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Magnetic field detrapping of polaronic electrons on films of liquid helium

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We have applied an extension of the Feynman path-integral formalism to the problem of noninteracting polaronic electrons on the surface of a liquid-helium film in a perpendicularly applied magnetic field. The ground-state energy, the Feynman-model mass, the magnetization, and the susceptibility of the system are calculated. We find that at a certain magnetic field value, the polaronic electron undergoes a transition from a heavy-mass, self-trapped state to a quasifree Landau state. This detrapping mechanism would have a dramatic effect in a cyclotron-resonance experiment.

I. INTRODUCTION

The system of electrons on the surface of liquid helium is a fascinating system. Because the electrons above the surface fall into image-potential bound states relative to motion perpendicular to the surface, at low temperatures, the system forms an almost ideal two-dimensional (2D) electron gas. For most areal densities achieved to date, the system acts as a classical 2D electron gas. Wigner crystallization has been observed in this system for electrons on bulk helium.[1] A desire to study 2D melting, related phase-diagram phenomena, and the modes of the helium itself, has led to an interest in electrons on *films* of liquid helium. Thin films allow a higher areal density of electrons[2,3] to be sustained on the helium surface, which is necessary to study quantum-mechanical melting[4] and to allow stabilization of surface modes.[2,3] But for thin enough films ($d \leq 1000$ Å), new, nonperturbative ("polaronic") effects[5-8] appear due to the strong electron-surface (i.e., electron-ripplon) coupling due to image-potential coupling of the electron not only to the helium itself, but to the substrate as well. What makes this system particularly interesting as a 2D polaron problem is the variability of the electron-ripplon coupling (by changing the liquid-helium film thickness or the substrate).

In a previous calculation, Jackson and Platzman[5-7] formulated this problem (in the low-density limit $n_s \leq 10^8/\text{cm}^2$) as a 2D polaron problem, with the Hamiltonian, describing the interaction of a single electron with the ripplon modes of the liquid-helium surface,

$$H=\frac{\vec{p}^{\,2}}{2m}+\sum_{\vec{k}} a^{\dagger}_{\vec{k}} a_{\vec{k}} \hbar\omega_k + U \;, \tag{1}$$

where (in the presence of a perpendicular pressing field $eE_\perp$)

$$U=\frac{1}{A^{1/2}}\sum_{k}(a_{\vec{k}}+a^{\dagger}_{-\vec{k}})e^{i\vec{k}\cdot\vec{r}}Q(k) \;, \tag{2}$$

$$\omega_k=\left[\left(g'k+\frac{\sigma}{\rho}k^3\right)\tanh(kd)\right]^{1/2} , \tag{3}$$

and

$$Q(k)=\left[\frac{\hbar k\tanh(kd)}{2\rho\omega_k}\right]^{1/2} eE_\perp \;. \tag{4a}$$

In general $eE_\perp$ is a function of k, due to the image potential from the liquid helium, and is a function of ϵ_s, the dielectric constant of the substrate. In the simplified treatment of Ref. 5 a linearized cutoff, ripplon spectrum was used, i.e., $\omega_k=sk$, where $s=\sqrt{g'd}$ for $k<k_c$, g' is the van der Waals acceleration of the liquid helium, i.e., $g'=g[1+(3c/\rho g d^4)]$ with c equal to the van der Waals coupling of the helium to the substrate, g equal to the acceleration of gravity, d equal to the film thickness, and k_c $[=(\rho g'/\sigma)^{1/2}]$ is the capillary constant (where ρ and σ are the density and surface tension of the helium, respectively). An electron-ripplon coupling constant can be defined,

$$\alpha=\frac{(eE_\perp)^2}{8\pi\sigma}\bigg/\frac{\hbar^2k_c^2}{2m} , \tag{4b}$$

with

$$eE_\perp=eE_{\perp\text{ext}}+\frac{e^2}{4d^2}\left[\frac{\epsilon_s-1}{\epsilon_s+1}\right] , \tag{5}$$

where the major contribution (for thin films) comes from the substrate image potential. The coupling constant is variable by changing the film thickness (in the presence of an applied field), or by changing the underlying substrate. On the basis of this model, using Feynman path-integral techniques, Jackson and Platzman[5] predicted the formation of a polaron state on the surface, characterized by a jump in mass of the electron of at least 5 orders of magnitude at a particular value of α (in the $T=0$ limit).

Recent measurements of Andrei[9] appear to corroborate this prediction. Rapid changes in the low-frequency response have been observed at certain film thicknesses. The experiments measure the spring constant K, characterizing the coupling of an electron to ripplons, and a mobility μ, measuring damping in the system, as a function of helium film thickness d. The experimental results appear to bear out in a qualitative way the predictions of the single-particle polaron treatment of this problem, i.e., the order-of-magnitude changes in the restoring force (spring) constant K, and in μ (Refs. 9 and 10) are correct. The actual measured values of K and the apparent binding energy ($E_B \gtrsim 1$ K) indicate that other effects (perhaps Coulomb) play a role.

This suggests the appropriateness of considering the effects of other probes of the single-particle polaronic state. To this end we consider in this paper the effect of a magnetic field, applied perpendicular to the helium surface, on the polaronic state of a 2D electron on the surface. In earlier papers, Peeters and Devreese[11] investigated, for the Fröhlich polaron, the free energy of a gas of noninteracting polarons in a magnetic field. Using an extension of the Feynman path-integral formalism, they calculated the free energy, magnetization, susceptibility, internal energy, entropy, and specific heat for various values of electron-phonon coupling. They extended the formalism to define variational parameters for a mass parallel ($m_{\parallel}$) and perpendicular ($m_{\perp}$) to the applied magnetic field. They found that, in the direction perpendicular to the applied field, the polaron transformed from a true polaron state ($m_{\parallel} \cong m_{\perp}$) to an almost-free ($m_{\parallel} \gg m_{\perp} \sim 1$) Landau state. In this problem the phonon frequency was fixed and it was what set the scale of energy in the problem. In the electron on helium system the phonons become ripplons and are acousticlike. There have been previous studies[12–14] on the problem of the cyclotron resonance of a 2D electron on a liquid-helium surface in a magnetic field, but most previous studies considered the case of bulk helium (not thin films) and the case where the size of the deformation on the surface in which the electron sits (R_p) is large compared to the magnetic length (R_c), i.e., $R_p \gg R_c$. These studies indicated a shift up in the cyclotron-resonance frequency ω_c when the deformation of the helium was present.

In this paper we consider the thin-film case and the case where the localization length of the particle on the surface and the magnetic length are comparable ($R_p \lesssim R_c$). We find a phase transition from strong electron-ripplon coupling where the particle goes from a large mass, self-trapped state to a free Landau-type electron state. This phase transition is also reflected in the behavior of the magnetization and the susceptibility. We discuss these results in the light of recent experiments and their connection with previous calculations. In Sec. II we formulate the problem of a polaronic 2D electron on the surface of a liquid-helium film in a magnetic field. In Sec. III we present our results and compare them with previous results. We conclude in Sec. IV with a discussion of the significance of the results and discuss experimental implications.

II. POLARONIC ELECTRONS IN A MAGNETIC FIELD

For a polaronic electron in a magnetic field B, perpendicular to the electron layer, the Hamiltonian becomes[11]

$$H=\frac{1}{2m}\left[\vec{p}+\frac{e}{c}\vec{A}\right]^2+\sum_{\vec{k}} a^{\dagger}_{\vec{k}} a_{\vec{k}} \hbar\omega_k + U \tag{6}$$

[where U is given in Eq. (2)], with resultant action (after elimination of the ripplon coordinates),

$$S=S_e+S_I \,, \tag{7}$$

where S_I is the usual electron-ripplon interaction piece of the action and

$$S_e=-\frac{1}{2m}\int_0^{\beta} d\tau\{[\dot{r}(\tau)]^2+i\omega_c[x(\tau)\dot{y}(\tau)-y(\tau)\dot{x}(\tau)]\} \tag{8}$$

[we choose the symmetrical Coulomb gauge $\vec{A}=\frac{1}{2}B(-y,x)$]. In his original formulation of the polaron problem, Feynman[15] used the Jensen inequality with respect to a trial action, i.e.,

$$\int \exp(S)Dr(\tau) \geq \exp(\langle S-S_0\rangle)\int \exp(S_0)Dr(\tau) \,, \tag{9}$$

to derive a variational upper bound on the ground-state energy

$$E_g \leq E_0 - A - B \,, \tag{10}$$

where E_0 (the ground-state energy of trial action), A, and B depend on the variational parameters of the trial action S_0. Since the Jensen inequality was proven to hold only for real actions, and the action S is complex, there is a question of whether we can, in fact, perform such a variational calculation of the energy. However, in their papers on the properties of the Fröhlich polaron in a magnetic field, Peeters and Devreese[11] presented an argument which suggests the validity of the Feynman-Jensen inequality for the magnetic field case, based on the Bogoliubov inequality[16]

$$F \leq F_0 + \langle H-H_0\rangle \,. \tag{11}$$

The Bogoliubov inequality can be derived from the Jensen inequality [Eq. (9)] provided the Hamiltonians H and H_0 are Hermitian and local in time. Since, in the presence of a magnetic field, H and the trial Hamiltonian H_0 are Hermitian (even though the actions S and S_0 are complex), the Jensen inequality should still hold. Even though S and S_0 in the Jensen inequality are nonlocal in time, they are derived from Hamiltonians which, before elimination of the ripplon variables, are local in time.

Assuming then that the Jensen inequality holds in this instance, we choose as a trial Hamiltonian

$$H_0=\frac{1}{2m}\left[\vec{p}+\frac{e}{c}\vec{A}\right]^2+\frac{(\vec{p}\,')^2}{2}M+\tfrac{1}{2}K(\vec{r}-\vec{r}\,')^2 \,. \tag{12}$$

This allows us to define the usual Feynman variational parameters v,w in terms of K and M [i.e.,

$v^2/w^2=(M+m)/m$, $K/m=v^2-w^2$]. The trial Hamiltonian in diagonalized form is

$$H_0=\sum_{i=1}^{3}\hbar s_i(c_i^\dagger c_i+\tfrac{1}{2}) \tag{13}$$

with the eigenfrequencies s_i determined by[11] [$\omega_c=(eB/mc)$]

$$s_i(s_i^2-v^2)+(-1)^i\omega_c(s_i^2-w^2)=0\ . \tag{14}$$

If we introduce

$$d_i^2=\frac{1}{2s_i}\frac{s_i^2-w^2}{3s_i^2+2(-1)^i\omega_c s_i-v^2}\quad (i=1,2,3) \tag{15}$$

and

$$\frac{\partial s_i}{\partial v}=\frac{2vs_i}{3s_i^2+2(-1)^i\omega_c s_i-v^2}\quad (i=1,2,3)\ , \tag{16}$$

then the variational ground-state ($T=0$) energy is

$$E=\left[\frac{1}{2}\sum_{i=1}^{3}s_i-w\right]-\frac{v^2-w^2}{4v}\sum_{i=1}^{3}\frac{\partial s_i}{\partial v}-A\ , \tag{17}$$

where

$$A=\frac{1}{2\pi}\int_0^\infty d\tau\int_0^\infty dk\,k\,|Q(k)|^2e^{-\omega_k\tau}e^{-k^2D(\tau)} \tag{18}$$

and

$$D(\tau)=\sum_{i=1}^{3}d_i^2(1-e^{-s_i\tau})\ . \tag{19}$$

For $\omega_c\to 0$, the above expression reduces to the expression for the ground-state energy given in Eqs. (6) and (7) of Jackson and Platzman.[5] We now minimize the expression for E with respect to the variational parameters v,w for various values of the magnetic field to determine the mass $m_0=v^2/w^2$ of the Feynman polaron model, the magnetization M, and the susceptibility χ. We again find that for thin films a good approximation to the ripplon dispersion relation is $\omega_k\approx sk$ for $l\le k_c$.

III. RESULTS AND DISCUSSION

For $\omega_c=0$, we reproduce the zero magnetic field results of Jackson and Platzman[5–7] and others[8] where, for $\alpha\approx\frac{1}{2}$, there is a sudden jump in the mass of 5 orders of magnitude, i.e., $m_0/m\approx 10^5$. For finite magnetic field $\omega_c\neq 0$, our results are summarized in Figs. 1–5. The results for the energy, the magnetization, and the susceptibility are referred to the respective results for a free electron in a magnetic field, i.e., $\Delta E=E-\hbar\omega_c/2$. For $\alpha<\frac{1}{2}$ and $\omega_c/\omega_0\ll\eta$, the ground-state energy (in units of $\hbar\omega_0$) is

$$E=-2\eta\alpha+\tfrac{1}{2}\omega_c-\omega_c\frac{\alpha}{(1+\eta)^2} \tag{20a}$$

$$=-2\eta\alpha+\frac{1}{2}\frac{\omega_c}{\omega_0}\cdot\frac{m}{m^*}\ , \tag{20b}$$

where m^*/m (the model effective mass) is given by

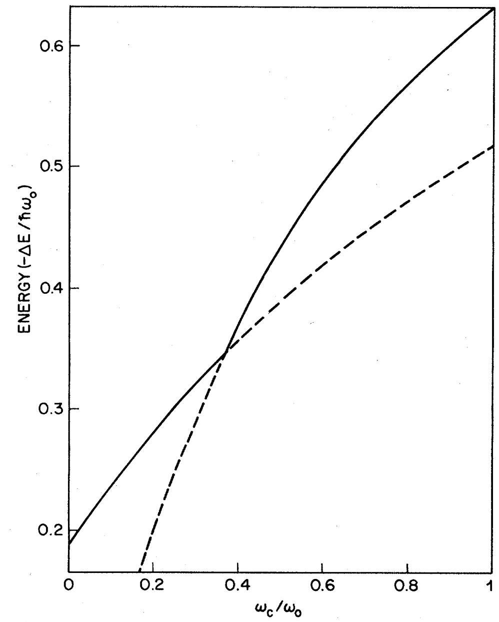

FIG. 1. Energy of polaronic electron vs magnetic field (in units of $\hbar\omega_0=\hbar^2k_c^2/2m$) for $\alpha=1$ (solid lines represent the stable phase).

$$\frac{m^*}{m}=m_0=1+\frac{2\alpha}{(1+\eta)^2} \tag{21}$$

and where

$$\eta=\frac{sk_c}{\omega_0}\approx 2.5\times 10^{-3} \tag{22}$$

is the adiabaticity parameter noted in previous work.[5] The first term in Eq. (20b) is the energy shift due to

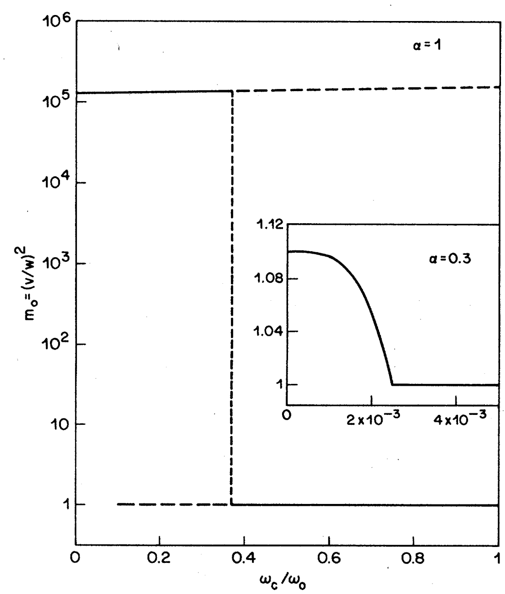

FIG. 2. Effective mass vs magnetic field (solid lines represent the stable phase).

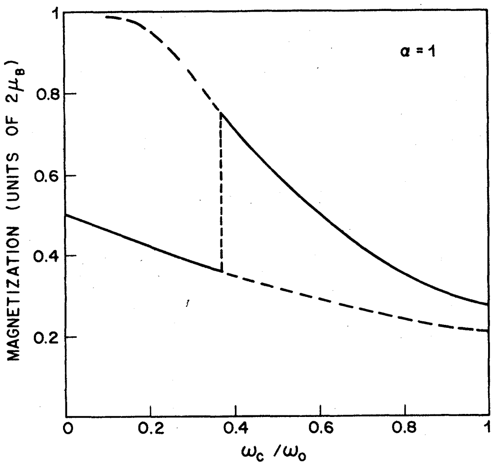

FIG. 3. Magnetization vs magnetic field (solid lines represent the stable phase).

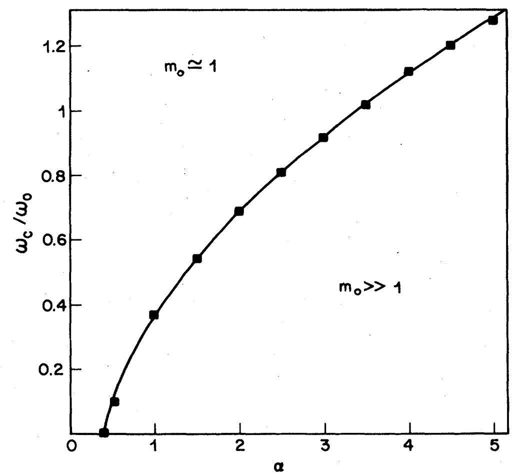

FIG. 5. Value of magnetic field needed to cause detrapping vs electron-ripplon coupling constant.

electron-ripplon interaction and the second term is the energy of a particle in the lowest Landau level modified by the mass renormalization due to ripplons. It is interesting to note the inset in Fig. 2 for $\alpha=0.3$, where m_0 $[=(v/w)^2]$ becomes $m_0=1$ at $\omega_c/\omega_0=\eta=2.5\times10^{-3}$. So for small values of electron-ripplon coupling the adiabaticity parameter (or ripplon frequency) sets the energy scale, as found in the zero magnetic field case.[5] In Figs. 1–4, we have performed our variational calculation for an electron-ripplon coupling value that is above the zero-field self-trapping value, i.e., $\alpha=1$. We find that at a certain value of the magnetic field strength, $\omega_c/\omega_0=0.37$, we have a transition from a self-trapped to a free-electron state (note that $\hbar\omega_0=\hbar^2k_c^2/2m$). This transition appears in all the calculated quantities, ground-state energy E,

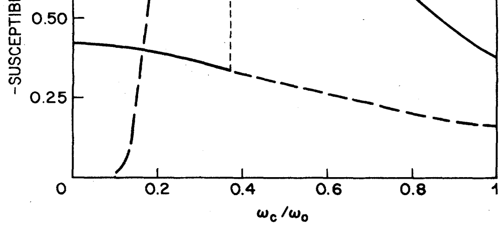

FIG. 4. Susceptibility vs magnetic field (solid lines represent the stable phase).

Feynman-model mass v^2/w^2, but especially the magnetization $M=-\partial\Delta E/\partial(\omega_c/\omega_0)$ and the susceptibility $\chi=-\partial^2\Delta E/\partial(\omega_c/\omega_0)^2$, where $\Delta E=E-(\hbar/2)\omega_c$ (M and χ are defined in units of $2\mu_b=e\hbar/mc$ and $4\mu_B^2/\hbar\omega_0$, respectively). For a 100 Å film this occurs at a magnetic field $B\approx1$ kG. In Fig. 5 we plot the value of the magnetic field strength necessary to cause the first-order transition versus electron-ripplon coupling constant. Note that since

$$\alpha=\frac{(eE_\perp)^2}{8\pi\sigma}\Big/\frac{\hbar^2k_c^2}{2m}\ , \tag{4b}$$

the value of the magnetic field where the transition occurs is a function of film thickness d, since $E_\perp$ depends on d.

Figures 1–4 indicate a transition from a strongly coupled self-trapped state to a "quasifree" state at $\omega_c=0.37\omega_0$ for $\alpha=1$. In the strong-coupling regime ($\alpha=1$), the electron moves in a self-induced ripplon potential well with a characteristic energy proportional to $\alpha(\hbar^2k_c^2/2m)$ (which is essentially the binding energy of the self-trapped state). When the energy of the oscillatory motion imposed by the magnetic field approximately equals the self-trapping energy, we expect the self-trapping mechanism to break down. Looking at Eq. (17) for the energy, we see for the case $v=w$ (quasifree state),

$$\Delta E_{\mathrm{QF}}\approx-\frac{\alpha\omega_c}{\omega_0}(1-e^{-\omega_0/\omega_c})\ , \tag{23}$$

while

$$M=\alpha[1-(1+\omega_0/\omega_c)e^{-\omega_0/\omega_c}] \tag{24}$$

and

$$\chi=-\alpha\left[\frac{\omega_0}{\omega_c}\right]^3e^{-\omega_0/\omega_c}\ . \tag{25}$$

Therefore for $\omega_c/\omega_0>>1$,

$$\Delta E=-\alpha\ , \tag{26}$$

$$M=\frac{\alpha}{2}\left[\frac{\omega_0}{\omega_c}\right]^2 , \quad (27)$$

$$\chi=-\alpha\left[\frac{\omega_0}{\omega_c}\right]^3 , \quad (28)$$

while for $\eta \ll \omega_c/\omega_0 \ll 1$, we get

$$\Delta E=-\alpha\omega_c/\omega_0 , \quad (29)$$

$$M=\alpha , \quad (30)$$

$$\chi=0 , \quad (31)$$

where the maximum in χ occurs when $\omega_c=\frac{1}{3}\omega_0$. These results all agree well with Figs. 1–4. Here the energy scale is clearly set by the magnetic energy ω_c, although there is a lowering of the energy due to ripplon effects (i.e., the $-\alpha$ term). In the usual self-trapping limit $v \gg w$, we find, as usual,

$$E_{\mathrm{ST}}=\left[\frac{v}{2}-\alpha v(1-e^{-1/v})\right] +\frac{\omega_c}{m_0}\left[1-\frac{\alpha}{m_0\eta^2}v(1-e^{-1/v})\right] +\frac{\omega_c^2}{8v}\left\{\tfrac{3}{2}-\alpha\left[1-e^{-1/v}\left[1+\frac{1}{v}\right]\right]\right\} , \quad (32)$$

where E_{ST} and v are in units of ω_0. In the extreme strong-coupling limit ($\alpha\to\infty$) we have

$$E_{\mathrm{ST}}=-\alpha+\frac{\omega_c}{2m_0}+\frac{\omega_c^2}{8v} . \quad (33)$$

The first term in Eq. (33) is the ground-state energy in the limit $\alpha\to\infty$ without a magnetic field, the second term is the zero-point energy of a free particle with mass ($m_0=2\alpha/\eta^2$) in a magnetic field, and the third term is the diamagnetic shift in the energy because the electron is bound in a potential well with characteristic frequency v. Equations (23) and (32) suggest that a transition should occur when $E_{\mathrm{QF}}\approx E_{\mathrm{ST}}$, i.e., $\omega_c\approx\omega_0$. We find $\omega_c=0.37\omega_0$. There are contributions from the internal excited states in the effective potential well due to the combined effects of the ripplons and the magnetic field which are left out of the simple expressions in Eqs. (23) and (32). We would expect these contributions to affect the exact value of ω_c where the transition occurs. Nevertheless what we are observing, roughly speaking, is the electron going from seeing just the "polaron" well (radius equals R_p) to seeing the "magnetic" well [radius $\approx R_c=(\hbar/m\omega_c)^{1/2}$].

As mentioned previously, in a recent experiment Andrei[9] observed what is believed to be the polaronic or a related transition for electrons on a thin film of liquid helium. Despite evidence in her experiment that interaction effects play a role, the fundamental ideas of the single-particle picture appear to be correct. We can ask then, what experimental consequences emerge from our single-particle model of a polaronic electron in a magnetic field. While it is generally accepted that the Feynman-model mass is not necessarily the true particle mass, what its behavior indicates is the existence of a self-trapping (or detrapping) transition in an ideal polaron gas (gas of noninteracting polarons).

A more reasonable way to look at the polaronic transition is in terms of the effective coupling in the Feynman model $K/m=v^2-w^2$. This is, in fact, one of the parameters measured by Andrei.[9] She extracted it and a mobility, in the polaron regime, from expressions for the capacitance C and dissipation G in a capacitance bridge when a time-dependent electric field was applied parallel to the electron layer. She found[9]

$$G=\sum_q A_q sh(qd)\frac{\omega^2 e/\mu}{(K+K_q-m\omega^2)^2+\left[\frac{\omega e}{\mu}\right]^2} , \quad (34)$$

$$C=\sum_q A_q sh(qd)\frac{K_q-m\omega^2}{(K+K_q-m\omega^2)^2+\left[\frac{\omega e}{\mu}\right]^2} , \quad (35)$$

where μ (the mobility) is a measure of relaxation in the system. These calculations were done for a sheet of electrons where

$$K_q=m\omega_q^2 , \quad (36)$$

ω_q is the 2D plasmon frequency appropriate for her geometry, and the sum is over plasmon modes. We can examine what happens when a time-dependent electric field is applied parallel to the helium surface (in the x direction, for instance) and a magnetic field is applied perpendicular to the sheet of electrons in the symmetric Coulomb gauge [$\vec{A}=\frac{1}{2}B(-y,x)$] by studying the single-particle classical equation of motion. We take the plasmon frequency ω_q to be fixed (one mode) and represented by an effective spring constant $m\omega_0^2$. We take the electron to be coupled to another fictitious particle via a spring with spring constant K with a loss-term proportional to $\tau_1^{-1}v_e$ for the electron and $\tau_2^{-1}v_F$ for the fictitious particle. We calculate a conductivity and find

$$\mathrm{Re}\sigma_{xx}\propto\frac{(1/\tau_{\mathrm{eff}})(1/\tau_{\mathrm{eff}}^2+\tilde{\omega}^2-\omega_c^2)}{\left[\left[\frac{1}{\tau_{\mathrm{eff}}^2}+\omega_c^2-\tilde{\omega}^2\right]^2+\frac{4\tilde{\omega}^2}{\tau_{\mathrm{eff}}^2}\right]} , \quad (37a)$$

$$\mathrm{Im}\sigma_{xx}\propto\frac{\tilde{\omega}(1/\tau_{\mathrm{eff}}^2+\tilde{\omega}^2-\omega_c^2)}{\left[\left[\frac{1}{\tau_{\mathrm{eff}}^2}+\omega_c^2-\tilde{\omega}^2\right]^2+\frac{4\tilde{\omega}^2}{\tau_{\mathrm{eff}}^2}\right]} \quad (37b)$$

with

$$\tilde{\omega}=\left[\omega-\frac{\Omega^2+\omega_0^2}{\omega}\right] \quad (38)$$

and Ω^2 ($=K/m$) measures the restoring force (spring) constant and τ_{eff} is the (in general, frequency-dependent) relaxation time which, in the case where the motion of the fictitious particle is heavily damped, is given approximately by

$$1/\tau_{\mathrm{eff}}=1/\tau_1+\tau_2\frac{\omega_1^2\omega_0^2}{\omega^2} \quad (39)$$

(here, $\omega_1^2 = K/M$) [note the similarity of Eqs. (37a) and (37b) to Andrei's expressions, neglecting the sum on plasmon modes]. In Eqs. (37a) and (37b), $\omega_c = eB/mc$ is the cyclotron frequency (m is the electron bare mass). If we vary the frequency of the applied electric field, we might expect to see a cyclotron-resonance signal which varies with coupling of the electrons to the ripplons (e.g., with film thickness). The cyclotron linewidth is determined by $\omega_c \tau$, where τ is the relaxation time in the system. If $\omega_c \tau \geq 1$, we might expect to see a distinctive cyclotron-resonance signal. If the external perpendicular electric fields are such that $\alpha = 1$, for example, then our results suggest that a transition from a self-trapped to a detrapped state occurs when $B \approx 1$ kG or $\omega_c \approx 1.80 \times 10^{10}$ sec^{-1}. We assume that, at this transition, the mobility μ, or relaxation time τ, takes on comparable values (i.e., there is a comparable jump) as would be the case at the self-trapping transition in the absence of a magnetic field.[9,17] Since the apparent energy scale in the Andrei experiment[9] (for $d = 1000$ Å) is similar to our energy scale (for $d = 100$ Å), we assume that at this transition the values of μ, hence τ, are similar to those given by experiment at the lowest temperature $T = 0.4$ K (since our results are $T = 0$ results).[17] Then, in the self-trapped state $\omega_c \tau = 3 \times 10^{-5}$, while in the quasifree regime, $\omega_c \tau \approx 1.03$. This suggests that if the film thickness is such that we begin in the self-trapped state, there would be no cyclotron-resonance signal. Then, at a certain value of the magnetic field (≈ 1 kG for $d = 100$ Å), a clear cyclotron signal would suddenly appear. Alternatively, the film thickness could be varied as in the Andrei experiment[9] and a cyclotron signal would rapidly appear or disappear as the film thickness is varied. This is quite a different regime than that considered by Cheng and Platzman,[12] and by Shikin,[13] who found that the presence of a deformation under the electron (due to coupling to ripplons) caused a shift up in the cyclotron resonance. Here, the electron in the dimple state has a lower cyclotron-resonance signal than the quasifree electron. However, Cheng and Platzman,[12] as well as Shikin,[13] worked in a regime where the dimple radius R_p was much larger than the cyclotron radius, i.e., $R_p \gg R_c$. The polaron radius is defined by $R_P \approx (1/\alpha k_c)^{1/2}$. For our self-trapped state and reasonable fields ($B \lesssim 1$ kG), $R_P < R_c$ and at the point of transition, $R_P < R_c$. In fact for $B = 1$ kG, $R_c \approx 840$ Å, while for $\alpha = 1$, $d \approx 100$ Å, $R_P \approx 363$ Å. If B is increased past the point where the transition from the self-trapped to the quasifree state occurs, a regime could be reached where $R_c < R_P$ and a further shift up on the cyclotron-resonance signal is expected (as calculated by Cheng and Platzman[12] and by Shikin[13]).

IV. CONCLUSIONS

We have performed, for an electron coupled to ripplon modes in a magnetic field, a Feynman path-integral calculation of the ground-state energy, model mass, magnetization, and susceptibility for an ideal polaron gas. We find that at a certain value of the magnetic field (roughly proportional to the electron-ripplon coupling constant α), there is a transition from a self-trapped to a quasifree electron state, characterized by order-of-magnitude changes in the mobility μ and internal coupling K. Since the magnetic fields necessary to cause this phase transition are quite reasonable ($B \approx 1$ kG), our results suggest that a cyclotron-resonance experiment (or the Andrei-type experiment[9] with both magnetic and electric fields applied) would be an appropriate way to further probe the nature of the polaronic transition, particularly if done within an experimental arrangement where film thickness also can be measured.

Our variational calculation is mean-field-like in the sense that the coupling of the electron to a second fictitious particle simulates the effective potential in which the electron moves. However, it goes beyond the usual mean-field theory because the retarded form of the trial action incorporates memory effects, and, in that sense, takes some account of fluctuations. Our numerical results indicate the magnetic field detrapping transition to be first order. Since our results are for $T = 0$, at finite temperature, the apparent order of the transition may be changed. Our primary result, of a transition from a self-trapped to a Landau-type state at a particular value of the magnetic field, we expect to remain and to be observable experimentally.

The magnitude of the changes expected in the cyclotron-resonance signal due to the size of the changes in the quantities K and μ should lead to quite dramatic effects in any experiments in a magnetic field in which single-particle polaron effects play a dominant role. Finally, this calculation assumes that the phase transition in the presence of a magnetic field is independent of particle-number density. However, since we now know that interaction effects play a role (at least in setting the absolute energy scale), we expect these effects to play a role here. Indeed, if the density of particles is high enough that Wigner crystallization occurs, we expect to see no cyclotron signal and no sudden appearance or disappearance of one at a particular magnetic field value. This would then provide a test of whether a single-particle polaron state has formed or a multiparticle crystalline state has formed.

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