Diffusion-controlled deposition on surfaces: Cluster-size distribution, interface exponents, and other properties

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The diffusion-controlled deposition of particles onto surfaces has been simulated in two and three dimensions. The deposits generated in these simulations are substantially larger than those used in earlier work and allow the geometric scaling relationships which characterize the surface deposits to be more precisely determined. Several of these geometric scaling relationships are investigated in this paper. All of our results are consistent with the idea that surface deposits generated by diffusion-limited particle deposition have a fractal-like structure with a fractal dimensionality (D) equal to that of clusters generated by the Witten-Sander model for diffusion-limited aggregation. Our results are also consistent with the relationships between the cluster-size-distribution exponent (τ), the fractal dimensionality and the Euclidean dimensionality (d) [$\tau=1+(d-1)/D$] recently obtained by Rácz and Vicsek. Results are also presented for the way in which the size of the old-growth—new-growth interface depends on the size of the old growth.

INTRODUCTION

The formation and properties of random fractals[1] are a topic of considerable current interest which may find important applications in diverse areas such as material science, colloid science, and biology. One of the most important models which generates fractal structure is the Witten-Sander (WS) model for diffusion-limited aggregation.[2] The WS model has stimulated a considerable amount of theoretical work[3–11] and has encouraged the development of a number of more or less similar models[12–18] including diffusion-controlled deposition on fibers and surfaces.[19–21]

The main conclusion of our earlier work[19] on diffusion-controlled deposition on fibers and surfaces was that these processes are very similar to diffusion-controlled deposition on a single growth site (the original WS model) to the extent that the exponent describing the density profile is invariant to the geometry (boundary conditions) within the uncertainty of the simulations. Unfortunately, the results obtained from the two-dimensional (2D) and three-dimensional (3D) simulations seemed to be subject to substantial uncertainty due to finite-size effects. The results of our simulations of surface deposition have been compared with theoretical predictions,[11,20] but more accurate results are needed for this purpose.

In this paper we report the results obtained from a larger-scale simulation of diffusion-controlled deposition on surfaces in 2D and 3D systems. More accurate results are obtained for the density profile (root-mean-square thickness) exponents. However, the main objective of this paper is to present results for other properties which characterize the structure and growth of surface deposits. In particular, the size of the old-growth—new-growth interface and the cluster-size distribution are emphasized. Other properties such as the mean height of the upper surface of the deposit and the deviation of the height of the upper surface from the mean and the density-density correlation functions are also discussed.

It is hoped that the results presented in this paper will provide useful comparisons with theoretical results and will contribute to the development of a better understanding of diffusion-limited aggregation in general.

SIMULATION RESULTS

The procedures used to simulate diffusion-controlled deposition on surfaces have been discussed before. No significant modifications were made to our computer programs for the present study. The structure of clusters of occupied lattice sites (or particles) grown using the Witten-Sander model for diffusion-limited aggregation exhibit geometric scaling relationships which are characteristic of fractals and can be used to estimate an effective fractal dimensionality (D_c) which has a value of about $\frac{5}{3}$ for $d=2$ and about $\frac{5}{2}$ for $d=3$. Similarly, the structure of surface deposits formed by diffusion-limited aggregation can also be described in terms of an effective fractal dimensionality (D_s) which can also be determined from a variety of geometric scaling relationships such as

$$X \sim N^{1/(1-d+D_s)}, \tag{1}$$

where X is some measure of the deposit thickness, N is the mass of the deposit, and d is the Euclidean dimensionality of the simulation. If Eq. (1) is taken as a definition of D_s, our earlier results indicated that $D_s \simeq D_c$.

2D SIMULATIONS

In the 2D simulations particles were deposited on a "surface" (line) of 2048 growth sites. The deposition process was allowed to continue until the deposit had reached a height of 400 lattice units. A typical deposit contained about 50 000 particles or occupied lattice sites and eight such deposits were generated. These calculations were carried out using an IBM 3081 computer.

Cluster- (tree) size distribution

The 2D surface deposits consist of a number of naturally separated clusters or "trees" (see Fig. 3 of Ref. 19). Since the number of clusters or trees of size I [$N(I)$] is small for all but very small values of I, it is convenient to explore the cluster-size distribution by examining the total number of clusters of size $\geq I(NT_I)$ as a function of I. Figure 1 shows a plot of $\ln(NT_I)$ versus $\ln(I)$ averaged over all eight simulations; from this figure we can conclude that $NT_I \sim I^{\nu}$ ($\nu \simeq -0.55$). Additional results obtained from all eight of the 2D simulations are shown in Table I. The results given in Table I indicate that $\nu = -0.55 \pm 0.05$. Consequently, we conclude that the number of clusters of size I are related to I by the power law $N(I) \sim I^{-\tau}$ ($\tau = 1.55 \pm 0.05$).

Rácz and Vicsek[20] have deduced that the exponent τ is related to the Euclidean dimensionality d and the fractal dimensionality (D) by

$$\tau = 1 + (d-1)/D_s \ . \tag{2}$$

Assuming that $D_s = D_c$ and that D_c has the value of $\approx \frac{5}{3}$ (obtained from simulations using the WS model for $d=2$ and from recent theoretical results) then the predicted value of τ is 1.6, in good agreement with our results. Alternatively, we can use the measured value of τ to estimate D and obtain $D = (d-1)/(\tau - 1) = 1.8 \pm 0.17$. This does not seem to be a good way of determining D since quite accurate values of τ are needed and the cluster-size distribution contains only a small part of the total information concerning the aggregate structure.

Root-mean-square deposit thickness

In our earlier work on diffusion-limited deposition the only quantity which was quantitatively determined was the dependence of the root-mean-square (RMS) thickness (T) on the number of particles (occupied lattice sites) in the deposit (N). The results obtained indicated that T was related to N by a limiting ($N \to \infty$) power-law relationship

$$T \sim N^{\epsilon} \ . \tag{3}$$

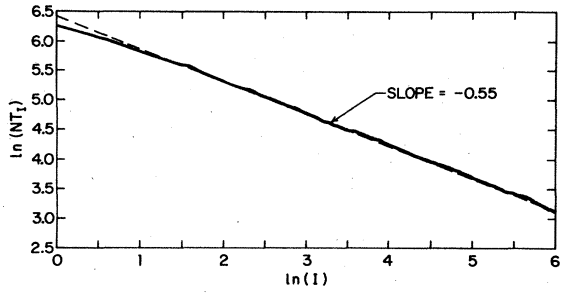

FIG. 1. This figure shows the dependence of $\ln(NT_I)$ on $\ln(I)$ for 2D surface deposits. NT_I is the total number of clusters (or trees) with size greater than or equal to I.

In order to estimate ϵ the dependence of T on N was fitted assuming that

$$T = A' N^{\epsilon'} + B' \tag{4}$$

or

$$T = A'' N^{\epsilon''} (1 + B'' N^{-y''}) \ . \tag{5}$$

The same procedures have been used for the larger scale simulations. Using Eq. (4) we find $\epsilon' = 1.36 \pm 0.05$ and from Eq. (5) $\epsilon'' = 1.55 \pm 0.1$. In our earlier simulations we found $\epsilon' = 1.31 \pm 0.06$ and a value of 1.33 was estimated for ϵ'' by fitting Eq. (5) to the dependence of T on N obtained by averaging all of our 2D simulations. Now that we have "better statistics" a value of ϵ'' can be obtained from each individual simulation and the statistical uncertainty (95% confidence limits) can be estimated.

The density profile in a 2D cluster grown using the WS model can be expressed as

$$\rho_c(r_c) = r_c^{-\alpha_c} \ , \tag{6}$$

where $\rho_c(r_c)$ is the mean density at a distance r_c from the origin and α_c is the fractal or Hausdorff codimensionality ($\alpha_c = d - D_c \simeq \frac{1}{3}$). If the 2D surface deposits have the same density profile

TABLE I. Values for the exponent ν, which relates the number of cluster of size $\geq I$ (NT_I) to I. The results were obtained by least squares fitting straight lines to the coordinates ($\ln(NT_I)$, $\ln(I)$) over the range $I_1 \leq I \leq I_2$.

I_1	5	10	5	20	50
I_2	500	100	50	200	500
	−0.584	−0.515	−0.550	−0.514	−0.630
	−0.537	−0.602	−0.566	−0.522	−0.541
	−0.572	−0.440	−0.471	−0.593	−0.618
	−0.556	−0.584	−0.500	−0.585	−0.543
	−0.542	−0.456	−0.534	−0.510	−0.577
	−0.562	−0.541	−0.508	−0.525	−0.590
	−0.564	−0.614	−0.562	−0.580	−0.544
	−0.531	−0.568	−0.630	−0.517	−0.516
Avg.	0.556±	0.540±	0.540±	0.547±	0.570±
	0.015	0.054	0.041	0.033	0.034

$$\rho_s(r_s)=r_s^{-\alpha_s}, \tag{7}$$

where $\rho_s(r_s)$ is the mean density at a distance r_s from the surface and α_s has a value of $\frac{1}{3}$ we expect to find that $T\sim N^{\epsilon}$ where the limiting $(N\to\infty)$ value for the exponent ϵ is given by

$$\epsilon=1/(1-\alpha_s)=1.5\ . \tag{8}$$

Our results seem to support the idea that $\alpha_s=\alpha_c$ or $D_s=D_c$.

If Eq. (8) is used to estimate a fractal dimensionality (D_s) for the surface deposit $(D_s=d-1+\epsilon^{-1})$, we find $D_s=1.74\pm0.02$ (from ϵ') and 1.645 ± 0.004 (from ϵ'').

Upper surface properties

By "upper surface" we mean the surface which an observer would see looking straight down at the deposit. The height of the surface at any position on the surface is either zero (the original surface with no deposit) or the maximum height of any particle (occupied lattice site) at that position. Figure 2 shows the dependence of $\ln(\bar{h})$ where $\bar{h}$ is the mean height of the upper surface on $\ln(N)$. It is apparent from Fig. 2 that $\bar{h}$ is related to N (in the large-N limit) by

$$\bar{h}\sim N^{\phi}\ . \tag{9}$$

The results shown in Fig. 2 are the average for all eight 2D simulations. The results obtained from the individual simulations are shown in Table II. These results indicate that the effective value of ϕ is 1.44 ± 0.03. Since the value of ϕ is not sensitive to the range of deposit sizes (thicknesses) used to obtain ϕ, we conclude that this value is probably a good approximation to the limiting $(N\to\infty)$ value. The exponent ϕ has also been obtained by fitting the dependence of $\bar{h}$ on N by functions of the form given in Eqs. (3) and (4). Using these methods we obtain the results $\phi'=1.44\pm0.03$ and $\phi''=1.47\pm0.04$. Taken together, these results indicate that $\phi=1.45\pm0.05$. Assuming that ϕ is related to the fractal dimensionality (D_s) in the same way that ϵ is related to D_s $(D_s=d-1+\phi^{-1})$, we obtain the estimate $D_s=1.690\pm0.025$.

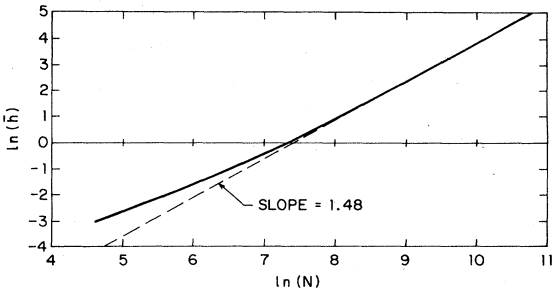

FIG. 2. Dependence of the mean upper surface height $(\bar{h})$ on the total number of particles deposited on a "surface" of 2048 lattice sites in 2D simulation of diffusion-limited deposition.

The RMS deviation of the upper surface height (h) from the mean upper surface height $(\bar{h})$ has also been measured. Figure 3 indicates that the RMS upper surface height deviation from $\bar{h}$ is related to N by

$$\langle(h-\bar{h})^2\rangle^{1/2}\sim N^{\omega} \tag{10}$$

for large N. The straight line in Fig. 3 has a slope of 1.28 and indicates that $\omega\simeq1.28$. However, the curvature apparent in Fig. 3 indicates that ω may be substantially larger than 1.28. Consequently, the limiting $(N\to\infty)$ value of ω has been obtained by least squares fitting the dependence of $\langle(h-\bar{h})^2\rangle^{1/2}$ on N by expressions of the form shown in Eqs. (3) and (4) $[\langle(h-\bar{h})^2\rangle^{1/2}=A'N^{\omega''}+B'$ and $\langle(h-h)^2\rangle^{1/2}=A''N^{\omega''}(1+B''N^{-\gamma''})]$. The results obtained in this way are $\omega'=1.32\pm0.08$. It was not possible to obtain ω'' for each individual simulation but a value of 1.50 was obtained using the averaged data shown in Fig. 3. These results indicate that $\langle(h-\bar{h})^2\rangle^{1/2}$ and $\bar{h}$ scale with N in the same way in the limit $N\to\infty$. This result is consistent with a surface deposit structure which can be mapped onto the surface deposit structure at later and earlier stages of growth (in a statistical sense) by a change of length scale.

TABLE II. Values obtained for the mean upper surface height exponent (ϕ) by least square fitting straight lines to the coordinates $(\ln(\bar{h}), \ln(N))$ over the range $N_1\le N\le N_2$.

2D Surface deposit size (N_{max})	$N_1=0.5N_{max}$ $N_2=N_{max}$	$N_1=0.25N_{max}$ $N_2=N_{max}$	$N_1=0.1N_{max}$ $N_2=N_{max}$	$N_1=0.25N_{max}$ $N_2=0.5N_{max}$
53 053	1.45	1.41	1.42	1.42
49 441	1.50	1.48	1.49	1.43
52 307	1.40	1.43	1.44	1.51
49 749	1.40	1.43	1.45	1.43
54 664	1.44	1.43	1.45	1.44
49 599	1.44	1.42	1.44	1.45
47 246	1.46	1.46	1.47	1.40
50 283	1.44	1.45	1.46	1.46
Avg. 50 793	1.44± 0.03	1.44± 0.02	1.45± 0.02	1.44± 0.03

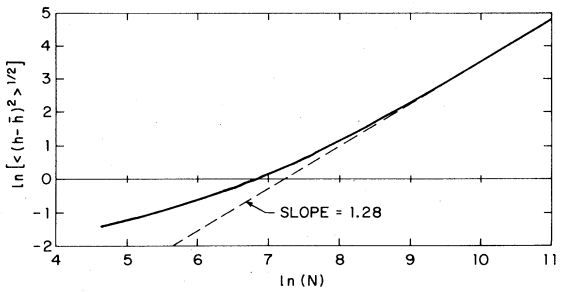

FIG. 3. This figure shows how the RMS deviation of the upper surface height $[\langle (h-\bar{h})^2 \rangle^{1/2}]$ depends on the total number of particles deposited in 2D simulations of diffusion-limited deposition on a surface (line).

Density-density correlation function

One of the most fundamental quantities characterizing the structure of random fractals is the density-density correlation function. For a structure with a fractal dimensionality of D and a Euclidean dimensionality of d we expect to find that the density-density correlation function $C(r)$ depends on distance (r) according to the power-law relationship

$$C(r) \sim r^{-\alpha}, \tag{11}$$

where α is the codimension $(d-D)$. In practice we expect that Eq. (11) will be accurate over only a limited range length scale with upper and lower cutoff lengths due to lattice effects and finite-size effects, respectively. In calculating the two-point density-density correlation function for the surface deposits, one of the two points in the correlation function was confined to lattice sites which had become occupied when the simulation was between $\frac{1}{3}$ and $\frac{2}{3}$ complete. This was done to avoid sampling the "anomalous" region adjacent to the original surface and the outer regions of the deposit which are not fully formed and are subject to additional growth. Table III shows estimates obtained for α by analyzing $C(r)$ over various length scale ranges. The results shown in Table III indicate that $\ln[C(r)]$ is linearly related to $\ln(r)$ over length scales ranging from a few lattice units to about 30 lattice units, and that the exponent α has a value of 0.38 ± 0.02 corresponding to an effective fractal dimensionality of $D_\alpha = 1.62 \pm 0.02$.

Old-growth—new-growth interface

The old-growth—new-growth interface consists of the points of contact between particles in a deposit containing M particles and additional particles added at a latter stage of growth to give a total of N particles. The size of the interface can be defined as the number of "new-growth" particles occupying lattice sites adjacent to lattice sites occupied by "old-growth" particles. In this paper we explore how the size of the interface (in the limit $N \to \infty$ where the interface is saturated) depends on M and how many old-growth particles must be added to complete half of the old-growth—new-growth interface. The same quantities have recently been investigated by Meakin and Witten for diffusion-limited aggregation on a single growth site in two and three dimensions.[22]

Figure 4 shows the dependence of N_I (the number of old-growth particles at the interface) on M. The results shown in Fig. 4 are the average for all eight 2D simulations. The results shown in Fig. 4 are given in the form of a log-log plot in Fig. 5. Figure 5 indicates that in the early stages of deposition N_I increases linearly with M but eventually crosses over to a much slower rate of increase ($N_I \sim M^\delta$, $\delta \simeq 0.11$). A more careful determination of the exponent δ by least squares fitting straight lines to the coordinates $(\ln(N_I), \ln(M))$ for each of the simulations gives the result $\delta = 0.161 \pm 0.032$ for $0.2N \leq M \leq 0.8N_{\max}$ and $\delta = 0.166 \pm 0.033$ for $0.25N \leq M \leq 0.75N_{\max}$ where $N_{\max}$ is the number of particles in a deposit at the end of a simulation.

The number of particle additions needed to half saturate or half complete the interface $(N_{1/2})$ has also been measured. Figure 6 shows the dependence of $N_{1/2}/N_I$ on the number of old particles (M). The results shown in Fig. 6 are averages for all eight simulations. $N_{1/2}/N_I$ starts off with a large value (mainly because N_I is small) and reaches a minimum at $N \sim 3000$ (about 1.5 particles per "surface" site). $N_{1/2}/N_I$ then grows and finally levels off and declines towards the end of the simulation. The final decrease in $N_{1/2}/N_I$ is a result of the fact that the growth of the interface is incomplete for $M \simeq N_{\max}$. It is not clear from the results shown in Fig. 6 if $N_{1/2}/N_I$

TABLE III. Density-density correlation-function exponents (α) obtained by least squares fitting straight lines to the coordinates $(\ln[C(r)], \ln(r))$ over the range $r_1 \leq r \leq r_2$.

$r_1=1$ $r_2=50$	$r_1=3$ $r_2=30$	$r_1=2$ $r_2=10$	$r_1=5$ $r_2=25$	$r_1=10$ $r_2=50$
0.377	0.362	0.359	0.365	0.394
0.415	0.394	0.385	0.383	0.469
0.403	0.380	0.362	0.377	0.457
0.413	0.390	0.361	0.386	0.476
0.420	0.389	0.367	0.385	0.500
Avg.				
0.406±	0.383±	0.367±	0.379±	0.457±
0.021	0.016	0.013	0.011	0.028

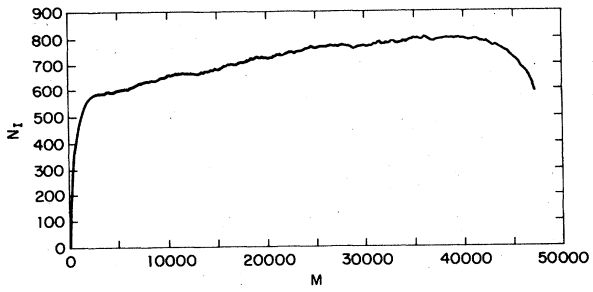

FIG. 4. Size of the old-growth—new-growth interface is shown as a function of the size of the old growth during 2D simulations of diffusion-limited deposition.

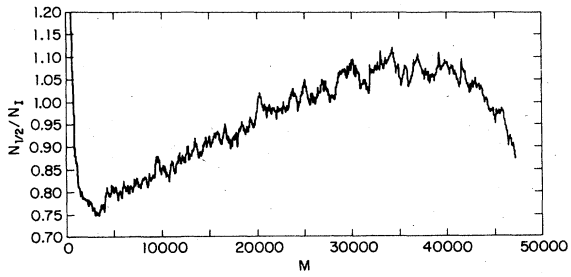

FIG. 6. This figure shows how the ratio between the number of added particles which half complete the old-growth—new-growth interface and the size of the interface depend on the old growth size during 2D simulations of diffusion-limited surface deposition.

would continue to increase as the deposit size increases or would level off at a value not much larger than 1.0.

3D SIMULATIONS

In the 3D simulations particles were deposited on a 200×200 surface until the deposit reached a maximum height of 100 lattice units. A typical deposit contains about 65 000 particles or occupied lattice sites. Seven deposits were generated using a CDC Cyber 205 computer.

Cluster- (tree) size distribution

The cluster or tree size distribution has been investigated in three dimensions using the same methods which were used to analyze the 2D surface deposits. Figure 7 shows the dependence of $\ln(NT_I)$ on $\ln(I)$ where NT_I is the total number of clusters of size $\geq I$. The slope of -0.84 indicates that the cluster-size-distribution exponent (τ) has a value of about 1.84. The results shown in Fig. 7 are averaged over all seven of the simulations. Table IV shows the results obtained from the individual 3D simulations. The results shown in Fig. 7 and Table IV indicate that NT_I is related to I by a power-law relationship ($NT_I \sim I^{\nu}$) where the exponent ν has an effective value of about -0.84 ± 0.08. However, the variation in the values obtained for ν over different ranges of cluster size is larger than would be expected from statistical uncertainties alone, indicating that our results may be subject to additional systematic errors. If we assume that the correct value[22] for D is 2.5 (the results obtained from diffusion-limited aggregation on a single growth site), then Eq. (1) indicates that the exponent $\tau(-(\nu+1))$ should have a value of 1.8. The result from our simulation $\tau=1.84\pm0.08$ is considered to be in good agreement with Eq. (1). If Eq. (1) is used to estimate the fractal dimensionality, we find $D=(d-1)/(\tau-1)=2/(0.84\pm0.08)=2.38\pm0.23$.

RMS deposit thickness

Assuming that the RMS deposit thickness (T) is related to the number of occupied lattice sites in the deposit (N) by $T\sim N^{\epsilon}$ (in the limit $N\rightarrow\infty$), Meakin[19] estimated a value of 1.70 ± 0.2 for the exponent ϵ using deposits grown to a maximum height of 60 lattice units on a 100×100 surface. The results obtained from our new simulation should provide a better estimate for ϵ because of improved statistics and a closer approach to the $N\rightarrow\infty$ limit. Using Eq. (3) to fit the dependence of T on N, we find that $\epsilon'=1.69\pm0.06$ (versus a value of 1.62 ± 0.06 in Ref. 18). Similarly, using Eq. (4) we find $\epsilon''=1.79\pm0.10$ (versus a single value of 1.816 obtained from the averaged data in Ref. 18). Overall, our results indicate that ϵ has a value of about 1.75 ± 0.15. With further increases in the scale of the computer simulations, it is probable that our estimates for ϵ would approach more closely the value of 2.0, corresponding to a density profile $\rho(r)$ of the form

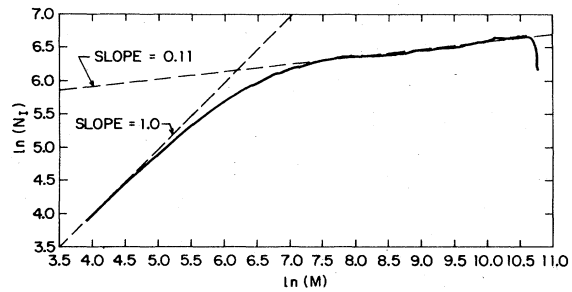

FIG. 5. This figure shows the dependence of the old-growth—new-growth interface size (N_I) on the size of the old growth (M) in the form of a log-log plot. Initially, $N_I\sim M$ (every old site is on the interface). At later stages in the growth process the interface grows much more slowly as M increases.

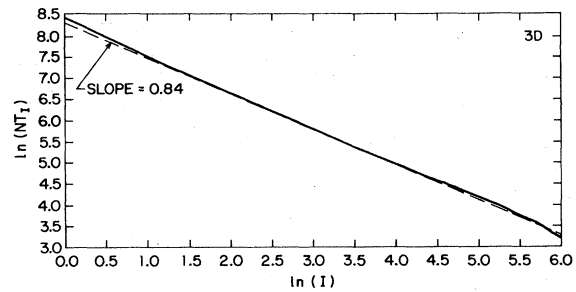

FIG. 7. The dependence of NT_I (the number of clusters of size greater than or equal to I) on I is shown in the form of a log-log plot.

TABLE IV. Values for the exponent ν which relates the number of clusters of size $\geq I$ (NT_I) to I for 3D surface deposits. The results were obtained by least squares fitting straight lines to the coordinates ($\ln(NT_I)$, $\ln(I)$) over the range $I_1 \leq I \leq I_2$.

I_1	5	10	5	20	50
I_2	500	100	50	200	500
	−0.863	−0.771	−0.808	−0.833	−0.917
	−0.921	−0.895	−0.914	−0.843	−0.953
	−0.826	−0.763	−0.839	−0.793	−0.856
	−0.892	−0.791	−0.808	−0.785	−0.955
	−0.875	−0.835	−0.869	−0.730	−0.941
	−0.905	−0.846	−0.864	−0.820	−0.967
	−0.832	−0.923	−0.847	−0.803	−0.818
Avg.	−0.873±	−0.832±	−0.850±	0.801±	0.915±
	0.033	0.058	0.035	0.034	0.052

$\rho(r)=\rho_0 r^{-0.5}$ found in Witten-Sander clusters with a fractal dimensionality of 2.5. The observed effective exponent of 1.75 can be interpreted in terms of an effective dimensionality of $D=2.57\pm0.1$, which is in reasonably good agreement with estimates of the fractal dimensionality of 3D Witten-Sander clusters[12,13,23–25] obtained from similar simulations in which particles are deposited on a single growth site.

Upper surface properties

The height of the upper surface (see above) ($\bar{h}$) has been determined during the growth of the 3D surface deposits. Figure 8(a) shows the dependence of $\ln(\bar{h})$ on $\ln(N)$ averaged over all seven simulations. Figure 8(a) shows a crossover from $\bar{h}\sim N$ (the expected behavior at low surface coverage) to $\bar{h}\sim N^{\phi}(\phi\approx2.0)$ for large N. An expansion of the upper right-hand corner of Fig. 8(a) is shown in Fig. 8(b). The results obtained by least squares fitting straight lines to the dependence of $\ln(h)$ on $\ln(N)$ over various ranges of deposit size (N) are shown in Table V. The results shown in Table V indicate that the limiting ($N\rightarrow\infty$) value for ϕ is probably about 2.1 ± 0.1. If an expression of the form shown in Eq. (3) is used to fit the dependence of $\bar{h}$ on N, we find $\phi=2.11\pm0.05$. If an expression of the form shown in Eq. (4) is used to fit the averaged data, we find $\phi''=2.02$. These results are in quite good agreement with the results we would find if the density profile in the surface deposit was the same as that in a 3D Witten-Sander cluster with a fractal dimensionality of 2.5 ($\phi=2.0$). If we used the result $\phi=2.1\pm0.1$ to estimate D_s for the deposit, the result is $D_s=2.475\pm0.025$.

Figure 9 shows the dependence of $\langle(h-\bar{h})^2\rangle^{1/2}$ on N. Figure 9 indicates that $\langle(h-\bar{h})^2\rangle^{1/2}\sim N$ for small N and $\langle(h-h)^2\rangle^{1/2}\sim N^{\omega}(\omega\sim1.7)$ for large N. However, because of the curvature apparent in Fig. 9, the limiting ($N\rightarrow\infty$) value for ω is probably larger than 1.7. The dependence of $\langle(h-\bar{h})^2\rangle^{1/2}$ on N was fitted using an expression of the form $\langle(h-\bar{h})^2\rangle^{1/2}=A''N^{\omega''}(1+B''N^{-\gamma''})$. From the results averaged for all seven 3D simulations the result is $\omega''\simeq1.82$. Overall, our results for the upper surface are consistent with the value of about 2.0 for both ϕ and ω. These results support the idea that the surface deposit formed by diffusion-limited aggregation is a sta-

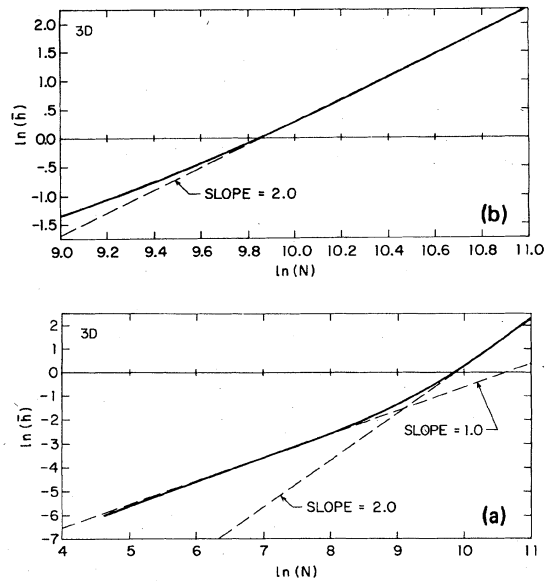

FIG. 8. (a) shows the growth of the mean upper surface height ($\bar{h}$) during 3D simulations of diffusion-limited deposition on a surface. The crossover from a linear to quadratic growth with increasing N can be clearly seen. (b) shows an expansion of part of (a) to show the dependence of $\ln(\bar{h})$ on $\ln(N)$ at large N.

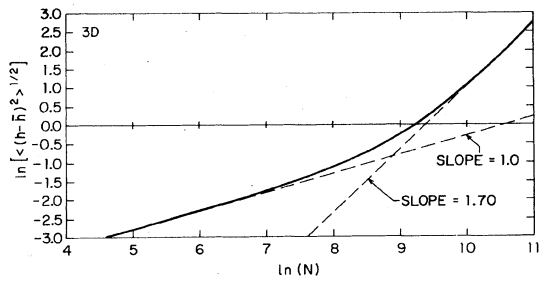

FIG. 9. Here the dependence of the RMS deviation of the upper surface height from the mean surface height on deposit size is shown. The results used in this figure were taken from seven 3D simulations.

TABLE V. Results obtained for the mean upper surface height exponent (ϕ) by least squares fitting straight lines to the coordinates ($\ln(\bar{h})$, $\ln(N)$) over the range $N_1 \leq N \leq N_2$. N is the number of particles deposited on a 200×200 surface in the 3D simulations.

Deposit size (N_{max})	$N_1=0.5N_{max}$ $N_2=1.0N_{max}$	$N_1=0.75N_{max}$ $N_2=1.0N_{max}$	$N_1=0.25N_{max}$ $N_2=0.5N_{max}$
67 816	2.04	2.06	2.01
54 721	2.18	2.15	1.87
75 663	2.04	2.06	1.91
68 991	2.06	2.05	1.93
68 782	2.09	2.18	1.93
55 580	2.02	2.05	1.84
67 249	2.07	2.10	1.87
Avg. 65 543	2.07± 0.05	2.09± 0.05	1.91± 0.05

tistically self-similar structure with a fractal dimensionality of about 2.5 and a mean density profile of the form $\rho(r)\sim r^{-\alpha_s}$ ($\alpha_s\simeq0.5$).

Old-growth—new-growth interface

Figure 10 shows the size of the old-growth—new-growth interface (N_I) as a function of the old-growth mass (M). The results shown in Fig. 10 are averaged over all of the 3D surface deposit simulations. The interface exponent (δ) was determined for each of the deposits by fitting straight lines to the coordinates ($\ln(N_I)$, $\ln(M)$) over the range $20\,000 < M < N_{max} - 20\,000$, where N_{max} is the maximum number of particles in the deposit. This procedure gives the result $\delta=0.047\pm0.028$.

The dependence of $N_{1/2}/N_I$ on the number of old-growth particles (M) which is qualitatively similar to that found in the 2D simulations is shown in Fig. 11. Here $N_{1/2}$ is the number of particle additions required to half complete the interface and N_I is the size of the old-growth—new-growth interface. The ratio $N_{1/2}/N_I$ at first falls rapidly and reaches a minimum when the old growth contains 15 000—20 000 particles (about 0.4 particles per surface site). The ratio $N_{1/2}/N_I$ seems to approach a limiting value of about 0.80—0.85. The apparent decrease in $N_{1/2}/N_I$ at the right-hand side of Fig. 11 results from the fact that the interface is not completely saturated towards the end of the simulation.

DISCUSSION

Except for the properties associated with the old-growth—new-growth interface, the quantities which we have determined for 2D and 3D surface deposits can be directly related to the fractal-like structure of the deposits. Our results give strong support to the idea that the structure of the random surface deposits can be described in terms of a fractal dimensionality of about $\frac{5}{3}$ for $d=2$ and about 2.5 for $d=3$. Consequently, the structure of the surface deposits is very similar to the structure of clusters grown by the Witten-Sander model for diffusion-limited aggregation which have the same fractal dimensionality (about $\frac{5}{3}$ for $d=2$ and about 2.5 for $d=3$).

One of the most interesting quantities which we have measured is the cluster-size-distribution exponent (τ). The measurement of τ was motivated by the importance of cluster-size distributions in models for critical phenomena[26–29] and the possibility that cluster-size distributions may be helpful in understanding the physics of nonequilibrium systems.[29,30] The cluster-size distribution in surface deposits has been investigated independently by Rácz and Vicsek.[20,31] They derived a simple relationship between the cluster-size distribution (τ) and the fractal dimensionality D_s describing the structure of the surface deposit [Eq. (1)]. According to Eq. (1) τ should have a value of 1.6 in 2D deposits and 1.8 in 3D deposits if $D_s=D_c=\frac{5}{3}$ for $d=2$ and $D_s=D_c=2.5$ for $d=3$.[20]

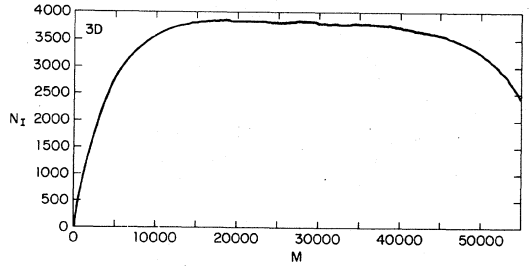

FIG. 10. This figure shows how the size of the old-growth—new-growth interface (N_I) depends on the number of particles in the old growth. The results shown in this figure are the average for seven 3D simulations.

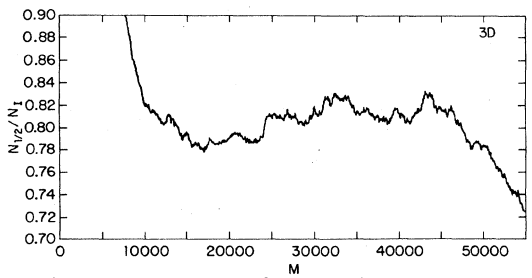

FIG. 11. The dependence of the ratio between the number of particles needed to half complete the old-growth—new-growth interface ($N_{1/2}$) and the size of the interface (N_I) on the mass of the old growth (M).

From their 2D simulations Rácz and Vicsek obtained the result $\tau=1.35\pm0.1$. Our result ($\tau=1.55\pm0.05$) obtained from larger scale simulations is in much better agreement with the results obtained using Eq. (1). Similarly, in three dimensions our value for 1.84 ± 0.08 is in good agreement with the value obtained from Eq. (1) assuming $D_s=2.5$.

The dependence of the size of the old-growth—new-growth interface (N_I) on the number of particles (mass) in the old-growth (M) has been investigated for 2D and 3D Witten-Sander clusters by Meakin and Witten.[22] They found that N_I is related to M by a power law ($N_I \sim M^{\delta}$) where the exponent δ has a value of about 0.60 for $d=2$ and about 0.74 for $d=3$. These results are consistent with a naive geometric interpretation in which $N_I \sim R^{d-1}$ or $N_I \sim M^{(d-1/d)}$. Here R is the cluster radius. If the results of Meakin and Witten are taken at face value, we might expect that the size of the old-growth—new-growth interface should be independent of the mass of the old growth ($\delta=0$) for surface deposits. Instead, we find $\delta\simeq0.16$ for $d=2$ and $\delta\simeq-0.05$ for $d=3$. The 2D and 3D surface deposits are considerably larger than the 2D and 3D Witten-Sander clusters used to obtain the interface exponent. However, the results obtained from the surface deposits are probably subject to larger uncertainties because of the large "anomalous" region near the original surface. As a result, the dependence of $\ln(N_I)$ on $\ln(M)$ is linear over a much smaller range of deposit sizes for the surface deposits. For the surface deposits, our results are in poor agreement with a simple mean-field theory[22] which leads to the prediction that

$$\delta=\frac{(D_s-d)}{2(1+D_s-d)}=-0.25$$

for $d=2$ and -0.5 for $d=3$. However, our results are consistent with the expectation that the mean-field theory should provide a lower limit for δ.

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