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Influence of nonlinear conductance and cosφ term on the onset of chaos in Josephson junctions

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Chaotic behavior in a Josephson junction is investigated. Threshold curves for the onset of chaos in the rf current-frequency plane are computed by means of Kolmogorov entropy. Both the nonlinear dependence of the quasiparticle current $I_N(V)$ and the cosφ term have been considered to account for previously reported experimental results.

Recently, a great interest has been addressed towards the study of the nonlinear system describing an rf-biased Josephson tunnel junction which can exhibit a chaotic behavior. As is well known this can be related to the appearance of a strange attractor in the phase space. The structure of the attractor, the power spectrum and the threshold curves on the rf current-frequency plane for the transition to chaos have been analyzed by both analog and digital simulation.[1-4] All these studies were performed assuming the simple resistively shunted junction (RSJ) model in which the Josephson tunneling junction[5] is described by an equivalent circuit consisting of the parallel of the junction capacitance C, the normal tunneling resistance R_N and the ideal Josephson element with a nonlinear current $I_C \sin\varphi$. In recent experiments[6] some discrepancies with theory were observed. It can be reasonable to ascribe such discrepancies to the differences between the actual Josephson junction and the simple RSJ model. The essential difference, as can be seen by the experimental current voltage characteristics, is the strongly nonlinear quasiparticle current branch $I_N(V)$.

In the current work we report new results on the occurrence of chaos in Josephson junctions taking into account such a nonlinearity and the effect of the phase-dependent contribution of the quasiparticle current, namely, the "cosφ term."[5]

We assume the expression of $I_N(V)$ suggested in Ref. 7:

$$I_N(V)=\frac{V}{R_N}\frac{V^{2n}}{V_0^{2n}+V^{2n}} \quad , \tag{1}$$

where V_0 is a characteristic voltage (related in our case to the gap voltage) and n the degree of nonlinearity. The best agreement with the experimental I-V curve of high-quality tunnel junction is found for $n\simeq 10$ and $V_0=2\Delta/e$. For low temperatures it can be assumed that $I_C R_N=(\pi/2)(\Delta/e)$.[5] The equation for an rf-driven Josephson junction can be expressed in terms of dimensionless variables as

$$\ddot{\varphi}+\frac{\dot{\varphi}}{\sqrt{\beta}}\left(\frac{\dot{\varphi}^{2n}}{[(16/\pi^2)\beta]^n+\dot{\varphi}^{2n}}+\epsilon\cos\varphi\right)+\sin\varphi=i_0+i_1\sin\omega t \quad , \tag{2}$$

where time is measured in a new scale $\omega_p^{-1}=(\hbar C/2eI_C)^{-1/2}$; $\beta=2eI_C C R_N^2/\hbar$ is the McCumber parameter; $i_0=I_0/I_C$ and $i_1=I_1 I_C$ are the normalized dc and rf current, respectively, and ω is the applied frequency normalized to the plasma frequency ω_p. This equation can be rewritten in the form of three first-order equations:

$$\begin{aligned}\dot{\varphi}&=v \quad ,\\ \dot{v}&=-\frac{v}{\sqrt{\beta}}\left(\frac{v^{2n}}{[(16/\pi^2)\beta]^n+v^{2n}}+\epsilon\cos\varphi\right)\\ &\quad-\sin\varphi+i_0+i_1\sin Z \quad ,\\ \dot{Z}&=\omega \quad .\end{aligned} \tag{3}$$

It is well known that the appearance of a strange attractor implies that phase trajectories must enter into the same phase volume where at the same time they diverge from each other.[8] For the nonlinear system (3) the former condition is fulfilled practically for all region of parameters of interest since the phase volume reduces to

$$\frac{\partial\dot{\varphi}}{\partial\varphi}+\frac{\partial\dot{v}}{\partial v}+\frac{\partial\dot{Z}}{\partial Z}<0 \quad . \tag{4}$$

As a criterion of divergence of the phase trajectories we chose the positive sign of Kolmogorov entropy[8] which is determined by

$$K=\lim_{\tau\to\infty}k(\tau),\ k(\tau)=\frac{1}{\tau}\ln\frac{\mathscr{D}(\tau)}{\mathscr{D}+(0)} \quad , \tag{5}$$

where $\mathscr{D}(\tau)=[(\varphi_1-\varphi_2)^2+(v_1-v_2)^2]^{1/2}$ is the distance between two phase points (φ_1, v_1) and (φ_2, v_2) at the time τ. $\mathscr{D}(0)$ indicates the distance of the two points at $\tau=0$. If there is chaos in the system neighboring points will separate at exponential rate $(K>0)$. In the other case $(K<0)$ we can have any periodic motion.

Solutions of Eq. (3) were generated numerically by the fourth-order Runge-Kutta method for various values of the parameters β, i_0, i_1, and ω. For all data discussed here there are 64 time steps per rf cycle. Starting from a given initial condition [usually $\varphi(0)=0$, $v(0)=0$], Eq. (3) is computed up to $t=30T$ $(T=2\pi/\omega)$. At least in nonchaotic solutions such a time length is enough to neglect relaxation processses. After that time a small perturbation of φ and v such that $\mathscr{D}(0)=10^{-3}$ is introduced and both perturbed and unperturbed solutions are computed which determine $\mathscr{D}(\tau)$ and $k(\tau)$.

The dependence of k on the number of periods N for $\omega=0.5$, $i_1=0.7$, $i_0=0$ are given in Fig. 1 for three types of initial conditions of $\varphi(0)$, $v(0)$. It can be seen that k tends to a value independent of initial conditions for large N. Such a value is independent of the perturbation $\mathscr{D}(0)$ as well. This implies that the distance between two points, ini-

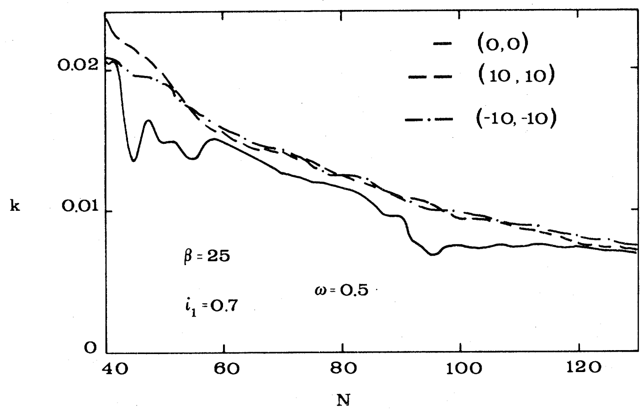

FIG. 1. Dependence of k on the number N of rf current periods for $\beta=25$, $\omega=0.5$, $i_1=0.7$, $\epsilon=0.5$, $n=10$, $\mathscr{D}(0)=10^{-3}$ and different initial conditions $\varphi(0)$, $v(0)$: (0,0) solid line, (10,10) dashed line, $(-10,-10)$ dashed-pointed line. All quantities are dimensionless.

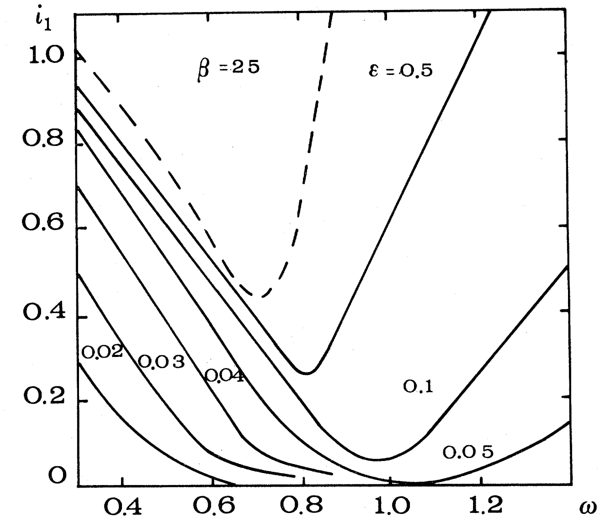

FIG. 3. Onset of chaos (threshold curves) for $i_0=0$, $\beta=25$, $n=10$, and different values of ϵ. Dashed line refers to the RSJ model ($n=0$, $\epsilon=0$).

tially close to each other, increases exponentially with time.

For the determination of the threshold curves in the $i_1-\omega$ plane we fix ω and *increase* i_1 by step $\Delta i_1=0.02$ and compute $k(N)$ at $N=100T$. To show how the positive sign for k implies the onset of chaos we compute the Poincaré section of the system. This is shown in Fig. 2 for 10^3 periods of the rf current. The very large number of distinct points in the phase plane corresponds to harmonics or subharmonics existing in the system. These do not lie on a line but in a region of the plane indicating the presence of chaos (see, for example, Ref. 1). It is interesting to observe that the introduction of additional nonlinearities in the system led to a higher degree of randomness in the position of the points on the phase plane.

In Fig. 3 threshold curves on the i_1-ω plane corresponding to $\beta=25$ and $n=10$ are reported for different values of ϵ. The result using the RSJ model ($n=0$, $\epsilon=0$) is also shown (dashed line). The corresponding curve lies above all others. The results of the computation for the RSJ model including the nonlinear dependence $I_N(V)$ only ($\epsilon=0$) do not agree with experiments. The corresponding threshold curves in fact would imply the onset of chaos at too low values of i_1. If we consider, in addition, the contribution of the cosine term the situation changes quite drastically. For values of ϵ within 0.1 and 0.5 good agreement is found with experimental results reported in Ref. 6. Let us observe that while the location of the threshold curves in the i_1-ω plane strongly depends on ϵ it is much less sensitive to n for $n\geqslant 2$.

In conclusion we have computed threshold curves for the onset of chaos using the concept of Kolmogorov entropy. The importance of considering a more realistic junction model which takes into account the nonlinear quasiparticle conductance has been demonstrated. Far from being conclusive, this investigation has shown that, for the agreement

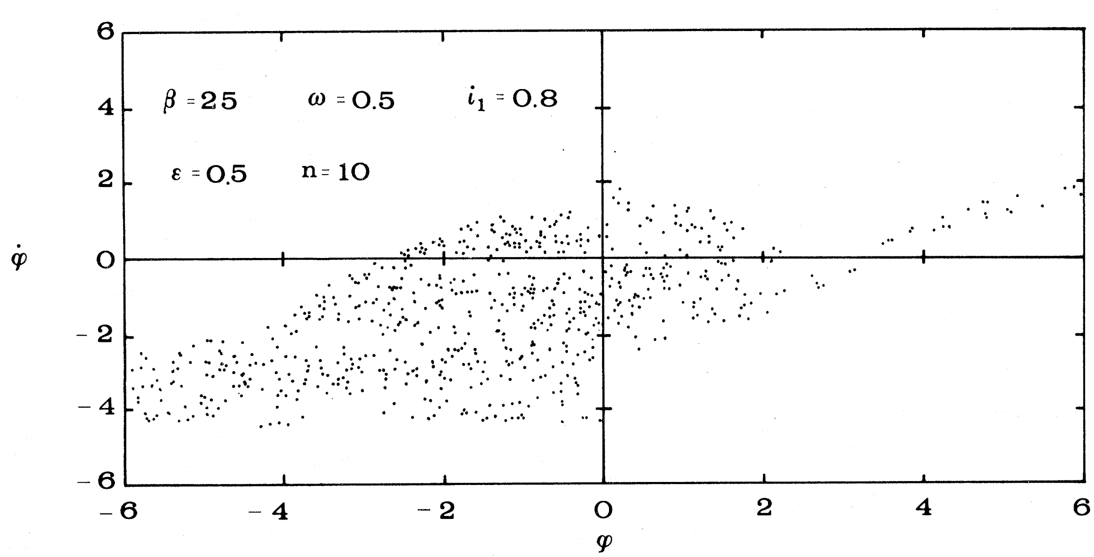

FIG. 2. Poincaré section for $\beta=25$, $\omega=0.5$, $i_1=0.8$, $\epsilon=0.5$, $n=10$. (Dimensionless quantities.)

with previously reported experimental results, the nonlinearity of $I(V)$ is not sufficient unless a contribution of the $\cos\phi$ term is taken into account. Further attention should be paid to the sign of $\cos\phi$, which has been for many years a debatable point (see Ref. 5 and references reported therein).

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