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Effect of gap distortion on collective modes in ^{3}He-B

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We investigate the influence of gap distortion, caused by the dipole interaction or by magnetic or electric fields, on the order-parameter collective modes in ^{3}He-B by the method of continual integration. The dipole-interaction-induced gap distortion splits the pair breaking, squashing, and real squashing modes at zero momentum $\vec{q}$. Furthermore, a branch crossing of these modes with different J_z appears at nonzero $\vec{q}$. Electric and magnetic fields also produce gap distortion with similar consequences.

I. INTRODUCTION

The condensate wave function of superfluid ^{3}He, or the order parameter (OP), determines all the main properties of the superfluid state. External fields and the dipole interaction cause a deformation of the OP, which leads to a number of interesting effects. Such phenomena as the longitudinal NMR in the A and B phases and the shift of the transverse NMR frequency in the A phase are connected with the dipole-interaction-induced distortion of the OP,[1] while the nonlinear field splitting of the real squashing mode in the B phase[2]—which has recently been observed[3]—is connected with the magnetic-field-induced distortion.

In this Rapid Communication, we use the continual integral method to investigate the influence of the OP distortion on collective modes (CM) in superfluid ^{3}He-B as caused by either dipole interaction (DI) or by magnetic field (MF) or electric field (EF) in the weak-coupling approximation.

We shall show that the gap distortion induced by dipole interaction splits the pair breaking $\epsilon=2\Delta$ mode at zero momentum $\vec{q}$ and makes possible the existence of pb (pair breaking) modes as resonances and the observation of them. A similar splitting takes place for the squashing and real squashing modes. Furthermore, a branch of these modes with different J_z appears at nonzero $\vec{q}$. In addition, this distortion generates gaps in the spectrum of the Goldstone modes associated with NMR phenomena. Electric and magnetic fields also produce gap distortion with similar consequences. In the case of the magnetic field we derive, in particular, the well-known nonlinear field splitting of the real squashing modes as well as of the squashing modes.

II. GAP DISTORTION BY DI, MF, AND EF

The OP in ^{3}He-B is proportional to the matrix R_{ij} which rotates the spin space with respect to the orbital space $A_{ij}=\Delta(T)R_{ij}(\hat{n},\theta)$. The energy gap $\Delta(T)$ is isotropic, whereas the rotation axis $\hat{n}$ and the rotation angle θ are arbitrary in the absence of DI and external fields.

The DI

$$F_D=g_D(A_{ii}A^*_{jj}+A_{ij}A^*_{ji}-\tfrac{2}{3}A_{ij}A^*_{ij})$$

introduces anisotropy in the energy gap, so that its values parallel (Δ_2) and perpendicular (Δ_1) to the direction of $\hat{n}$ (which remains arbitrary) are different:[1] $\Delta_1^2-\Delta_2^2=\frac{5}{2}\Omega_B^2$. The DI also fixes $\theta=\arccos[-\frac{1}{4}(\Delta_2/\Delta_1)]\approx\arccos(-\frac{1}{4})$. Here g_D is the dipolar constant and Ω_B is the longitudinal NMR frequency.

A moderately strong MF, $\vec{H}=H\hat{z}$ with

$$F_{\rm cond}>>F_Z=g_ZH_i(AA^\dagger)_{ij}H_j>>F_D\ ,$$

causes a similar anisotropy of the energy gap[4]

$$\Delta_1^2=\Delta^2+\tfrac{1}{2}(3\beta_{12}+\beta_{345})^{-1}\beta_{345}^{-1}\beta_{12}g_ZH^2\ ,$$

$$\Delta_2^2=\Delta^2-\tfrac{1}{2}(3\beta_{12}+\beta_{345})^{-1}\beta_{345}^{-1}(2\beta_{12}+\beta_{345})g_ZH^2\ ,$$

where $\Delta=(6\beta_{12}+2\beta_{345})^{-1}\alpha$ is the gap in the absence of the MF and $\beta_{ijk}=\beta_i+\beta_j+\beta_k$, β_i are the coefficients in the expansion of $F_{\rm cond}$. In addition to this, the MF fixes the direction of $\hat{n}\parallel\vec{H}$, because of the same corrections to the OP, and the DI fixes $\theta=\arccos(-\frac{1}{4})$.

An EF with energy

$$F_E=-g_E(E_iA_{ki}A^*_{kj}E_j-\tfrac{1}{3}|\vec{E}|^2A_{ki}A^*_{ki})$$

has a similar effect.[5] But, unlike the MF and the DI, the EF causes the gap to increase in the direction of the field $\vec{E}(\parallel z)$. In the case of strong EF ($F_E>>F_D$) one has

$$\Delta_1^2=\Delta^2-(6\beta_{345})^{-1}g_EE^2;\ \ \Delta_2^2=\Delta^2+(3\beta_{345})^{-1}g_EE^2\ .$$

The EF fixed $\hat{n}\perp\vec{E}$ and the DI now fixes

$$\theta=\arccos[-\Delta_1/2(\Delta_1+\Delta_2)]\simeq\arccos(-\tfrac{1}{4})\ .$$

One can describe the OP distortion for all three cases (DI, MF, and EF) by using a unified approach with the OP matrix in the form (for the cases of MF and EF only, see Ref. 5)

$$A_{ij}=[\Lambda^{1/2}R(\hat{n},\theta)]_{ij}\ .$$

Here Λ is a diagonal matrix with the elements $\lambda_1, \lambda_1, \lambda_2$,

$$\lambda_1=\Delta_1^2=\Delta^2+\Omega^2;\ \lambda_2=\Delta_2^2=\Delta^2+a\,\Omega^2\ .$$

For DI,

$$\Omega^2=\tfrac{5}{6}\Omega_B^2,\ a=-2,\ \hat{n}\parallel\hat{z}\ ;$$

for MF,

$$\Omega^2=(g_Z/10\beta_0)H^2,\ a=-4,\ \hat{n}\parallel\hat{z}\ ;$$

for EF,

$$\Omega^2=-(g_E/6\beta_0)E^2,\ a=-2,\ \hat{n}\perp\hat{z}\ .$$

For all three cases we put $\theta=\arccos(-\frac{1}{4})$. Furthermore, we work in the weak-coupling approximation:

$$-2\beta_1=\beta_2=\beta_3=\beta_4=-\beta_5=\beta_0$$
$$=7\zeta(3)(120\pi^2)^{-1}N(0)(k_BT_c)^{-2}\ .$$

III. CALCULATION OF THE SPECTRUM OF THE COLLECTIVE MODES

For calculating the collective modes in the B phase with a deformed OP, we have used the method developed earliei by us.[6] In this method, we describe the initial fermions by Fermi fields and transform to Bose fields $c_{ia}(\vec{x},\tau)$, which correspond to Cooper pairs. In terms of these Bose fields, we construct the functional of "hydrodynamical action"

$$S_h=g^{-1}\sum_{p,i,a}c_{ia}^\dagger(p)c_{ia}(p)+\tfrac{1}{2}\ln\det\hat{M}(c,c^\dagger)/\hat{M}(0,0)\ .$$

Here $c_{ia}(p)$ is the Fourier transform of $c_{ia}(\vec{x},\tau)$, a negative constant g is proportional to the pair scattering amplitude of the quasiparticles, and $\hat{M}$ is an operator dependent on the Bose fields and quasifermion parameters.

The functional S_h determines all physical properties of the system and, in particular, the spectrum of the collective excitations. In the region $T_c-T\simeq T_c$, we expand ln det in S_h in powers of the deviation of $c_{ia}(p)$ from the condensate wave function $c_{ia}^{(0)}(p)$. In the cases of DI and MF we obtain

$$c_{ia}^{(0)}(p)\sim\begin{pmatrix}\Delta_1\cos\theta & -\Delta_1\sin\theta & 0\\ \Delta_1\sin\theta & \Delta_1\cos\theta & 0\\ 0 & 0 & \Delta_2\end{pmatrix}$$

and EF

$$c_{ia}^{(0)}(p)\sim\begin{pmatrix}\Delta_1 & 0 & 0\\ 0 & \Delta_1\cos\theta & -\Delta_1\sin\theta\\ 0 & \Delta_2\sin\theta & \Delta_2\cos\theta\end{pmatrix}\ .$$

After the shift $c_{ia}(p)\to c_{ia}^{(0)}(p)+c_{ia}(p)$, one finds the Bose spectrum from the quadratic form of S_h

$$\sum_p A_{ijab}(p)c_{ia}^\dagger(p)c_{jb}(p)+\frac{1}{2}\sum_p B_{ijab}(p)[c_{ia}(p)c_{jb}(-p)+c_{ia}^\dagger(p)c_{jb}^\dagger(-p)]\ .$$

The equation for the spectrum is $\det Q=0$, where Q is the matrix of this quadratic form.

The tensor coefficients A_{ijab} and B_{ijab} are proportional to the integrals (sums) of the products of the Green's functions of the fermions and are given by

$$A_{ijab}=\delta_{ab}\left[\frac{\delta_{ij}}{g}+\frac{4Z^2}{\beta V}\sum_{p_1+p_2=p}n_{1i}n_{1j}\frac{(i\omega_1+\xi_1)(i\omega_2+\xi_2)}{[\omega_1^2+\xi_1^2+\Delta^2(\theta')][\omega_2^2+\xi_2^2+\Delta^2(\theta')]}\right]\ ,$$

$$B_{ijab}=-\frac{4Z^2}{\beta V}\sum_{p_1+p_2=p}n_{1i}n_{1j}\frac{(2f_af_b-f_i^2\delta_{ab})}{[\omega_1^2+\xi_1^2+\Delta^2(\theta')][\omega_2^2+\xi_2^2+\Delta^2(\theta')]}\ .$$

Here

$$\Delta^2(\theta)=\Delta_1^2(n_1^2+n_2^2)+\Delta_2^2n_3^2$$
$$=\Delta^2+\Omega^2[a+(n_1^2+n_2^2)(1-a)]\ ,$$

$$n_1^2+n_2^2=\sin^2\theta'\ ,$$

$$\vec{f}=\{\Delta_1(n_1\cos\theta+n_2\sin\theta);\Delta_1(-n_1\sin\theta+n_2\cos\theta);\Delta_2n_3\}$$

in the cases of DI and MF, while

$$\vec{f}=\{n_1\Delta_1;n_2\Delta_1\cos\theta+n_3\Delta_2\sin\theta;-n_2\Delta_1\sin\theta+n_3\Delta_2\cos\theta\}$$

in the case of EF; $\beta=T^{-1}$, V is the volume of the system, $\xi_l=v_F(k_l-k_F), n_l=k_l/k_F$, Z is normalization constant, and $\omega=(2n+1)\pi T$ is the Fermi frequency, $p\equiv(\vec{k},\omega)$.

IV. RESULTS (Gd—Goldstone, pb—pair breaking, sq—squashing, rsq—real squashing modes)

For the DI, the energies ϵ^2 are equal to the following.

Gd: $\frac{\Omega_B^2}{3}(1;\pm1;r)$, $\frac{2}{3}\Omega_B^2(1;0;r)$, $0(0;0;i)$;

pb: $4\Delta^2-\frac{\Omega_B^2}{3}(1;\pm1;i)$, $4\Delta^2-\frac{2}{3}\Omega_B^2(1;0;i)$, $4\Delta^2(0;0;r)$;

rsq: $\frac{8\Delta^2}{5}-\frac{\Omega_B^2}{3}(2;\pm1;r)$, $\frac{8\Delta^2}{5}+\frac{2\Omega_B^2}{3}(2;\pm2,0;r)$;

sq: $\frac{12\Delta^2}{5}+\frac{\Omega_B^2}{3}(2;\pm1;i)$, $\frac{12\Delta^2}{5}-\frac{2\Omega_B^2}{3}(2;\pm2,0;i)$.

In parenthesis we have given (J,J_z,r or i), where J is the total angular momentum, J_z is the projection of J on the z

axis, and r or i mean the real or imaginary parts of OP, respectively. Thus a gap of order Ω_B appears in the spectrum of the spin waves. One of these modes, the longitudinal spin wave $(1,0,r)$, can be excited by longitudinal NMR. No gap appears in the spectrum of Anderson-Bogoliubov sound $(0,0,i)$, that is a consequence of the broken gauge symmetry. As it was shown in Ref. 7, the velocity of Anderson-Bogoliubov sound is not changed by the DI, either.

The pb modes $\epsilon=2\Delta$ split into a set of three modes with energies lying between

$$2\Delta_{\max}=(4\Delta^2+\tfrac{10}{3}\Omega_B^2)^{1/2}\ ,$$

$$2\Delta_{\min}=(4\Delta^2-\tfrac{20}{3}\Omega_B^2)^{1/2}\ .$$

In A phase, where gap $\Delta=\Delta_{\max}\sin\theta$ is anisotropic too, the collective excitations with energies less than $2\Delta_{\max}$ attenuate moderately and can be regarded as resonances.[8] By the analogy with the A phase we can say that pb modes can exist as resonances. As Schopohl and Tewordt have shown[9] among pb modes only one mode, $(0,0,r)$, couples to density (a sound wave), and we can observe one peak in sound absorption experiments at $\epsilon=2\Delta$. In Ref. 10, it was shown that all imaginary $J=1$ modes (pb modes) couple to spin density (an electromagnetic wave) via particle-hole asymmetry (weak coupling). It means that we can observe two peaks in NMR absorption experiments near the continuum edge (at pressure $p\simeq 29$ bar and $T\lesssim 0.3T_c$, the energies of these modes $\lesssim 1.99\Delta$). Note that the observability of mode splittings is limited by the collision-induced line broadening. We recall that if the DI is not taken into account, the energies of all pb modes exactly equal 2Δ and the dispersion coefficients α are complex.[6] The physical cause is the possibility of the decay of the Bose excitation into two fermions.

The DI leads to a splitting of both the sq and the rsq modes. Modes of each type are obtained in two sets: two modes with $J_z=\pm 1$ and three modes with $J_z=0,\ \pm 2$. The dispersion law of these modes is $\epsilon^2=\Omega^2+\alpha q^2$ for nonzero momentum $\vec{q}$. Because[6] $\alpha(J_{z1})>\alpha(J_{z2})$ for $J_{z1}<J_{z2}$, the sq-mode branch $J_z=0$ crosses the branches $J_z=\pm 1$, and the rsq-mode branches $J_z=\pm 1$ cross the branches $J_z=\pm 2$ at nonzero $\vec{q}$.

The MF induced gap distortion leads to H^2-dependent corrections in the CM spectrum. For the $J=2$ modes, ϵ^2 takes the following values:

$$\text{rsq: } \frac{8\Delta^2}{5}-\tfrac{2}{5}\tilde{H}^2(2;\pm 1;r),\quad \frac{8\Delta^2}{5}+\tfrac{8}{5}\tilde{H}^2(2;\pm 2;r),\quad \frac{8\Delta^2}{5}+\tfrac{16}{15}\tilde{H}^2(2,0,r)\ ;$$

$$\text{sq: } \frac{12\Delta^2}{5}+\tfrac{2}{5}\tilde{H}^2(2;\pm 1;i),\quad \frac{12\Delta^2}{5}-\tfrac{8}{5}\tilde{H}^2(2;\pm 2;i),\quad \frac{12\Delta^2}{5}-\tfrac{16}{15}\tilde{H}^2(2;0;i)\ .$$

Here $\tilde{H}^2=(g_Z/10\beta_0)H^2$.

Thus the MF splits the rsq and sq modes into sets containing three modes each. This field splitting leads to two kinds of branch crossings for the modes. If we also include the linear MF splitting of the rsq and the sq modes,[2,11] in strong enough MF, we find crossing in the $J_z=-1,0$ branches and in the $J_z=2,1$ branches for the rsq modes as well as in the $J_z=0,1$ branches and in the $J_z=-1,-2$ branches for the sq modes. The crossing of rsq-mode branches with $J_z=1,2$ was predicted by Schopohl, Warnke, and Tewordt[2] earlier and was observed by Shivaram *et al.*[3]

Another kind of branch crossing of the rsq and sq modes occurs at nonzero $\vec{q}$. For the rsq mode, branches $J_z=\pm 2$ cross the branches $J_z=0,\ \pm 1$, and for the sq modes, the branch $J_z=0$ crosses the branches $J_z=\pm 1$.

The MF also splits the pb mode into a set of three modes with energies below $2\Delta_{\max}=(4\Delta^2+4\tilde{H}^2)^{1/2}$, that leads to the possibility of existance of pb modes as resonances. One has for ϵ^2

$$\text{pb: } 4\Delta^2+\tfrac{4}{3}\tilde{H}^2(0,0,r),\quad 4\Delta^2(1,0,i),\quad 4\Delta^2-2\tilde{H}^2(1;\pm 1;i)\ .$$

The MF creates also a gap ($\epsilon^2=2\tilde{H}^2$) in the spectrum of the transverse spin waves, but does not change the spectrum of the longitudinal spin waves.

The EF causes similar changes. The gap distortion by EF leads to the possibility of the existence of pb modes as resonances and to the possibility of their observation (for one of the pb modes, one has $\epsilon^2=4\Delta^2-\tfrac{2}{5}\tilde{E}^2$, where $\tilde{E}^2=g_E E^2/6\beta_0$). The EF also creates gaps in the spectrum of the Gd modes (for one of the spin waves $\epsilon^2\simeq\tfrac{2}{5}\tilde{E}^2$), and splitting the rsq and sq modes through the appearance of different corrections of order $\tilde{E}^2$. Note that within each kind of mode (pb, rsq, sq, or Gd) there are some modes which do not change in the EF case.

Thus the gap distortion induced by DI, MF, or EF leads to important changes in the spectrum of the OP collective modes.

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