Significance of the low-field magnetization maximum of a spin-glass near T_g

H. Bouchiat
Laboratoire de Physique des Solides, Université Paris—Sud, Bâtiment 510, F-91405 Orsay Cédex, France
(Received 27 February 1984)

When the low-field magnetization of a spin-glass is analyzed within the framework of a scaling assumption for the order parameter, three kinds of very different qualitative behaviors are found for the maximum of the magnetization, depending on the value of the critical exponent β. These different behaviors seem to be found experimentally among the various existing spin-glass systems. The ability of scaling hypotheses to describe the spin-glass transition is questionable as long as physical reasons are not given to justify the absence of universality for the critical behavior of these systems (even as near to one another as *Ag*Mn to *Cu*Mn).

The existence of a line in the plane (H,T) separating the spin-glass phase (where the irreversibility occurs) from the paramagnetic phase has been experimentally found in most of the known spin-glasses.[1–3] Furthermore, the low-field behavior of this irreversibility line in most cases is found to coincide with the mean-field-theory result[4]

$$H^2 \propto \tau^3, \quad \tau = |T-T_g|/T_g .$$

This even occurs for systems where the determination of critical exponents characteristic of the transition gives rise to values very far from those given by mean-field theory.

We have pointed out this paradox in Ref. 5. As a preliminary approach of the problem, for a non-mean-field spin-glass, we have analyzed the variation in field of the maximum of $M(T)$ which coincides with the de Almeida—Thouless instability line in the mean-field theory. However, the question of the coincidence between the power laws characterizing on the one hand the occurrence of irreversibility and on the other hand, the variation with field of the maximum of $M(T)$ remained unanswered, for a non-mean-field system. A further step has been made by Malozemoff, Barnes, and Barbara in Ref. 6 (which we will refer to as MBB in the following). In this paper the following identification is made between the irreversibility line and the crossover line: $H^2=\tau^{\beta+\gamma}$ when a scaling assumption

$$q=\tau^{\beta}F(H^2/\tau^{\beta+\gamma}) \tag{1}$$

is made for the order parameter. β and γ are defined by

$$q=a\tau^{\beta} \text{ for } H=0,\ T<T_g , \tag{2}$$

$$q=H^2/\tau^{\gamma} \text{ for } H\neq 0,\ T>T_g . \tag{3}$$

The connection between this so-called crossover line[6] (MBB approach) and the low-field behavior of the maximum of $M(T)$, which is the approach to the problem we have made in Ref. 5, is still obscure in our opinion, however.

Here we try to clear up whether or not, and for what values of the critical exponents, a coincidence between the crossover line and the field variation of the maximum of $M(T)$ is expected in a non-mean-field spin-glass. This leads us to a precise description of the behavior of the low-field magnetization in the neighborhood of T_g both above and below T_g. This behavior is found to be very strongly dependent on whether the value of the critical exponent β is smaller, equal to, or larger than 1. We then show that this discussion is not a purely academic one since both behaviors either characteristic of $\beta<1$ or $\beta>1$ are apparently found to be experimentally in different systems.

MAXIMUM OF $M(T)$ FOR $T<T_g$

For the mean-field Ising case, Parisi and Toulouse[7] have shown that the position of the maximum of $M(T)$ occurring below T_g follows a $H^{2/3}$ law identical with the low-field equation of the instability de Almeida—Thouless line.[6] In Ref. 5, assuming a scaling behavior for the order parameter, it has been shown that the magnetization below T_g can be written as the following:[8,9]

$$M=\frac{H}{1-\tau}-H\left[a\tau^{\beta}+\tau^{\beta}f\left[\frac{H^2}{\tau^{\beta+\gamma}}\right]\right], \tag{4}$$

$$M=H\left[1+\tau-a\tau^{\beta}-\tau^{\beta}f\left[\frac{H^2}{\tau^{\beta+\gamma}}\right]\right], \tag{5}$$

with $f(x)\rightarrow x$, when $x\rightarrow 0$ and $f(x)\rightarrow x^{\beta/(\beta+\gamma)}=x^{1/\delta}$, when $x\rightarrow\infty$. In the cases where $\beta<1$, or $\beta=1$ and $a>1$, $M(T)$ has a maximum at T_m below T_g with

$$(T_g-T_m)/T_g \propto H^{2/(\beta+\gamma)} . \tag{6}$$

For this case the exponent describing the temperature of the maximum is identical with the one characterizing the low-field equation of the so-called crossover line by MBB.[6] However, a different behavior can be expected in the case where $\beta=1$ and $a=1$ (independent of the value of the other critical exponents; this case is less restrictive than the mean-field one). As is obvious in (5), the first term of the development of q cancels exactly the term coming from the Curie law in M. Then one must consider higher-order terms in the development of q in zero field, and the following

$$(T_g-T_m)/T_g \propto H^{2/(\gamma+2)} \tag{7}$$

TABLE I. Discussion of the maximum of $M(T)$ and T_m for the different values of the parameters a and β characterizing the order parameter $q=a[(T_g-T)/T_g]^{\beta}=a\tau^{\beta}$ below T_g in zero magnetic field.

Existence of a maximum in $M(T)$ for:	$\beta<1$ [see Fig. 1(a)]	$\beta=1,\ a>1$ [see Fig. 1(b)]	$\beta=1,\ a=1$ [see Fig. 1(b)]	$\beta=1,\ a<1$ [see Fig. 1(c)]	$\beta>1$ [see Fig. 1(c)]
$T=T_g$ $H=0$	Yes	Yes		No	
$T'>T_g$ T'_m increases with H	Yes: $H^2\propto\tau^{\gamma+1}$	No		No	
$T<T_g$ T_m decreases with H	Yes: $H^2\propto\tau^{\gamma+\beta}$	Yes $H^2\propto\tau^{\gamma+\beta}$	Yes $H^2\propto\tau^{\gamma+2}$	?	

variation for the maximum of $M(T)$ is obtained.

Contrary to a previous statement by MBB,[6] we do not give any conclusion in Ref. 5 concerning whether this cancelation should occur in non-mean-field systems (even if β is equal to the mean-field value). To make this discussion complete one must also consider the cases where $\beta>1$, or $\beta=1$ and $a<1$. In those cases [see Eq. (5)] the contribution of the order parameter is not sufficient to compensate for the increase of the magnetization due to the Curie law, in the zero-field limit at $T=T_g$. There is no maximum in the zero-field limit of $M(T)/H$ at $T=T_g$ [see Fig. 1(c)] without more assumption about the zero-field development of $q(\tau)$; it is not possible to discuss the eventual existence of a maximum in $\lim_{H\to 0}M(T)/H$ at $T<T_g$. Furthermore the validity of (4) becomes doubtful when attention is turned to a neighborhood not close to T_g.

MAXIMUM OF $M(T)$ FOR $T>T_g$

In the vicinity above T_g the magnetization can be developed as

$$M/H=1/(1+\tau)-H^2/\tau^{\gamma}+\tau^{\beta}O((H^2/\tau^{\beta+\gamma})^2)\ . \qquad (8)$$

For

$$H^2<\tau^{\beta+\gamma} \qquad (9)$$

this development can be restricted to the first two terms. A maximum at T'_m of $M(T)$ is found above T_g as already shown in Ref. 10:

$$T'_m-T_g\propto H^{2/(\gamma+1)}\ . \qquad (10)$$

However, it is very important to note that this result is only valid if T'_m also satisfies the condition $T'_m-T_g>H^{2/(\beta+\gamma)}$ in order to be compatible with (9). This occurs only if β is strictly <1.[11] For $\beta>1$, one must consider the expression of the scaling function in the limit where $H^2/\tau^{\beta+\gamma}\gg 1$:

$$\frac{M}{H}=\frac{1}{T}-\frac{H^{2/\delta}}{T}\left[1+O\left[\frac{\tau}{H^{2/(\beta+\gamma)}}\right]\right],$$
$$\frac{M}{H}=(1-H^{2/\delta})-\tau(1-aH^{2(\beta-1)/(\beta+\gamma)}-H^{2/\delta})\ . \qquad (11)$$

When $\beta>1$, the variation of M/H with the temperature is dominated by the Curie law, so there is no maximum in $M(T)$ in the region $H^2/\tau^{\beta+\gamma}\gg 1$. The case where $\beta=1$ is more delicate because the position of the maximum when it exists would coincide with the point where $H^2\sim\tau^{\beta+\gamma}$, at which point the scaling hypotheses do not yield any small-parameter expansion of the scaling function. However, when one makes a reasonable, smooth extrapolation for this scaling function between its two asymptotic forms, one finds[11] that for $\beta=1$, dM/dT is remaining negative for every temperature above T_g. Hence we find that the existence or absence of a maximum in $M(T)$ above T_g moving up in temperature with increasing magnetic field can be used as a criterion to characterize a system whose critical exponent β is strictly less than or greater than 1. A summary of this whole discussion is given in Table I.

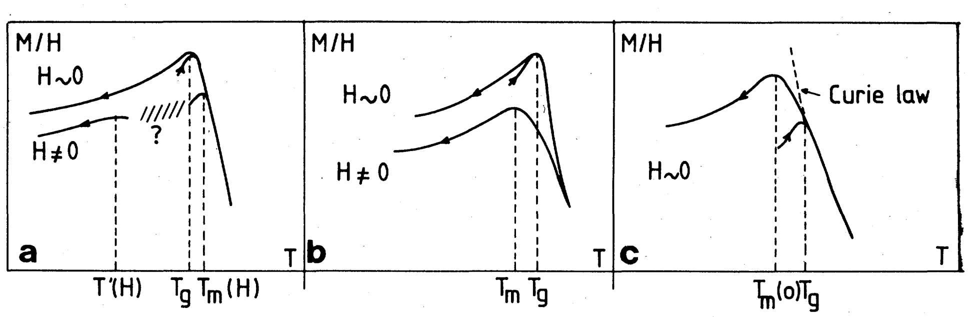

FIG. 1. Low-field magnetization M/H for (a) $\beta<1$; (b) $\beta=1$ and $a>1$; and (c) $\beta=1$ and $a<1$ or $\beta>1$. [These sketches should be only taken as a qualitative guide. The transition temperature T_g is defined as the branching point between the low-field limit of the $M(T)$ curve obtained after zero-field cooling and the $M(T)$ curve obtained after field cooling. Question mark denotes uncertainty.]

We achieve an unexpected result for $\beta < 1$: the existence of two maxima in the $M(T)$ curve, one above T_g, the other one below T_g. A simple scaling hypothesis is not sufficient to perform the calculation of $M(T)$ completely between T_m and T'_m. However, one can easily verify both relations:

$$M(Tm) \simeq 1 - aH^{2/\gamma} \simeq M(T_g) ,$$

but

$$M(T'_m) \simeq 1 - bH^{2/\gamma} > M(T_g) .$$

The maximum at T'_m is hence more pronounced than the maximum at T_m and this can perhaps explain why the existence of two maximum in $M(T)$ has not been observed so far in a spin-glass.

EXAMPLES OF EXPERIMENTAL DATA

The distinction we have made among three kinds of qualitative behaviors of the low-field magnetization, either characteristic of $\beta < 1$, $\beta = 1$ and $a > 1$, $\beta = 1$ and $a < 1$, or $\beta > 1$, are apparently found among the various existing spin-glass systems.

(a) Low-field studies on *Gd*Al (Ref. 10), *Cu*Mn (Ref. 10), and *Au*Fe (Ref. 12) have shown the existence of a maximum in $M(T)$ moving up in temperature with increasing field (behavior characteristic of $\beta < 1$).

(b) On the contrary, systems such as *Ag*Mn (Ref. 1) and *Au*Mn (Ref. 13) show a *decrease* with magnetic field of the maximum T_m of $M(T)$. The low-field limit of this maximum coincides with the temperature characterizing the occurrence of irreversibility within an accuracy of 1% (behavior characteristic of $\beta = 1$ and $a \geq 1$).

Such different behaviors observed in the twin systems *Cu*Mn and *Ag*Mn are striking when it is related to values of critical exponents different in the two systems. However, one must confirm that these differences are physical and not due to differences, for instance, in the metallurgical states of these two systems.[14]

(c) The third type of behavior characteristic either of $\beta > 1$ or $\beta = 1$ and $a < 1$ (Ref. 15) is also found in other systems such as *Y*Er (Ref. 16) and $CsNiFeF_6$,[17] where the irreversibility in the low-field magnetization occurs at a temperature T_g which is above the temperature of the maximum of $M(T)$: T_m, the quantity $(T_g - T_m)/T_g$ varying from 5% in *Y*Er systems to more than 10% in those of $CsNiFeF_6$.[17]

The conclusion of this analysis is that the diversity of the low-field magnetization maximum behaviors found in the different kind of systems called spin-glasses may reflect the same diversity in the values of the critical exponents β in these systems when a scaling assumption is made for the order parameter of the spin-glass transition. If this experimental diversity is confirmed and if one wants to prove in the future that this scaling assumption is not absurd (i.e., the spin-glass transition is a real phase transition) one will have to justify the physical origins (range of interaction, nature of anisotropy, etc.) of different classes of spin glasses.

ACKNOWLEDGMENTS

I thank P. Monod for all the stimulative help he has given to this work. I also thank J. Ferré, T. Garel, and C. Pappa for fruitful discussions, and P. M. Levy for his critical reading of the manuscript.

[1]R. V. Chamberlin, M. Hardiman, L. A. Turkevich, and R. Orbach, Phys. Rev. B **25**, 6720 (1982).

[2]N. Bontemps, J. Rajchenbach, and R. Orbach, J. Phys. (Paris) Lett. **44**, L47 (1983).

[3]J. L. Tholence and M. B. Salamon, J. Magn. Magn. Mater. **31**, 1340 (1983).

[4]J. R. L. de Almeida and D. J. Thouless, Phys. Rev. Lett. **32**, 1792 (1974).

[5]H. Bouchiat, J. Phys. C **16**, L145 (1983).

[6]A. P. Malozemoff, S. E. Barnes, and B. Barbara, Phys. Rev. Lett. **51**, 1704 (1983).

[7]G. Parisi and G. Toulouse, J. Phys. (Paris) Lett. **43**, L45 (1980); J. Vannimenus, G. Toulouse, and G. Parisi, J. Phys. (Paris) **42**, 565 (1981).

[8]For Heisenberg spins the situation is more complex because of the occurrence of the symmetry replica breaking along the transverse freezing line. However, there is still a maximum of $M(T)$ following a $H^{2/3}$ law like the de Almeida-Thouless line (occurrence of strong longitudinal irreversibility); see D. Elderfield and D. Sherrington, J. Phys. C **15**, L783 (1982).

[9]We have chosen to consider here exclusively the case where the average of the magnetic interactions is $J_0 = 0$. As in the mean-field case, the properties of the maximum of $M(T)$ are qualitatively the same for $J_0 \neq 0$, than for $J_0 = 0$ as long as $J_0 < 1$.

[10]B. Barbara and A. P. Malozemoff (unpublished); A. P. Malozemoff, B. Barbara, and Y. Imry, Phys. Rev. Lett. **47**, 1852 (1981); B. Barbara and A. P. Malozemoff, J. Less-Common Met. **94**, 45 (1983).

[11]A reasonable extrapolation of $f(x)$ between its two limits is given in Ref. 10 by

$$f(x) = g_1 x^{1/\delta} / [x^{-1/\phi} + (g_1/g_2)^{1/\gamma}]^{\gamma} ,$$

g_1 and g_2 are constants verifying both $g_1 \sim g_2 < 1$. For $\beta = 1$

$$\frac{dM}{dT} = -1 + \frac{\gamma g_1}{[(\tau/H)^{2/\phi} + (g_1/g_2)^{1/\gamma}]^{\gamma+1}}$$

the existence of a maximum in $M(T)$ varying like

$$\tau/H^{2/\phi} = (\gamma g_1)^{1/(\gamma+1)} - (g_1/g_2)^{1/\gamma}$$

is possible only if the quantity $(\gamma g_1)^{1/(\gamma+2)} - (g_1/g_2)^{1/\gamma}$ is positive which is not the case when one assumes that $g_1 \approx g_2$ and $\gamma g_1 \ll 1$.

[12]L. Lundgreen, P. Svedlindh, and O. Beckman, Phys. Rev. B **26**, 3990 (1982).

[13]S. P. Mcalister and C. M. Hurd, J. Magn. Magn. Mater. **15**, 169 (1980).

[14]R. V. Chamberlin, M. Hardiman, and R. Orbach, J. Appl. Phys. **52**, 1771 (1981).

[15]One must, however, be extremely cautious concerning the interpretation of experiments in which the irreversibility occurs at a temperature above the maximum of $M(T)$. It is shown indeed in Ref. 14 that inhomogeneities of concentration in

*Cu*Mn can be at the origin of such a behavior in this system. However, in this last case [contrary to the observations in *Y*Dy, *Y*Er (Ref. 16) where the irreversibility occurs at a very well-defined temperature] the spin-glass transition is very ill defined.

[16]K. Baberschke, P. Pureur, A. Fert, R. Wendler, and S. Senoussi (unpublished); H. Bouchiat and P. Monod (unpublished).
[17]C. Pappa, J. Phys. C (to be published).