\lambda}\Gamma\left[\frac{1-\lambda^2}{\lambda^2}\right]\left[\Gamma\left[\frac{1-\lambda^2}{\lambda^2}+\frac{3}{2}\right]\right]^{-1}\,.$$

In the limit $\lambda^2\rightarrow 0$, $\Delta E_{\rm CR}\rightarrow-(\pi/8)\alpha\lambda^2$, so that, adding the unperturbed cyclotron energy, λ^2, the total cyclotron energy is

$$[1-(\pi/8)\alpha]\lambda^2\,. \tag{12}$$

Thus we can identify $[1+(\pi/8)\alpha]m$ with the two-dimensional polaron effective mass in the weak-coupling limit. This is to be compared with the better known three-dimensional polaron effective mass, $(1+\frac{1}{6}\alpha)m$. [A similar calculation of the ground-state energy of the two-dimensional polaron as $\lambda^2\rightarrow 0$ gives $-(\pi/2)\alpha$ compared to the bulk value of $-\alpha$.]

Although Eq. (11) is in a form convenient for numeri-

cal integration for $\lambda^2<1$ and $1-\lambda^2$ not too small, it is not correct for $\lambda^2>1$ and not convenient when λ^2 is near 1. When $\lambda^2>1$ the energy denominator in the $n=0$ term becomes negative. We treat this term separately, subtracting off its contribution in Eq. (11) and adding on the closed-form expression for it, given by Eq. (13), in $\Delta E_{\rm CR}$. Evaluating the $n=0$ term directly, we obtain

$$\frac{\lambda\alpha}{\lambda^2-1}\int_0^\infty dx\, x^2 e^{-x^2}=\frac{\sqrt{\pi}\lambda\alpha}{4(\lambda^2-1)}\ . \tag{13}$$

Thus a form valid for all λ^2 is

$$\Delta E_{\rm CR}=-\sqrt{\pi}\lambda\alpha\left\{\int_0^\infty dt\, e^{-t}\times\left[\sinh^2\left[\frac{\lambda^2 t}{2}\right](1-e^{-\lambda^2 t})^{-3/2}-\tfrac{1}{4}e^{\lambda^2 t}\right]-\frac{1}{4(\lambda^2-1)}\right\}. \tag{14}$$

Since the integrand in Eq. (14) at large t decays like e^{-t} rather than $e^{-(1-\lambda^2)t}$, as in Eq. (11), Eq. (14) is also more convenient than Eq. (11) for evaluating $\Delta E_{\rm CR}$ when λ^2 is near 1.

Examination of Eq. (14) indicates that $\Delta E_{\rm CR}$ becomes proportional to $(\lambda^2-1)^{-1}$ as $\lambda^2\to 1$; one knows that this divergence is not a feature of the actual polaron-resonance phenomenon, but rather an artifact of the approximations leading to the RSPT.

A theoretical description which gives the polaron corrections exact to order α at $\lambda^2=1$ can be found in Ref. 9. In that reference the $n=1$ polaron wave function exact to order $\alpha^{1/2}$ is expanded in the form

$$\Phi=\left[c\psi_{1,0}+\sum_{n,\vec{k}}c_n(\vec{k})\psi_{n,-k_x}b^\dagger_{\vec{k}}+\sum_{n,\vec{k},\vec{l}}d_n(\vec{k},\vec{l})\psi_{n,-k_x-l_x}b^\dagger_{\vec{k}}b^\dagger_{\vec{l}}\right]|0\rangle\ . \tag{15}$$

Here, it is anticipated that the result is independent of the q_x quantum number associated with the zero-phonon state and q_x is set to zero. By requiring $H\Phi=E\Phi$, we obtain

$$(E-\tfrac{3}{2}\lambda^2)c=\sum_{\vec{k}}\nu_k c_n(\vec{k})M_{1n}(\vec{k},-k_x)\ , \tag{16a}$$

$$D_n c_n(\vec{k})=c\nu_k M_{n,1}(-\vec{k},0)+2\sum_{n',\vec{l}}\nu_l d_{n'}(\vec{k},\vec{l})M_{nn'}(\vec{l},-k_x-l_x)\ , \tag{16b}$$

$$\mathscr{D}_n d_n(\vec{k},\vec{l})=\tfrac{1}{2}\nu_l\sum_m c_m(\vec{k})M_{nm}(-\vec{l},-k_x)+\tfrac{1}{2}\nu_k\sum_m c_m(\vec{l})M_{nm}(-\vec{k},-l_x)\ , \tag{16c}$$

where, in accordance with the convention of Eq. (3),

$$M_{mn}(\vec{k},-q_x)=\langle\psi_{m,-q_x+k_x}|e^{i\vec{k}_\perp\cdot\vec{r}}|\psi_{n,-q_x}\rangle\ ,$$

and, in two dimensions,

$$D_n=E-\tfrac{1}{2}\lambda^2-n\lambda^2-1\ ,$$
$$\mathscr{D}_n=E-\tfrac{1}{2}\lambda^2-n\lambda^2-2\ .$$

There are two regions of interest for E in Eqs. (16). If $E-\frac{1}{2}\lambda^2-1=O(\alpha^0)$, then, since $\nu_k=O(\alpha^{1/2})$, solutions exist in which $c=O(\alpha^0)$, $c_n(k)=O(\alpha^{1/2})$, and $d_n(\vec{k},\vec{l})=O(\alpha)$. This is the situation away from the polaron resonance. It is permissible to neglect the two-phonon amplitudes d in this case. However, suppose that $E-\frac{1}{2}\lambda^2-1=O(\alpha^{1/2})$, which, it turns out, occurs for $\lambda^2\simeq 1$, the polaron-resonance region. Then $c=O(\alpha^0)$, $c_0(k)=O(\alpha^0)$, $c_n(k)=O(\alpha^{1/2})$ for $n>0$, and $d_n(\vec{k},\vec{l})=O(\alpha^{1/2})$. It is no longer permissible in this case to ignore the two-phonon amplitudes in calculating energies exact to order α. This motivates retention of the d amplitudes in our calculations.

Since for $m>0$, c_m is always of order $\alpha^{1/2}$ or smaller, it is permissible to set $c_{m>0}=0$ in Eq. (16c). Substituting back into Eq. (16b) gives the integral equation

$$(E-\tfrac{1}{2}\lambda^2-1)c_0(\vec{k})=c\nu_k M_{0,1}(-\vec{k},0)+c_0(\vec{k})\sum_{\vec{l},n'}\nu_l^2 M_{n',0}(-\vec{l},-k_x)M_{0,n'}(\vec{l},-k_x-l_x)/\mathscr{D}_{n'}+T\ ,$$

where

$$T=\nu_k\sum_{n',\vec{l}}\nu_l c_0(\vec{l})M_{n',0}(-\vec{k},-l_x)M_{0,n'}(\vec{l},-k_x-l_x)/\mathscr{D}_{n'}$$

which can be rewritten from Eq. (3),

$$(E-\tfrac{1}{2}\lambda^2-1)c_0(\vec{k})=c\nu_k M_{0,1}(-\vec{k},0)+c_0(\vec{k})\sum_{\vec{l},n}\nu_l^2 e^{-l_\perp^2/\lambda^2}(l_\perp^2/\lambda^2)^n/(n!\mathscr{D}_n)+T\ , \tag{17a}$$

where

$$T=\nu_k\sum_{n,\vec{l}}\left[\nu_l c_0(\vec{l})e^{in(\phi_l-\phi_k)}\left[\frac{k_\perp l_\perp}{\lambda^2}\right]^n\exp\left[-\frac{k_\perp^2+l_\perp^2}{2\lambda^2}\right]\exp\left[-\frac{2i}{\lambda^2}(k_x l_y-k_y l_x)+\frac{i}{\lambda^2}(k_y k_x-l_y l_x)\right]\right]\frac{1}{n!\mathscr{D}_n}\ , \tag{17b}$$

with

$$\cos\phi_k = k_x/k_\perp\,,\quad \sin\phi_k = k_y/k_\perp\,.$$

Since, for $n>0$, $D_n = O(\alpha^0)$, $c_n(k)$ in (16b) be approximated by

$$c_n(\vec{k}) = c\nu_k M_{n,1}(-\vec{k},0)/D_n \quad (n>0)\,. \tag{17c}$$

In the Appendix we show how (17a) can be reduced to a one-dimensional integral equation and solved efficiently by numerical methods. All previous treatments of the three-dimensional analog of this equation start from the approximation $T=0$, even though T is of order α when c_0 is of order α^0. Clearly, setting $T=0$ provides a great simplification since one no longer has to solve an integral equation; however, the energy is then not guaranteed to be correct to order α for $\lambda^2 \simeq 1$.

Let us now discuss various approximations to Eqs. (16) and (17). If one makes no further approximations beyond ignoring T then, combining (17a), (17c), and (16a) one obtains the energy expression

$$E = \tfrac{3}{2}\lambda^2 + \sum_{\vec{k}} \nu_k^2 \,|\, M_{1,0}(\vec{k}) \,|\,^2/(E - E_{\rm GS} - \delta - 1) + \sum_{n=1}^{\infty} \sum_{\vec{k}} \nu_k^2 \,|\, M_{1n}(\vec{k}) \,|\,^2/(E - E_{\rm GS} - n\lambda^2 - 1)\,, \tag{18}$$

where δ is defined by Eq. (A4) in the Appendix. We have replaced $\frac{1}{2}\lambda^2$ in the energy denominator $(E - \frac{1}{2}\lambda^2 - n\lambda^2 - 1)$ for $n>0$ on the right-hand side of (18) by $E_{\rm GS}$, the ground-state energy exact to order α, where

$$E_{\rm GS} = \tfrac{1}{2}\lambda^2 - \sum_{\vec{k}} \nu_k^2 \left[\frac{k_\perp^2}{\lambda^2}\right]^n e^{-k_\perp^2/\lambda^2}/[n!(n\lambda^2+1)]\,.$$

$$= \tfrac{1}{2}\lambda^2 - \frac{\alpha\pi}{2\lambda}\frac{\Gamma(1/\lambda^2)}{\Gamma(1/\lambda^2+\frac{1}{2})}\,.$$

Since, as implied in the Appendix, δ is expected to be small when E is close to $1+E_{\rm GS}$, and, for E far from $1+E_{\rm GS}$, δ is unimportant, being only of order α, it is a reasonable approximation to set $\delta=0$ in (18), giving

$$E = \tfrac{3}{2}\lambda^2 + \sum_{n=0}^{\infty} \sum_{\vec{k}} \nu_k^2 \,|\, M_{1n}(\vec{k}) \,|\,^2/(E - E_{\rm GS} - 1 - n\lambda^2)\,. \tag{19}$$

If one omits all two-phonon terms in Eq. (15), setting $d_n(\vec{k},\vec{l}\,)=0$ for all n, $\vec{k}$, and $\vec{l}$, the equation for E becomes

$$E = \tfrac{3}{2}\lambda^2 + \sum_{n=0}^{\infty} \sum_{\vec{k}} \nu_k^2 \,|\, M_{1n}(\vec{k}) \,|\,^2/(E - \tfrac{1}{2}\lambda^2 - 1 - n\lambda^2)\,, \tag{20}$$

which is the same as Eq. (19) except that $E_{\rm GS}$ is replaced by the unperturbed ground-state energy $\frac{1}{2}\lambda^2$ in the energy denominator. Equation (20) is the equation for second-order Wigner-Brillouin perturbation theory (WBPT). If only the resonant term ($n=0$) is kept, one obtains the approximation of DM for infinitely sharp levels,

$$E = \tfrac{3}{2}\lambda^2 + \sum_{\vec{k}} \nu_k^2 \,|\, M_{10}(\vec{k}) \,|\,^2/(E - \tfrac{1}{2}\lambda^2 - 1) = \tfrac{3}{2}\lambda^2 + \alpha\lambda\sqrt{\pi}/4(E - \tfrac{1}{2}\lambda^2 - 1)\,, \tag{21}$$

whereas if E in the energy denominator of Eq. (20) is replaced by $\frac{3}{2}\lambda^2$, its unperturbed value, Eq. (20), becomes the expression for E in RSPT.

Equation (21) is a quadratic equation for E which, when solved, leads to two separate curves for $E(\lambda^2)$. One of these curves always lies below and the other above $E = \frac{1}{2}\lambda^2 + 1$; these are denoted, respectively, the lower and upper branches of E. Their splitting at $\lambda^2 = 1$ is $\pi^{1/4}\alpha^{1/2}$, which is of order $\alpha^{1/2}$. Analogous solutions have been obtained for Eqs. (19) and (20), where the energy dividing

TABLE I. Corrections to the unperturbed cyclotron-resonance frequency of the two-dimensional polaron, λ^2, as calculated for $\alpha=0.06$ by various methods. The $n=0$ energy is taken to be the perturbed $n=0$ ground-state energy in RSPT, except for $\Delta E_{\rm CR}$ [$n=1$ calculated from Eq. (21)] where it is the unperturbed energy, $\lambda^2/2$. Columns labeled "Lower" and "Upper" refer to the lower and upper branches, respectively, of the $n=1$ Landau levels. $\Delta E_{\rm CR}$ [$n=1$ calculated from Eq. (A8)] is believed to be the most accurate for $\alpha=0.06$. All energies are in units of the long-wavelength LO-phonon energy.

	$\Delta E_{\rm CR}$ [a]		$\Delta E_{\rm CR}$ [b]		$\Delta E_{\rm CR}$ [c]		$\Delta E_{\rm CR}$ [d]	
λ^2	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper
0.4	−0.0160	0.630	−0.0161	0.626	−0.0151	0.625	−0.0268	0.627
0.6	−0.0342	0.452	−0.0345	0.447	−0.0327	0.444	−0.0462	0.446
0.8	−0.072 15	0.293	−0.0733	0.288	−0.0699	0.283	−0.0838	0.284
1.0	−0.153	0.178	−0.157	0.173	−0.152	0.166	−0.163	0.163
1.2	−0.289	0.1175	−0.296	0.114	−0.291	0.106	−0.298	0.0978
1.4	−0.459	0.0913	−0.467	0.0878	−0.463	0.0795	−0.467	0.0673

[a] Perturbed $n=1$ level calculated from Eq. (A8).
[b] Perturbed $n=1$ level calculated from Eq. (18).
[c] Perturbed $n=1$ level calculated from Eq. (19).
[d] Perturbed $n=1$ level calculated from Eq. (21).

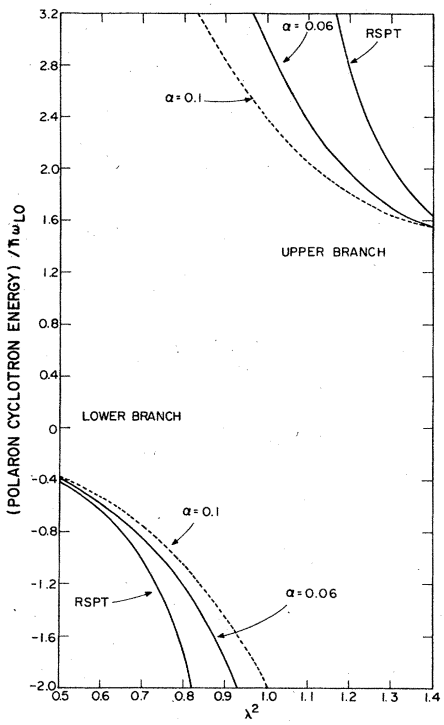

FIG. 1. Comparison of results of calculations of the polaron shifts of the $n=0\rightarrow n=1$ cyclotron transition in the two-dimensional, zero-density limit. The curves marked RSPT are calculated by second-order Rayleigh-Schrödinger perturbation theory. The remaining curves represent solutions of Eqs. (A7) and (A8).

the upper and lower branches is $E_{\rm GS}+1$ and $\frac{1}{2}\lambda^2+1$, respectively. Similar branches are found also in the solutions of Eqs. (18) and (A8).

We shall call the energy dividing the lower and upper branches the "pinning energy"; this energy is a kind of asymptote, being approached from below by the lower branch as $\lambda^2\rightarrow\infty$ and from above the upper branch as $\lambda^2\rightarrow 0$ and $\alpha\rightarrow 0$.

In Table I we compare energies on both the upper and lower branches as calculated exactly to order α from (A8) and from the various approximations (18), (19), and (21) for $\alpha=0.06$, which is a value believed close to the bulk value for GaAs. Rather than recording E we have entered the deviation between the cyclotron-resonance energy calculated from the various approximations and the unperturbed cyclotron energy λ^2. For all calculations except (21) the cyclotron energy is taken to be $E-E_{\rm GS}$; for the approximation (21), $E-\frac{1}{2}\lambda^2$ is the assumed cyclotron energy. The results presented indicate that inclusion of the term T in (17a) is not very important in the region of λ^2 tabulated.

In Fig. 1 we compare the cyclotron-resonance correction predicted by RSPT and that calculated from Eq. (A8) for $\alpha=0.06$ and 0.10. As expected, the RSPT is found to exaggerate the polaron corrections near the resonance region, but is accurate sufficiently far away. The smaller the value of α, the larger the region over which RSPT is useful.

We have concentrated on the two-dimensional polaron at some length because of its mathematical simplicity, which has allowed us to solve the cyclotron-resonance problem to order α in the polaron-resonance region ($\lambda^2\approx 1$). As an exercise in polaron theory one can apply the entire array of methods originally developed for the three-dimensional polaron to the two-dimensional polaron. It is natural to ask, however, how nearly two dimensional the polaron behavior is for a given confining potential $V(z)$. We take up this question in the next section.

B. Cyclotron resonance of polarons confined to a slab of nonzero thickness

1. *Harmonic-oscillator confinement potential*

A more general model than that of the two-dimensional polaron just discussed is provided by considering the cyclotron resonance of a polaron confined by a one-dimensional harmonic-oscillator potential, given in ordinary units by $v(z)=\frac{1}{2}m\omega_{\rm HO}^2 z^2$, where $\omega_{\rm HO}$ is the classical harmonic-oscillator frequency associated with v. In polaron units $v(z)$ becomes $V(z)$, where

$$V(z)=\tfrac{1}{4}\eta^4 z^2,\quad \eta^2=\omega_{\rm HO}/\omega_{\rm LO}\,. \tag{22}$$

An η increases, the thickness of the slab in which the electron is confined decreases. By varying η^2 from values which are less than unity to values considerably greater than unity we can explore the transition, within the harmonic-oscillator model, from bulk to two-dimensional behavior.

In this model the subband energies E_s and the squares of the intersubband matrix elements, $|\mathcal{M}_{0s}(k_z)|^2$, become

$$E_s=s\eta^2 \text{ and } |\mathcal{M}_{0s}(k_z)|^2=\frac{1}{s!}(k_z/\eta)^{2s}\exp(-k_z^2/\eta^2)\,. \tag{23}$$

Substituting Eqs. (23) into Eq. (4) and performing the n summation as previously described gives

$$\Delta E_{\rm CR}=-4\sum v_k^2\int_0^\infty dt\,e^{-t}\sum_s\frac{1}{s!}\left[\left[\frac{k_z}{\eta}\right]^2 e^{-\eta^2 t}\right]^s \sinh^2\left[\frac{\lambda^2 t}{2}\right]\{x^2\exp[-x^2(1-e^{-\lambda^2 t})]\}\,.$$

Performing the summation on s, we obtain

$$\Delta E_{\rm CR}=-\frac{8\alpha}{\pi\lambda^2}\int_0^\infty dt\,e^{-t}\sinh^2\left[\frac{\lambda^2 t}{2}\right]\int_0^1 d\cos\theta\sin^2\theta\int_0^\infty dk\,k^2\exp[-k^2(B\sin^2\theta+A\cos^2\theta)]\,,$$

$$A=(1-e^{-\eta^2 t})/\eta^2,\quad B=(1-e^{-\lambda^2 t})/\lambda^2\,. \tag{24}$$

The two innermost integrals can be performed analytically, yielding

$$\Delta E_{\rm CR}=-\frac{2\alpha}{\sqrt{\pi}\lambda^2}\int dt\, e^{-t}\sinh^2\left[\frac{\lambda^2 t}{2}\right]\frac{A^{1/2}/B-I_1}{A-B},$$

where

$$I_1=\begin{cases}\ln\{[(A-B)^{1/2}+A^{1/2}]/B^{1/2}\}/(A-B)^{1/2} & \text{for } \lambda^2>\eta^2,\\ \sin^{-1}[(B-A)^{1/2}/B^{1/2}]/(B-A)^{1/2} & \text{for } \lambda^2<\eta^2.\end{cases}$$

(25)

Equation (25) is the generalization of Eq. (11) to the case of an arbitrary harmonic-oscillator confining potential, and like Eq. (11), is valid only for $\lambda^2<1$. In the limit of strong confinement ($\eta^2\to\infty$), $A\to 0$ and

$$\Delta E_{\rm CR}\to-\frac{\alpha\pi^{1/2}}{\lambda^2}\int_0^\infty dt\, e^{-t}\sinh^2\left[\frac{\lambda^2 t}{2}\right]\Big/B^{3/2},$$

which is identical to Eq. (11). In the bulk limit ($\eta^2\to 0$), $A\to t$ and $\Delta E_{\rm CR}$ becomes equivalent to Eq. (29) of Ref. 5. Figure 2 displays plots of $\Delta E_{\rm CR}$ as a function of λ^2, computed from Eq. (25), for several values of η^2. Surprisingly large values of η^2 appear necessary for approximating two-dimensional behavior.

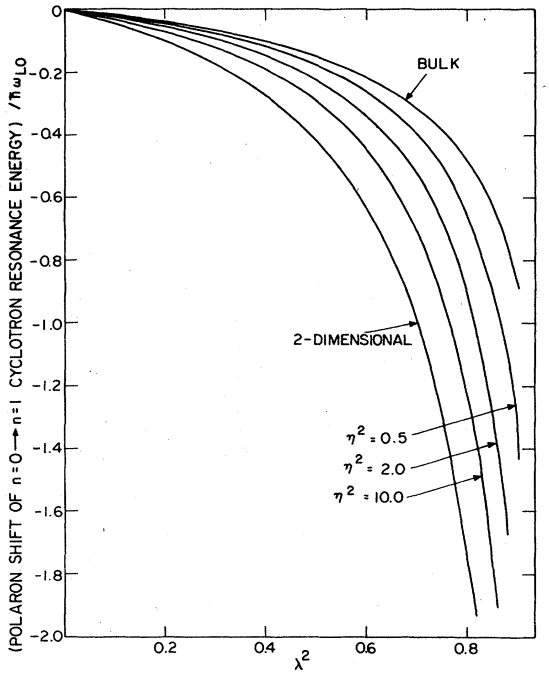

FIG. 2. Comparison of polaron shifts of the lower branch of the $n=0\to n=1$ cyclotron transition for various strengths of the harmonic-oscillator confining potential. The confinement of the electron to the surface increases with η^2, defined in Eq. (22).

2. *Interpolation formula for cyclotron resonance in a general confining potential*

The harmonic-oscillator potential does not represent very realistically the Hartree confining potential at, say, a heterojunction. The actual Hartree potential is more nearly triangular in shape. In general, it would seem prohibitively difficult to evaluate $\Delta E_{\rm CR}$ exactly from Eq. (4) for subband states appropriate to a realistic potential. On the other hand, it is usually not very difficult to find an accurate approximation to the ground-state wave function of the actual confining potential. We seek a formula which may give a good approximation to $\Delta E_{\rm CR}$ defined by Eq. (4) for a realistic Hartree potential and which requires only knowledge of the ground-subband-state wave function.

Since we do not know in advance how strongly confining the potential may be, we shall require that our formula go over to the three-dimensional limit for weak confinement and to the two-dimensional limit for strong confinement.

To guarantee correct strong-confinement behavior we extract from Eq. (4) the dominant terms in the strong-confinement limit, namely the $s'=0$ terms, and evaluate them exactly. The remaining terms in Eq. (4) are then approximated by replacing $E_{s'}$ (for $s'>0$) in the energy denominator by k_z^2, a replacement which would be exact for these terms in the limit of zero confinement, but which is only approximate otherwise.[10] Once this replacement is made, the sum on s' in Eq. (4) can be evaluated by completeness using

$$\sum_{s'}|\mathcal{M}_{0,s'}(k_z)|^2=1$$

for all k_z if the sum is taken over all s' (including $s'=0$). Thus our approximate polaron correction, denoted $\Delta E_{\rm CR}^{(\rm A)}$, is given by

$$\Delta E_{\rm CR}^{(\rm A)}=-\sum_{\vec{k}}v_k^2|\mathcal{M}_{00}(k_z)|^2\sum_{n=0}^{\infty}\frac{g_n(x)}{(n-1)\lambda^2+1}-\sum_{\vec{k}}v_k^2(1-|\mathcal{M}_{00}(k_z)|^2)\sum_{n=0}^{\infty}\frac{g_n(x)}{(n-1)\lambda^2+1+k_z^2}.$$

(26)

The term

$$-\sum_{n=0}^{\infty}v_k^2\frac{g_n(x)}{(n-1)\lambda^2+1+k_z^2}$$

is just the RSPT bulk cyclotron-resonance correction, which we denote $\Delta E_{\rm CR}^{(\rm bulk)}$. Extracting this term and performing the n summation and an integration over $k_\perp$ in the remaining terms, we obtain

$$\Delta E_{\rm CR}^{(\rm A)}=\Delta E_{\rm CR}^{(\rm bulk)}-\frac{2\alpha}{\pi\lambda^2}\int_0^\infty dt\, e^{-t}\sinh^2\left[\frac{\lambda^2 t}{2}\right]\int_{-\infty}^{\infty}dk_z F(k_z^2)|\mathcal{M}_{00}(k_z)|^2(1-e^{-k_z^2 t}),$$

(27)

where $F(k_z^2)=1/B-k_z^2 e^{Bk_z^2}E_1(Bk_z^2)$, B is defined in Eqs. (24), and $E_1(Bk_z^2)=\int_{Bk_z^2}^{\infty}(e^{-v}/v)dv$ is an exponential integral

which can be readily evaluated numerically by quite simple algorithms.[11] Equation (27) is the suggested approximate RSPT polaron correction to the cyclotron energy for an arbitrary confining potential in the low-density limit.

A better approximation than Eq. (27) can presumably be obtained by, for example, treating exactly the $s'=1$ terms in Eq. (4) as well as the $s'=0$ terms. The approximate expression so derived would be valid for $\lambda^2<1+E_1$.

We have compared the exact harmonic-oscillator RSPT results obtained from Eq. (25) to approximate results obtained by evaluating Eq. (27) for a harmonic-oscillator potential. Table II compares values obtained for $\eta^2=2$. Similar agreement is found for other values of η^2.

The good agreement between exact RSPT and approximate RSPT values of $\Delta E_{\rm CR}$ indicated in Table II gives some confidence in the accuracy of the approximation method.

Energies more accurate than those given by RSPT for λ^2 near 1 can readily be obtained by generalizing Eq. (27) along the lines already described for the two-dimensional polaron. We introduce a general integral for which the right-hand side of Eq. (27) is a special case:

$$C_1(\xi)=-\frac{2\alpha}{\pi\lambda^2}\int_0^\infty dt\, e^{-(1+\xi)t}\sinh^2\left[\frac{\lambda^2 t}{2}\right]\int_{-\infty}^{\infty} dk_z F(k_z^2)[\,|\mathcal{M}_{00}(k_z)|^2(1-e^{-k_z^2t})+e^{-k_z^2t}]\,, \tag{28a}$$

and also

$$C_0(\xi)=-\frac{\alpha}{2\pi}\int_0^\infty dt\, e^{-(1+\xi)t}\int_{-\infty}^{\infty} dk_z e^{Bk_z^2}E_1(Bk_z^2)[\,|\mathcal{M}_{00}(k_z)|^2(1-e^{-k_z^2t})+e^{-k_z^2t}]\,. \tag{28b}$$

Here, $C_1(0)=\Delta E_{\rm CR}^{(\rm A)}$ [compare (27)] and $C_0(0)$ is the approximate RSPT correction to the ground-state energy calculated by the approximation leading to Eq. (27). Then an approximate equation analogous to, for example, Eq. (19), is

$$E=\tfrac{3}{2}\lambda^2+C_1(\xi)+C_0(\xi)\,,$$

or

$$E-E_{\rm GS}=\lambda^2+C_1(\xi)+C_0(\xi)-C_0(0)\,, \tag{29}$$

where

$$\xi=\lambda^2-(E-E_{\rm GS}),\quad E_{\rm GS}=\tfrac{1}{2}\lambda^2+C_0(0)\,.$$

The required lowest-branch cyclotron energy, $E-E_{\rm GS}$, is to be found by solving Eq. (29).

II. EFFECTS OF NONZERO ELECTRON DENSITY

In the preceding sections we have implicitly assumed that at sufficiently low electron densities each polaron acts as if no other polarons were present. We now investigate the dependence of the polaron shifts of the cyclotron-resonance frequency upon the density of electrons at the surface in two dimensions, neglecting the Coulomb repulsion $\frac{1}{2}\sum'_{i,j} e^2/\epsilon_0 r_{ij}$ between electrons.

TABLE II. Comparison of exact RSPT and approximate RSPT polaron corrections, $\Delta E_{\rm CR}$ and $\Delta E_{\rm CR}^{(\rm A)}$, respectively, to the $n=0\rightarrow n=1$ cyclotron-resonance transition energy of electrons bound in the one-dimensional harmonic-oscillator potential of (25) with $\eta^2=2$. Formulas for $\Delta E_{\rm CR}$ and $\Delta E_{\rm CR}^{(\rm A)}$ used in the computations are given by (25) and (27), respectively. The filling factor is zero.

λ^2	$\Delta E_{\rm CR}/\alpha$	$\Delta E_{\rm CR}^{(\rm A)}/\alpha$
0.2	−0.0552	−0.0575
0.4	−0.1489	−0.1549
0.6	−0.3383	−0.3503
0.8	−0.9082	−0.9316

The polaron cyclotron energy depends upon the electron density because of the Pauli principle, which forbids, in perturbation theory, electronic intermediate states which are already occupied, and also because of polaron-polaron interaction.

The primary effect of the Pauli principle on the cyclotron resonance is to reduce the number of $n=0$ Landau levels to which an electron in the $n=1$ Landau level can couple via the electron-phonon interaction, since a fraction ν of the $n=0$ Landau levels will already be occupied.

The quantity ν will play a central role in our discussion. If ρ is the number of electrons per unit area in ordinary units, then

$$\nu=2\pi\rho r_c^2=4\pi\rho r_0^2/\lambda^2\,, \tag{30}$$

where r_c is the cyclotron radius ($r_c^2=\hbar/m\omega_c$). With this definition $\nu=1$ corresponds to an electron density just sufficient to fill one Landau level with electrons of a particular spin. We shall, for greatest simplicity, restrict our discussion to the region $\nu<2$. It is convenient to introduce $\tilde{\nu}$ determined by $\tilde{\nu}=\nu$ for $\nu<1$ and $\tilde{\nu}=\nu-1$ for $1\leq\nu<2$. In a noninteracting two-dimensional electron gas at zero temperature with $1<\nu<2$, the lower spin state of the $n=0$ Landau level will be filled and the upper spin state of the same level partially filled. Since the electron–LO-phonon interaction is essentially spin independent, all $n=0$ intermediate states will be forbidden to an electron in the $n=1$ Landau level and lower spin state, but only the fraction $\tilde{\nu}$ of $n=0$ Landau levels of the upper spin state will be forbidden to an electron in the $n=1$ Landau level and upper spin state. From this consideration, it is expected that the cyclotron-resonance corrections for spin-up and spin-down electrons should differ when $\nu>1$ and ν is not an integer, giving rise to a "spin splitting" of the cyclotron resonance. This effect should be particularly pronounced for $\lambda^2=1$ where the

cyclotron-resonance conditions are relatively sensitive to the $n=1-n=0$ coupling. (A spin splitting of the cyclotron resonance can be expected for odd-integer values of ν because the highest Landau levels occupied by electrons of opposite spin are different for ν odd.)

It is less easy to guess, prior to calculation, how polaron-polaron interactions will affect cyclotron resonance. At large separations r two polarons will attract each other with the potential

$$-\left[\frac{1}{\epsilon_\infty}-\frac{1}{\epsilon_0}\right]e^2/r \tag{31}$$

(or, equivalently, $-2\alpha/r$ in polaron units) due to the electron—LO-phonon interaction, and repel each other with the potential $e^2/\epsilon_\infty r$, giving a net Coulomb repulsion of $e^2/\epsilon_0 r$.[12] In our model we ignore this net repulsion. It does not, by itself, affect the cyclotron frequency of the gas. However, when the electronic wave functions of two polarons begin to overlap there develops a short-range interaction between them, resulting from exchange of virtual LO phonons, which cannot be represented by a potential of the form $U(r)$ in general, and which can, in principle, affect the cyclotron resonance. We shall shortly consider the interaction of an electron in a $n=1$ Landau level with the sea of electrons in the $n=0$ Landau levels. This interaction can be expected to be relatively important for $\lambda^2\sim 1$ since in that field region the $n=1$ electron has a relatively high probability of emitting a virtual phonon. In addition, interaction effects are expected to be favored by a relatively high density of electrons in the $n=0$ Landau levels, since high densities provide a relatively high probability of wave-function overlap between the electrons.

To treat the cyclotron-resonance problem quantitatively, it is convenient to write the Fröhlich Hamiltonian in terms of Landau-level creation and annihilation operators. Let $C^\dagger_{n,q_x}$ create an electron in the Landau state ψ_{n,q_x} given by Eq. (2). If we take as our zero of energy the unperturbed $n=0$ Landau-level energy, then the required Hamiltonian is

$$H=H_0+H'+V\ ,$$

where

$$H_0=\lambda^2\sum_{n,q_x} nC^\dagger_{n,q_x}C_{n,q_x}+\sum_{\vec{k}} b^\dagger_{\vec{k}}b_{\vec{k}}\ ,$$

$$H'=\sum_{n,m,q_x,\vec{k}} \nu_k[M_{nm}(-\vec{k},q_x)C^\dagger_{n,q_x-k_x}C_{m,q_x}b^\dagger_{\vec{k}} + M_{mn}(\vec{k},q_x)C^\dagger_{m,q_x+k_x}C_{n,q_x}b_{\vec{k}}]\ , \tag{32}$$

$$V=\sum_{\substack{n,m,n',m'\\ k,q_x,q_x'}} \nu_k^2 M_{nm}(\vec{k},q_x')M_{n'm'}(-\vec{k},q_x) \times C^\dagger_{n,q_x'+k_x}C^\dagger_{n',q_x-k_x}C_{m',q_x}C_{m,q_x'}\ ,$$

with

$$V=\alpha\sum_{i,j}{}' r_{ij}^{-1}=\sum_{i,j}{}' \nu_k^2\cos[\vec{k}\cdot(\vec{r}_i-\vec{r}_j)]\ .$$

[To see how V arises in Eq. (32) note that the more realistic Hamiltonian with electron-electron repulsion included is the same as Eq. (32), except that V is replaced by the interaction $\frac{1}{2}\sum'_{i,j}e^2/\epsilon_\infty r_{ij}$ written in polaron units. Upon subtracting $\frac{1}{2}\sum' e^2/\epsilon_0 r_{ij}$ pursuant to our neglect of this interaction, we are left with $\frac{1}{2}(1/\epsilon_\infty-1/\epsilon_0)\sum' e^2/r_{ij}$, which, expressed in polaron units in Eq. (32), is V.] Here, $\sum'$ denotes summation over all pairs such that $i\neq j$.

It is convenient to break up H', which we shall treat as a perturbation on eigenstates of H_0, in the following way:

$$H'=H_1+H_2+H_3+H_4\ ,$$

$$H_1=\sum_{n,q_x,\vec{k}} \nu_k[M_{n0}(-\vec{k},q_x)C^\dagger_{n,q_x-k_x}C_{0,q_x}b^\dagger_{\vec{k}}+\text{H.c.}]\ ,$$

$$H_2=\sum_{q_x,\vec{k}} \nu_k[M_{01}(-\vec{k},q_x)C^\dagger_{0,q_x-k_x}C_{1,q_x}b^\dagger_{\vec{k}}+\text{H.c.}]\ , \tag{33}$$

$$H_3=\sum_{\substack{n,q,\vec{k}\\ n>0}} \nu_k[M_{n1}(-\vec{k},q_x)C^\dagger_{n,q_x-k_x}C_{1,q_x}b^\dagger_{\vec{k}}+\text{H.c.}]\ .$$

If we denote the electron vacuum state by $|0)$, then the initial unperturbed states of interest have the following form:

$$|\Phi_0(N))\,|0\rangle=\prod_{i=1}^{N} C^\dagger_{0,q_{ix}}\,|0)\,|0\rangle\ \ \text{(ground state)}\ ,$$

$$|\Phi_1(N))\,|0\rangle=C^\dagger_{1,q_x}\,|\Phi_0(N-1))\,|0\rangle\ . \tag{34}$$

In lowest order, H_1 affects only electrons in the $n=0$ Landau level, whereas H_2 and H_3 affect only the electron in the $n=1$ level. Clearly, H_2 is responsible for the polaron-resonance phenomenon. Let us therefore investigate corrections to the energy of $|\Phi_1(N))\,|0\rangle$ due solely to H_2, neglecting, for the moment, $H_1+H_3+H_4+V$. Treating H_2 in second-order Wigner-Brillouin perturbation theory gives

$$E\simeq\lambda^2+(E-1)^{-1}\sum_{\vec{k}}\nu_k^2\,(\Phi_0(N-1)|(1-C^\dagger_{0,q_x-k_x}C_{0,q_x-k_x})|\Phi_0(N-1))\,|M_{01}(\vec{k})|^2\ , \tag{35}$$

where we have used $\{C_{0,q_x-k_x},C^\dagger_{0,q_x-k_x}\}=1$. The exact value of the summation in Eq. (35) depends upon the set of q_x

values describing the occupied $n=0$ Landau levels in $|\Phi_0(N-1)\rangle$. However, for the overwhelming majority of possible initial states $|\Phi_0(N-1)\rangle$, these values are sufficiently uniformly distributed that we can replace $C^\dagger_{0,q_x-k_x}C_{0,q_x-k_x}$ in Eq. (35) by $\tilde{\nu}$, which is the probability that the expectation value of $C^\dagger_{0,q_x-k_x}C_{0,q_x-k_x}$ will be 1. (We shall use this replacement henceforth wherever required.[13]) Thus, Eq. (35) becomes

$$E=\lambda^2+\frac{1-\tilde{\nu}}{E-1}\sum_{\vec{k}}v_k^2(k_\perp^2/\lambda^2)\exp(-k_\perp^2/\lambda^2)\ ,$$

which, as before, in the two-dimensional limit, becomes

$$E\simeq\lambda^2+\frac{1-\tilde{\nu}}{E-1}\alpha\int_0^\infty dk_\perp(k_\perp^2/\lambda^2)\exp(-k_\perp^2/\lambda^2)=\lambda^2+\frac{1-\tilde{\nu}}{E-1}\frac{\alpha\lambda\sqrt{\pi}}{4}\ . \tag{36}$$

Equation (36) is the analog of Eq. (21), the difference between them being the factor $1-\tilde{\nu}$, which is a consequence of the Pauli exclusion principle. [In Eq. (36) the zero of energy is taken at $\frac{1}{2}\lambda^2$ per electron above the lowest subband level at zero magnetic field.] As in the zero-density case, the splitting of the $n=1$ level at $\lambda^2=1$ is $O(\alpha^{1/2})$. However, if the perturbation theory (WBPT) is continued to higher orders, one finds that at $\lambda^2=1$ every order of perturbation theory contributes also in $O(\alpha^{1/2})$, so that one must sum all terms to obtain the correct resonance behavior in the present model.

To see how this comes about consider the next order (fourth order) in perturbation theory. Here, an electron in an $n=1$ level (ψ_{1,q_x}) emits a virtual phonon, dropping into an unoccupied $n=0$ level (ψ_{0,q_x-k_x}); the phonon is absorbed by an electron in an occupied $n=0$ level (ψ_{0,s_x}), exciting it to an $n=1$ level (ψ_{1,s_x+k_x}). A second phonon is then emitted and reabsorbed to return to the initial state. The contribution is

$$(E-1)^{-1}(E-\lambda^2)^{-1}(E-1)^{-1}\sum_{\substack{\vec{k},\vec{k}',\\ s_x}}v_k^2v_{k'}^2\delta_{k_x,k_x'}M_{10}(\vec{k}',q_x-k_x)M_{01}(-\vec{k}',s_x+k_x)M_{10}(\vec{k},s_x)M_{01}(-\vec{k},q_x)$$
$$\times\langle\Phi_0(N-1)|(1-C^\dagger_{0,q_x-k_x}C_{0,q_x-k_x})C^\dagger_{0,s_x}C_{0,s_x}|\Phi_0(N-1)\rangle\ , \tag{37}$$

where $\vec{k}'$ and $\vec{k}$ are the wave vectors of the two virtual phonons participating. It is easy to evaluate this contribution if the sum on s_x is done first. The dependence of the product of the four matrix elements M in Eq. (37) on s_x is found from Eq. (3) to be

$$\exp\left[\frac{2}{\lambda^2}i(k_y-k_y')s_x\right]\ .$$

Summing this on s_x after replacing $C^\dagger_{0,s_x}C_{0,s_x}$ by ν in Eq. (37) gives[13]

$$\frac{L}{2\pi}\int_{-\infty}^{\infty}ds_x\exp\left[\frac{2}{\lambda^2}i(k_y-k_y')s_x\right]=\frac{L}{2}\lambda^2\delta(k_y'-k_y)\ . \tag{38}$$

The sum on k_y' is now trivial, and the sums on k_z and k_z' can be performed in the two-dimensional limit as before. (Notice that for the virtual phonon of wave vector $\vec{k}'$, although $k_x'=k_x$ and $k_y'=k_y$, k_z' is independent of k_z.) What remains is

$$(E-\lambda^2)^{-1}(E-1)^{-2}\alpha^2\lambda^2(1-\tilde{\nu})\nu\frac{1}{4\pi}\int_0^{2\pi}d\phi_k\int_0^\infty\frac{dk_\perp}{k_\perp}(k_\perp^2/\lambda^2)^2\exp(-2k_\perp^2/\lambda^2)=(\alpha^2\lambda^2(1-\tilde{\nu})\nu/16)(E-\lambda^2)^{-1}(E-1)^{-2}\ . \tag{39}$$

When $\lambda^2=1$ this term is also $O(\alpha^{1/2})$ because each energy denominator is of that order. Higher-order terms are found to obey the recursion relation

$$S(I+2)=\tfrac{1}{8}(\nu\alpha\lambda)^2(1+2/I)^{-I/2-1}S(I)/[(E-\lambda^2)^2(E-1)^2]\ , \tag{40a}$$

where, from Eq. (36),

$$S(1)=(1-\tilde{\nu})\alpha\lambda\pi^{1/2}/[4(E-1)]\ , \tag{40b}$$

and, from Eq. (39),

$$S(2)=((1-\tilde{\nu})\nu\alpha^2\lambda^2/16)(E-\lambda^2)^{-1}(E-1)^{-2}\ . \tag{40c}$$

The perturbed energy E is found by solving

$$E-\lambda^2=\lim_{N\to\infty}\sum_{I=1}^{N}S(I)\ . \tag{41}$$

Notice that if $E-\lambda^2=O(\alpha^{1/2})$ and $E-1=O(\alpha^{1/2})$ (λ^2 near 1), or if $E-\lambda^2=O(\alpha)$ and $E-1=O(\alpha^0)$ (E well away from the pinning region), every term $S(I)$ in Eq. (41) is of the same order in α.

The series in Eq. (41) can be evaluated numerically by

summing it exactly from $I=1$ to $I=N$, where N is a sufficiently large integer, and then approximating the remaining infinite series by a geometric series. It is more instructive, however, to choose a constant R such that Eqs. (40) can be approximated by

$$S(I+1)\simeq RS(I)/z\ ,\quad z=(E-\lambda^2)(E-1)\ .$$

[For large I, $R^2\simeq(\nu\alpha\lambda)^2/8e$.] Then Eq. (41) can be written as

$$z\simeq G\sum_{I=0}^{\infty}(R/z)^I=\frac{G}{1-R/z}\ , \tag{42}$$

where $G=(E-1)S(1)$. Note that G is proportional to $1-\tilde{\nu}$ and independent of E. The solution of Eq. (42) is

$$z\simeq G+R\ . \tag{43}$$

A remarkable feature of Eq. (41) as $\tilde{\nu}$ approaches 1 is that, although, on one hand,

$$\lim_{N\to\infty}\lim_{\tilde{\nu}\to 1}\sum_{I=1}^{N}S(I)=0\ , \tag{44a}$$

on the other hand,

$$\lim_{\tilde{\nu}\to 1}\lim_{N\to\infty}\sum_{I=1}^{N}S(I)\simeq R\ . \tag{44b}$$

The result expressed in Eq. (44b) follows from Eq. (43) and the fact that $G\propto 1-\tilde{\nu}$.

The theory just presented can be "improved" by taking into account, in second-order RSPT, $H'-H_2$ from Eq. (33) and V defined in Eq. (32). For $\nu<2$ these terms shift the unperturbed cyclotron energy from λ^2 to $\tilde{\lambda}^2$, where

$$\tilde{\lambda}^2=\lambda^2+\Delta E_{\rm CR}-\alpha\lambda\sqrt{\pi}/4(\lambda^2-1)-\nu\frac{1+2\lambda^2}{1+\lambda^2}\alpha\lambda\sqrt{\pi}/4\ , \tag{45}$$

with $\Delta E_{\rm CR}$ defined in Eq. (14). Note that, because the term proportional to $(\lambda^2-1)^{-1}$ cancels out the divergence in $\Delta E_{\rm CR}$ as $\lambda^2\to 1$, $\tilde{\lambda}^2$ is well behaved near $\lambda^2=1$. In obtaining Eq. (45), $H'-H_2$ is treated in second-order RSPT, and V is treated in first-order perturbation theory.

A more accurate version of Eq. (41), which includes effects due to terms which are nonresonant at $\lambda^2=1$, can be obtained simply by replacing $E-\lambda^2$ whenever it occurs by $E-\tilde{\lambda}^2$ in Eqs. (40) and (41). In Fig. 3 we compare the cyclotron-resonance energy for $\lambda^2=1$ as calculated from the lowest-order resonant term according to

$$E-\lambda^2=S(1)\ ,$$

and from Eq. (41) with λ^2 replaced by $\tilde{\lambda}^2$ as described above. Both upper and lower branches are indicated. [For $\lambda^2\simeq 1$, $E-\lambda^2=O(\alpha^{1/2})$, so the off-diagonal part of V contributes in order $\alpha^{3/2}$ and can therefore be neglected. However, for λ^2 away from 1 where $E-\lambda^2=O(\alpha)$, certain off-diagonal parts of V contribute in order α. These terms can be taken into account by removing them to lowest order by a unitary transformation.]

The large energy shifts induced by terms $S(I)$ in Eq. (41) for $I>1$, which are indicated in Fig. 3, arise from the shrinking of the energy denominators $E-\tilde{\lambda}^2$ and $E-1$ as $\tilde{\nu}\to 1$. This shrinkage enhances the contribution of

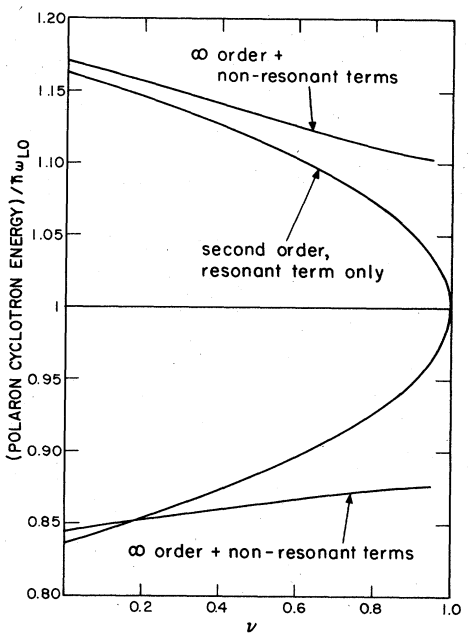

FIG. 3. Comparison of results of two calculations of the two-dimensional polaron cyclotron energy at $\lambda^2=1$ for both upper and lower branches as a function of the filling factor ν. The curve marked "second order" is a plot of E solving Eq. (36). The remaining curves include higher-order terms in ν as well as nonresonant zero-density terms as described in the text.

higher-order terms relative to lower-order terms not only in the energy but in the perturbed wave function. Although it is difficult to calculate the strength of the cyclotron transition using the perturbed wave function, one can expect that with increasingly strong admixture of unperturbed states (which admixed states do not connect to the ground state by an optical matrix element), the cyclotron transition should weaken appreciably as $\tilde{\nu}$ approaches 1. Thus the above discussion leads to the conclusion that the cyclotron transition should become weak for ν close to integer values.

There appears to be no experimental evidence for such an effect.[14,15] This suggests that the theoretical model leading to Eq. (41) is at fault.

The problem with our model does not appear to reside in the assumption of strict two-dimensional confinement since recalculation of $S(I)$ in Eq. (41) in the harmonic-oscillator confinement model described earlier gives results similar to the two-dimensional case, although both G and R are smaller than in two dimensions.[16]

More likely to cause trouble is our neglect of Coulomb repulsion between electrons. Near integer values of ν the actual excited cyclotron state may, in the absence of polaron effects but in the presence of interelectron Coulomb repulsion, be a kind of exciton consisting of an excited $n=1$ electron and its "left-behind" $n=0$ hole in a spatially correlated state.[17] Unlike the free-electron Landau levels, the exciton states are not completely degenerate, but form, rather, a band of nonzero width.[17] For this reason it seems possible that high-order terms in perturba-

tion theory using excitons as unperturbed states may be much less important than terms of similar order in Eq. (41).

In this regard we note that impurity potentials can broaden the cyclotron-resonance transition, an effect which, by smearing out λ^2 in the product of energy denominators z appearing in the $I>1$ terms of Eq. (41), may also reduce the importance of these terms relative to $I=1$.

Finally, it can be expected that, unlike the free-electron Landau levels postulated in our model, Landau levels localized by impurity potentials will display polaron pinning effects even at $\tilde{\nu}=1$ in second-order perturbation theory. This occurs because a localized $n=1$ state which has been optically excited from a nearby localized $n=0$ state (like bound impurity levels) has a nonzero polaron shift in second-order perturbation theory, even when the coupling is restricted exclusively to the single emptied $n=0$ level (in addition to LO phonons). The second-order perturbation-theory coupling between a free-particle $n=1$ Landau level and any other single free-particle $n=0$ Landau level (in addition to LO phonons), on the other hand, vanishes in the infinite-volume limit. Thus an $n=1$ Landau electron excited from a filled $n=0$ level displays no polaron resonance in second order, as is evident from the fact that G in Eq. (42) is proportional to $1-\tilde{\nu}$.

CONCLUSIONS

At low densities, $\nu \ll 1$, one can expect that polaron corrections to the cyclotron resonance of mobile electrons can be calculated quite well over a large range of magnetic fields from a simple Fröhlich-type model with electron-impurity and electron-electron interactions neglected. However, unless the spacing of the two lowest subband levels is very large compared to $\hbar\omega_{\rm LO}$, the two-dimensional approximation significantly overestimates polaron shifts.

At higher densities, $\nu \gtrsim 1$, it appears that important errors may be introduced by ignoring electron-electron correlations and/or electron-impurity interactions in solving the polaron cyclotron-resonance problem. This is because of the enormous degeneracy of the eigenstates of free, noninteracting electrons in a magnetic field and the fact that the electron-phonon interaction couples these degenerate states significantly if ν is not small. Both electron-electron and electron-impurity interactions remove some of this degeneracy and can thereby be expected to affect the polaron corrections in an important way.

In cyclotron-resonance experiments, the electron density is usually held fixed and the magnetic field varied. At low fields ν, which, from Eq. (30), is inversely proportional to the magnetic field, tends to be large. The small-ν regime (say, $\nu \lesssim 0.2$) for interfaces with $\rho \sim 10^{11}$ e/cm^2 is achieved at magnetic fields $\gtrsim 20$ T. Thus it would seem that the zero-density theory presented in this paper may be applicable for analyzing cyclotron resonance in extremely high fields and especially the polaron resonance in GaAs/(GaAl)As interfaces of low electron density. As of this writing the polaron-resonance experiment in these interfaces remains to be done.

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APPENDIX

We wish to transform the integral equation defined by Eqs. (17a) and (17b) into a form suitable for efficient numerical solution. A solution of the form

$$c_0(\vec{k})=c\nu_k M_{01}(-\vec{k},0)f(k_\perp)/(k_\perp/\lambda) \tag{A1}$$

is assumed, where $f(k_\perp)$ is to be determined by substituting (A1) into (17) and solving. This substitution yields

$$(E-E_{\rm GS}-\delta-1)f(k_\perp)=k_\perp/\lambda+\sum_{\vec{l}}\sum_{n=0}^{\infty} I_n(\vec{l})/(E-\tfrac{1}{2}\lambda^2-n\lambda^2-2)\,, \tag{A2}$$

$$I_n(\vec{l})=\frac{1}{n!}\nu_l^2\left[\frac{k_\perp l_\perp}{\lambda^2}\right]^n e^{-l_\perp^2/\lambda^2} f(l_\perp)\exp\left[i(n-1)(\phi_l-\phi_k)-\frac{2ik_\perp l_\perp}{\lambda^2}\sin(\phi_l-\phi_k)\right], \tag{A3}$$

$$\delta=-E_{\rm GS}+\tfrac{1}{2}\lambda^2+\sum_{\vec{l}}\sum_n \nu_l^2\,|M_{0n}(l)|^2/(E-\tfrac{1}{2}\lambda^2-n\lambda^2-2)\,. \tag{A4}$$

Note that $\delta=0$ for $E=1+\frac{1}{2}\lambda^2$ since the rightmost term in (A4) is equal in that case to the RSPT correction to the ground-state energy. In general, δ is given by

$$\delta=0.5\alpha\lambda\sqrt{\pi}\int_0^\infty dt(\exp\{-[2-(E-\tfrac{1}{2}\lambda^2)]t\}-\exp(-t))/(1-e^{-\lambda^2 t})^{1/2}\,.$$

More challenging is the evaluation of the sum on n in (A2). Our approach is to assume that for large n, say $n>N$, we can approximate $E-\frac{1}{2}\lambda^2-n\lambda^2-2$ by replacing $n\lambda^2$ by $l_\perp^2$; then we can rewrite the rightmost term in (A2) in the approximate form

$$\sum_{\vec{l}}\sum_{n=0}^{\infty} I_n(\vec{l})/(E-\tfrac{1}{2}\lambda^2-l_\perp^2-2)+\sum_{\vec{l}}\sum_{n=0}^{N} I_n(\vec{l})[1/(E-\tfrac{1}{2}\lambda^2-n\lambda^2-2)-1/(E-\tfrac{1}{2}\lambda^2-l_\perp^2-2)]\,. \tag{A5}$$

Making the replacement

$$\sum_{\vec{l}} v_l^2 \rightarrow \frac{\alpha}{2\pi}\int_0^\infty dl_\perp \int_0^{2\pi} d(\phi_l-\phi_k)$$

and summing over n, we obtain

$$\sum_{\vec{l}}\sum_{n=0}^{\infty} I_n(\vec{l})=\alpha\int_0^\infty dl_\perp e^{-l_\perp^2/\lambda^2} f(l_\perp)\left[\frac{1}{2\pi}\int_0^{2\pi} d\phi \exp\left[\frac{k_\perp l_\perp}{\lambda^2}e^{-i\phi}\right]e^{-i\phi}\right].$$

The integrand of the ϕ integral can be expanded in powers of $e^{-i\phi}$. Since all terms vanish upon ϕ integration, we have

$$\sum_{\vec{l}}\sum_{n=0}^{\infty} I_n(\vec{l})=0\,. \tag{A6}$$

We can also evaluate $\sum I_n(\vec{l})$ for each n, obtaining

$$\sum_{\vec{l}} I_n(\vec{l})=\frac{\alpha}{n!}\int_0^\infty dl_\perp\left[\frac{k_\perp l_\perp}{\lambda^2}\right]^n e^{-l_\perp^2/\lambda^2} f(l_\perp)\left[\frac{1}{2\pi}\int_0^{2\pi} d\phi \exp\left[i(n-1)\phi-\frac{2ik_\perp l_\perp}{\lambda^2}\sin\phi\right]\right].$$

Determining the square-bracketed integral by applying the expansion

$$\exp(-iz\sin\phi)=\sum_{m=-\infty}^{\infty}\exp(-im\phi)J_m(z)\,,$$

we have

$$\sum_{\vec{l}} I_n(\vec{l})=\frac{\alpha}{n!}\int_0^\infty dl_\perp\left[\frac{k_\perp l_\perp}{\lambda^2}\right]^n e^{-l_\perp^2/\lambda^2} f(l_\perp)J_{n-1}\left[\frac{2k_\perp l_\perp}{\lambda^2}\right].$$

Our integral equation becomes, finally,

$$(E-E_{\rm GS}-\delta-1)f(k_\perp)=k_\perp/\lambda+\alpha\sum_{n=0}^{N}\frac{1}{n!}\int_0^\infty dl_\perp\left[\frac{k_\perp l_\perp}{\lambda^2}\right]^n e^{-l_\perp^2/\lambda^2} \times f(l_\perp)J_{n-1}\left[\frac{2k_\perp l_\perp}{\lambda^2}\right][1/(E-\tfrac{1}{2}\lambda^2-n\lambda^2-2)-1/(E-\tfrac{1}{2}\lambda^2-l_\perp^2-2)]\,. \tag{A7}$$

This equation can be solved for $f(k_\perp)$, at given E, by iteration, at least for the range of λ^2 in Table I. In obtaining the results reported in Table I, we have replaced $E-\frac{1}{2}\lambda^2$ in the energy denominators occurring on the right-hand side of (A7) and (A4) by $E-E_{\rm GS}$, a replacement which does not affect to order α the final value of E. The same replacement is made in the integral defining δ. We find that $N=10$ more than suffices for reaching five-place accuracy in E.

From Eqs. (16a), (17c), and (A1) we obtain

$$E-E_{\rm GS}=\lambda^2+\tfrac{1}{2}\lambda^2-E_{\rm GS}+\alpha\sum_{n=1}^{\infty}\int_0^\infty dk_\perp\,|M_{1n}(\vec{k})|^2/(E-E_{\rm GS}-n\lambda^2-1)+\alpha\int_0^\infty dk_\perp\frac{k_\perp}{\lambda}e^{-k_\perp^2/\lambda^2}f(k_\perp)\,, \tag{A8}$$

which can be solved self-consistently with (A7) to find E on either the lower or upper branch. Results are presented in Table I.

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[3]M. Horst, U. Merkt, and J. P. Kotthaus, Phys. Rev. Lett. **50**, 754 (1983).

[4]See, for example, the review given by D. M. Larsen, in *Proceedings of the 10th International Conference on the Physics of Semiconductors, Cambridge, Mass., 1970,* edited by S. P. Keller, J. C. Hensel, and F. Stern (U.S. AEC, Oak Ridge, 1970), p. 145.

[5]S. Das Sarma and A. Madhukar, Phys. Rev. B **22**, 2823 (1980).

[6]In this paper, $V(z)$ is not calculated but taken as given. Thus the average Coulomb repulsion can be implicitly taken into account by the choice of a realistic $V(z)$.

[7]D. M. Larsen, Phys. Rev. **142**, 428 (1966) [see especially Sec. V. Note that a factor $1/z$ multiplying the logarithm on the right-hand side of Eq. (29) there should be inserted]; **135**, A419 (1964).

[8]Because ϕ_0 is normalized, it follows from Eq. (5) that $|\mathcal{M}_{00}(0)|^2=1$. Since, by assumption, $|\phi_0(z)|^2$ decreases very rapidly from its maximum value, the magnitude of its Fourier transform, $|\mathcal{M}_{00}(k_z)|$, should vary very slowly with k_z for k_z small. This gives Eq. (9). Whether effective-mass theory, upon which our treatment is based, is valid in real materials in the two-dimensional limit remains an open question.

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[10]The replacement of $n\lambda^2$ by l_1^2 in Eq. (A5) and of $E_{s'}$ in Eq. (4) are closely related to the approximation described in Ref. 6 and in *Polarons in Ionic Crystals and Polar Semiconductors* [edited by J. T. Devreese (North-Holland, Amsterdam, 1972), p. 285], in which $n\lambda^2$ is replaced by k_1^2 in the bulk-polaron cyclotron-resonance problem.

[11]M. Abramowitz and I. A. Stegun, *Handbook of Mathematical Functions* (National Bureau of Standards, Washington, D.C., 1964), p. 231. Prepackaged subroutines are also available.

[12]See, for example, D. M. Larsen, Phys. Rev. B **23**, 628 (1981).

[13]In Eq. (35) the sum is over all $n=0$ states with the same spin as the initial $n=1$ electron. We are implicitly assuming that for $\nu<1$ the $n=1$ electron is in the lower spin state, and for $1>\nu>2$ the $n=1$ electron is in the upper spin state. When the summation is over $n=0$ levels of *both* spins, $C^{\dagger}_{0,q_x-k_x}C_{0,q_x-k_x}$ should be replaced by ν. Note that H_2 does not connect electron states of different spins.

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[15]Z. Schlesinger, S. J. Allen, J. C. M. Hwang, D. M. Platzman, and N. Tzoar (unpublished).

[16]To calculate $S(I)$ in the harmonic-oscillator confinement model, we have assumed that all intermediate states are in the ground subband state. This should be a good approximation except for weak confinement. In the bulk limit (zero confinement) we find that high-order terms in the sum on $S(I)$ can be neglected, and both orders of limits of Eqs. (44a) and (44b) give zero.

[17]K. Kallin and B. Halperin (unpublished).