Second sound, osmotic pressure, and Fermi-liquid parameters in ^{3}He-^{4}He solutions

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Second-sound velocities and osmotic pressures are analyzed to obtain the first experimental values for the Landau compressibility parameter F_0^s in ^{3}He-^{4}He solutions. Data are presented as a function of pressure and ^{3}He concentration, and are compared to theoretical predictions. The square of the second-sound velocity at finite temperature is found to be accurately proportional to the internal energy of a perfect Fermi gas. Using inertial effective masses given by the Landau-Pomeranchuk theory, the square of the velocity is found to separate into two parts: a temperature-dependent part characterized completely by ideal Fermi-gas behavior and a temperature-independent part containing all the Fermi-liquid corrections. This is related to a similar separation found in the osmotic pressure.

I. INTRODUCTION

In most respects ^{3}He-^{4}He solutions at low temperatures behave like a weakly interacting Fermi gas. The known Landau Fermi-liquid parameters are small. At zero pressure, only F_0^a and F_1^s are known; both are of order 0.1 or less in magnitude, depending on concentration. The one exception to this rule is the quasiparticle compressibility, which is predicted[1,2] to show large deviations from ideality. This is reflected in the Landau parameter F_0^s, given by Fermi-liquid result

$$1+F_0^s=\frac{K_{\text{ideal}}}{K}\ , \tag{1}$$

where K is the compressibility of the Fermi component of the mixture, and K_{ideal} is the compressibility of an ideal Fermi gas having the same effective mass as the Fermi liquid. Bardeen *et al.*[1] and Owen[2] have predicted that F_0^s will be negative and as large as -0.4. This is substantial, since $F_0^s>-1$ must be true to assure mechanical stability.

Experimental values of F_0^s can be obtained from the velocity of second sound, which, at low temperatures, is a compressional wave in the ^{3}He quasiparticles which leaves the total pressure of the solution constant. It therefore measures the quasiparticle compressibility alone, with no contribution from that of the ^{4}He. For this reason second sound is a much more sensitive measure of this compressibility in mixtures than is first sound. The mode is analogous to first sound in pure liquid ^{3}He, which is the most accurate way to obtain F_0^s in that system. Independent values of this parameter can also be obtained from Eq. (1) if the osmotic pressure is known as a function of concentration at $T=0$. This paper presents the first experimental values of F_0^s, obtained using both techniques, as a function of concentration and pressure.

The temperature dependence of the velocity of second sound in ^{3}He-^{4}He solutions at low temperatures has been analyzed previously by Brucker *et al.*[3] in terms of an effective interaction theory, and also by Bashkin.[4] The close similarity between the behavior of the second-sound velocity and that expected for first sound in an ideal Fermi gas was first pointed out by Greywall and Paalanen,[5] for concentrations below 1%. This similarity is shown to extend over the full range of accessible concentrations and pressures, and takes a particularly simple form for inertial effective masses given by the Landau-Pomeranchuk theory.

II. ANALYSIS OF DATA

A. Compressibility parameter F_0^s

Khalatnikov[6] has derived an expression relating F_0^s and the velocity of second sound at $T=0$:

$$u_2^2(T=0)=\frac{v_F^2}{3}(1+F_0^s)\left[1+\frac{F_1^s}{3}\right]\left\{1-\frac{n_3m^*}{(\rho-n_3m_3)(1+F_1^s/3)}\left[\alpha_1\left[1+\frac{F_1^s}{3}\right]+\frac{\delta m}{m^*}\right]^2\right\},$$

where v_F is the Fermi velocity, m^* is the ^{3}He effective mass, m_3 and m_4 are the atomic masses of ^{3}He and ^{4}He, $\alpha_1 m^*/m_4=v_3/v_4$, the ratio of ^{3}He and ^{4}He atomic volumes, ρ is the total density, n_3 is the ^{3}He quasiparticle number density, and $\delta m=m^*-m_3(1+F_1^s/3)$. To first order in the ^{3}He mole fraction x, this can be written

$$u_2^2(T=0)=\frac{v_F^2}{3}(1+F_0^s)\left[1+\frac{F_1^s}{3}\right]\times\left[1-x\frac{m_4}{m_i}\left[1+\alpha+\frac{m_i-m_3}{m_4}\right]^2\right], \quad (2)$$

where the Bardeen-Baym-Pines parameter $\alpha=v_3/v_4-1$, and m_i, the so-called "inertial" ^{3}He effective mass, is equal to $m^*/(1+F_1^s/3)=\rho_n/n_3$. Here ρ_n is the normal fluid density. With the exception of the quantity in square brackets, this is the same relation that is obtained in pure liquid ^{3}He between F_0^s and the square of the first-sound velocity u_1 at $T=0$. The quantity in square brackets reflects the fact that the ^{3}He and ^{4}He atomic volumes are different, and that most of the ^{3}He effective mass is due to the inertia of the ^{4}He through which it moves.

Second-sound velocities obtained in this laboratory[7] have been analyzed as a function of temperature to obtain limiting values at $T=0$ for use in Eq. (2). Values published previously by Brubaker *et al.*,[8] Greywall,[9] Greywall and Paalanen,[5] and de Voogt and Kramers,[10] have been similarly analyzed. With the exception of de Voogt and Kramers, who used heater and bolometer for second-sound generation and detection, all the data were obtained using porous-membrane capacitance transducers. The square of the second-sound velocity was plotted as a function of the internal energy U of an ideal Fermi gas having the same Fermi temperature as the solution, as tabulated by Stoner.[11] This yields a remarkably accurate linear fit, shown in Fig. 1. In calculating the Fermi temperature $T_F=\hbar^2(3\pi^2 n_3)^{2/3}/2m^*k_B$, data for α and the ^{4}He molar volume V_4 were taken from Watson *et al.*[12] Specific-heat effective masses were taken at $P\cong 0$ from an average of the data of Anderson *et al.*[13] and preliminary

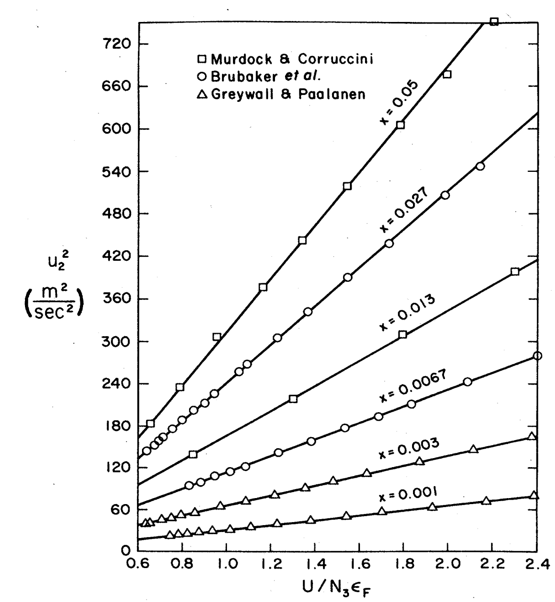

FIG. 1. Square of the second-sound velocity u_2 plotted as a function of the internal energy U of an ideal Fermi gas with the same Fermi temperature as the solution. The data shown are from Refs. 5, 7, and 8. $T=0$ corresponds to $U=0.6N_3\epsilon_F$.

TABLE I. Some parameters used in the analysis of second-sound velocity.

P (atm)	α	$V_4\left[\frac{\text{cm}^3}{\text{mol}}\right]$	$\frac{m_0^*}{m_3}$
0	0.284	27.580	2.34
10	0.207	25.180	2.64
20	0.176	23.744	2.90

values of Mueller *et al.*,[14] linearly interpolated as a function of concentration; and at $P=10$ and 20 atm from Mueller *et al.* and the measured and extrapolated values of Polturak and Rosenbaum.[15] In Eq. (2), the inertial effective mass m_i was taken to be given, within the variation of published values,[9,16,17] by the Landau-Pomeranchuk assumption $m_i=m_0^*$. Here m_0^* is the specific-heat effective mass (obtained from the sources above) extrapolated to $x=0$. Values used for α, V_4, and m_0^* for three of the pressures analyzed here are given in Table I.

At temperatures above approximately 0.5 K, u_2^2 falls systematically below the straight-line fit, due presumably to phonon excitations taking part in the wave. This is visible in the data plotted for $x=0.05$. Below 0.5 K the linearity of these data is not unexpected, for two reasons. First, it is what one would expect to see in the case of a perfect gas, where the velocity of sound can be expressed[5] as

$$c^2=\frac{10}{9}\frac{U(T)}{Nm}, \quad (3)$$

independent of statistics, where $U(T)$ is the internal energy, N is the number of particles, and m is the particle mass. Second, a similar linear behavior has been observed in the osmotic pressure $\pi(T)$ by Landau *et al.*[18] Because the second-sound velocity and π are related by a derivative,[19]

$$u_2^2\cong\frac{1}{\rho_n/n_3}\left.\frac{\partial\pi}{\partial n_3}\right|_{S,P}$$

one might expect this linearity to persist in u_2^2.

To obtain experimental values of F_0^s, u_2^2 was extrapolated to $T=0$ ($U=0.6N_3\epsilon_F$, where N_3 is the number of ^{3}He quasiparticles and ϵ_F is the Fermi energy), using a least-squares fit, and the resulting intercepts were employed in Eq. (2). The results are shown as a function of ^{3}He concentration x in Fig. 2. The solid curve for $P\cong 0$ is the original prediction of Bardeen, Baym, and Pines[1] (BBP), multiplied by $(\frac{3}{4})^{1/2}$ to reflect subsequent corrections to the calculated spin-diffusion coefficient which they used.[20] According to BBP, $F_0^s=2N(0)V_0+F_0^a$, where $N(0)$ is the density of states at the Fermi surface and V_0 is the effective ^{3}He-^{3}He interaction at $k=0$. For small concentrations x, this formula predicts that F_0^s is proportional to $x^{1/3}$. The solid curves for $P=10$ and 20 atm are similar predictions based on the BBP theory, using spin-diffusion data obtained previously in this laboratory at elevated pressure.[21]

The uncertainty in F_0^s, shown in Fig. 2 by the error

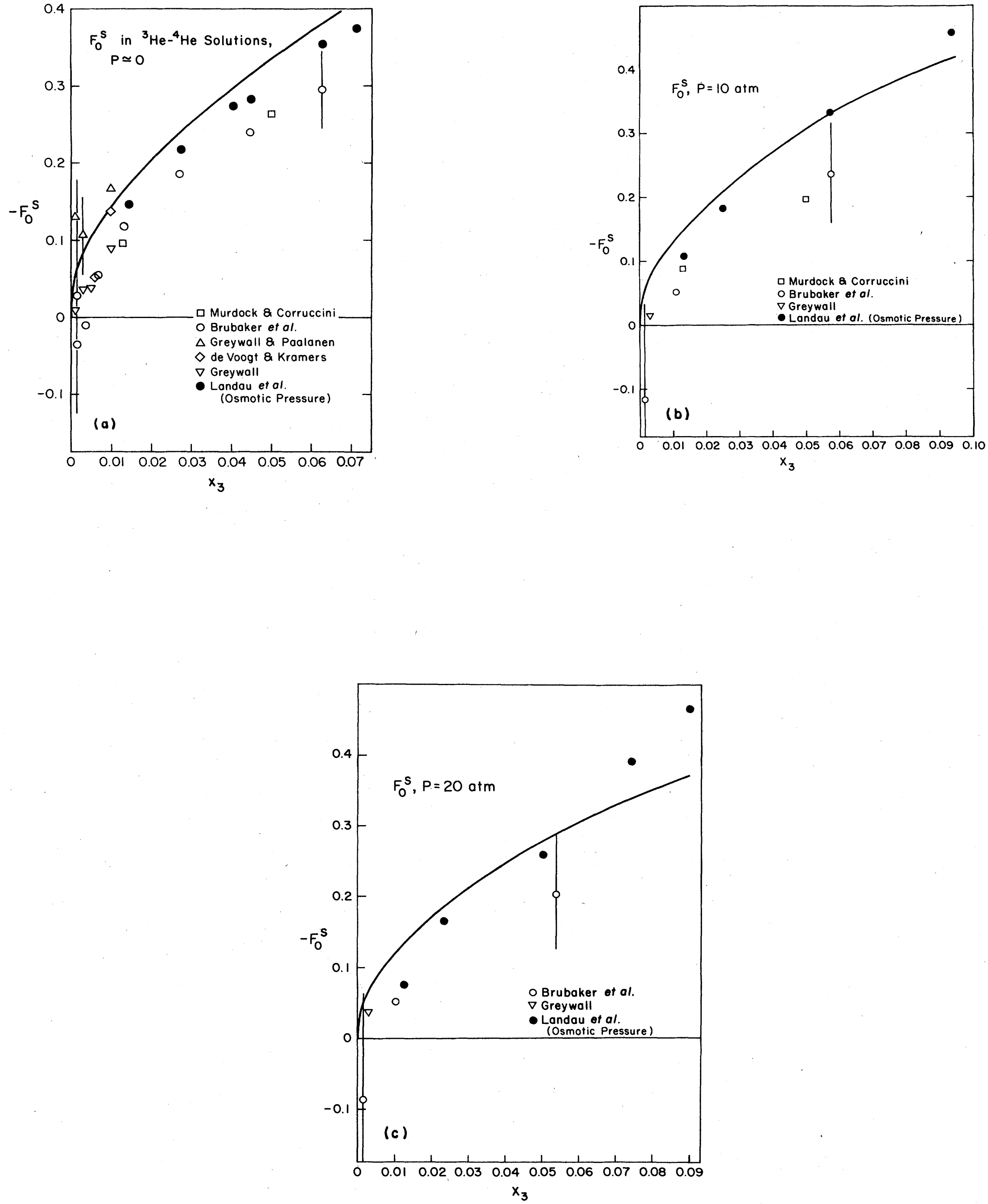

FIG. 2. Fermi-liquid compressibility parameter F_0^s as a function of ^{3}He concentration x for three pressures. The values were obtained by extrapolating u_2 to $T=0$, as shown in Fig. 1, and using these values in Eq. (2). The values based on osmotic pressure were obtained as described in the text. The solid lines are predictions based on the BBP theory, Ref. 1, fitted to the spin-diffusion data of Refs. 13 and 21.

bars, is due at higher concentrations primarily to the variation in published values of m_i (for u_2 values), and to a lesser extent the published uncertainty in m^*, α, and V_4. At the lowest concentrations below 1%, the error is typically much larger and is due primarily to uncertainty in the extrapolation to $T=0$, from scatter in the u_2 data. Within the experimental uncertainty, all the F_0^s are negative, as predicted by BBP, but are generally smaller in magnitude than that theory predicts. They are at least 35% smaller than the recent predictions of Owen.[2] As a function of pressure, F_0^s declines slowly about 25% in magnitude from $P \cong 0$ to $P=20$ atm. This is explained by the BBP theory as a consequence of the weakening with pressure of the effective ^{3}He-^{3}He interaction, $V_0 \cong -\alpha^2 m_4 s^2/n_4$, where s is the velocity of sound in ^{4}He, and n_4 is the number density of pure ^{4}He. The negative sign of F_0^s is a reflection of the attractive nature of V_0.

With some loss in accuracy, this parameter can also be obtained from measured values of the osmotic pressure π, directly from the Fermi-liquid result

$$1+F_0^s=\frac{K_{\text{ideal}}(T=0)}{K(T=0)} . \tag{1'}$$

Here

$$K=-\frac{1}{V}\frac{\partial V}{\partial \pi}=\frac{1}{n_3}\frac{\partial n_3}{\partial \pi} .$$

Following Landau *et al.*,[18] π may be separated into parts due to the kinetic pressure of a perfect Fermi gas and the (attractive) interactions between ^{3}He quasiparticles:

$$\pi=\pi_{\text{ideal}}+\pi_{\text{int}} . \tag{4}$$

Therefore

$$1+F_0^s=\frac{\left.\frac{\partial}{\partial n_3}(\pi_{\text{ideal}}+\pi_{\text{int}})\right|_{T=0}}{\left.\frac{\partial \pi_{\text{ideal}}}{\partial n_3}\right|_{T=0}} ,$$

$$F_0^s=\frac{\left.\frac{\partial \pi_{\text{int}}}{\partial n_3}\right|_{T=0}}{\left.\frac{\partial \pi_{\text{ideal}}}{\partial n_3}\right|_{T=0}} . \tag{5}$$

To obtain F_0^s from experimental values of π one must interpolate an equation of state for $\pi_{\text{int}}(T=0)$ as a function of n_3. This was done as follows. When the osmotic-pressure data of Landau *et al.* are plotted against the ideal internal energy U, calculated using the same specific-heat effective masses used for u_2, they again fall on a linear curve and can be accurately extrapolated to $T=0$. Least-squares values of $\pi(T=0)$ obtained in this way differ by less than 1% from those of Ref. 18. They were used in Eq. (4) to obtain $\pi_{\text{int}}(T=0)$, along with values of the ideal kinetic Fermi-gas pressure

$$\pi_{\text{ideal}}(T)=\frac{2}{3}\frac{U(T)}{V} ,$$
$$=\frac{2}{3}\frac{xU(T)}{N_3 v_4(1+\alpha x)} , \tag{6}$$

or

$$\pi_{\text{ideal}}(T=0)=\frac{2}{5}\frac{x\epsilon_F}{v_4(1+\alpha x)}\left(=\tfrac{2}{5}n_3\epsilon_F\right) .$$

The Fermi energy ϵ_F is equal to $\hbar^2 k_F^2/2m^*$, and $k_F=(3\pi^2 n_3)^{1/3}$. For purposes of calculating the derivative $\partial\pi_{\text{int}}/\partial n_3$ in Eq. (5), $\pi_{\text{int}}(T=0)$ was empirically fit over the concentration range of interest to a simple power law, $\pi_{\text{int}}(T=0)=An_3^y$, where A and y are constants. Then $\partial\pi_{\text{int}}/\partial n_3=y\pi_{\text{int}}(T=0)/n_3$. The accuracy of this fit was better than 3% at $P=0.26$ and 10 atm, and for $x \geq 0.0504$ at 20 atm; for $x \leq 0.0504$ at $P=20$ atm, the accuracy was only 10%. Least-squares values of y at $P=0.26$, 10, and 20 atm were found to be 2.22, 2.39, and 2.51 (± 0.05), respectively. Combined with the Fermi-gas result

$$\frac{\partial \pi_{\text{ideal}}(T=0)}{\partial n_3}=\tfrac{2}{3}\epsilon_F=\frac{5}{3}\frac{\pi_{\text{ideal}}(T=0)}{n_3} ,$$

Eq. (5) yields

$$F_0^s=\tfrac{3}{5}y\left.\frac{\pi_{\text{int}}}{\pi_{\text{ideal}}}\right|_{T=0} . \tag{7}$$

Experimental values of $\pi_{\text{int}}(T=0)$ were used in (7) to obtain F_0^s. These values are shown with those obtained from second-sound velocities in Fig. 2. As a check, they were also compared with values obtained by a point-by-point differentiation of π_{int}; within experimental scatter, they agree. Landau *et al.*[18] used a slightly different form for π_{ideal}, and different effective masses, to derive values of π_{int}. If these values of $\pi_{\text{int}}(T=0)$ are used instead in this analysis, the resulting values of F_0^s are increased in magnitude over the osmotic-pressure values shown in Fig. 2. The amount of increase is approximately 22% at $P=0.26$ atm, and approximately 30% at both $P=10$ and 20 atm. These values are outside the experimental uncertainty in the values obtained from second-sound velocity. Because determination of F_0^s from π involves the derivative of the difference between two quantities of comparable size, this method is rather sensitive to the form chosen for π_{ideal}, and should probably be considered less reliable than determination from u_2.

The osmotic-pressure values of F_0^s shown in Fig. 2 are all somewhat larger in magnitude than those obtained from second-sound velocity. If the inertial effective masses of Sherlock and Edwards[16] are used in the analysis of u_2, rather than m_0^*, the second-sound values of F_0^s are all increased in magnitude and most of this discrepancy is removed. This is, perhaps, to be expected, since these inertial masses were obtained in the first place by a self-consistent analysis of the second-sound velocities of Brubaker *et al.*[8] and the osmotic pressures of Landau *et al.*[18]

A list comparing various experimental quantities in

^{3}He-^{4}He solutions is shown in Table II, for $x=0.013$ and $x=0.05$ at $P\cong 0$, where a large body of data exists.

One consequence of the negative sign of F_0^s is that zero sound is unlikely to exist in ^{3}He-^{4}He mixtures. In mixtures zero sound is high-frequency *second sound* (first sound in the quasiparticle gas) with $\omega\tau \gg 1$. The approximate condition for its existence is

$$F_0^s+\frac{F_1^s}{1+F_1^s/3}>0 .$$

The large relative magnitude of F_0^s indicates the mode probably will not propagate. It is interesting that this conclusion is also predicted by the viscoelastic theory of zero sound, both as proposed by Rudnick[28] and as modified by Bedell and Pethick.[29] Rudnick's prediction is

$$u_0^2=(K_0+\tfrac{4}{3}K_\eta)/\rho ,$$

where K_0 is the bulk modulus and $K_\eta\tau=\eta$, the viscosity. It is not quantitatively correct in the limit of weak interactions, as discussed in Ref. 29; nevertheless, it yields a velocity for ^{3}He-^{4}He mixtures which is real but less than v_F, implying overdamping from quasiparticle excitations. The modifications of Bedell and Pethick lead to an imaginary velocity, as in the Fermi-liquid theory.

Mermin[30] has established that any Fermi liquid must support either longitudinal zero sound or longitudinal spin waves. The fact that $F_0^s<0$ would then indicate that Landau spin waves should propagate in mixtures at zero magnetic field.

TABLE II. Measured and derived quantities for ^{3}He-^{4}He solutions at $x=0.013$, 0.05, at $P\cong 0$. D, κ, and η are, respectively, the coefficients of spin diffusion, thermal conductivity, and viscosity. C_p denotes specific heat.

	x_3 0.013	0.05
m^*/m_3 (C_p)	$2.38\pm.04$[a] $2.40\pm.05$[b]	$2.46\pm.04$[a] $2.47\pm.02$[b] $2.45\pm.12$[c]
F_0^s	$-0.10\pm.05$[d] $-0.12\pm.05$[e]	$-0.26\pm.05$[d]
$u_2(T=0)$ (m/sec)	9.79 ± 0.1[d] 9.69 ± 0.1[e]	12.95 ± 0.1[d]
F_0^a	$0.09\pm.03$[a]	$0.08\pm.03$[a] $0.03\pm.02$[f]
$\dfrac{1}{1+F_0^a}-\dfrac{1}{1+F_1^a/3}$		$0.028\pm.003$[g,h]
DT^2 $(10^{-6}\text{cm}^2\,\text{K}^2/\text{sec})$	18 ± 3[i] 17.2 ± 1.7[a]	74.9 ± 8[i] 90 ± 9[a]
κT (erg/sec cm)	11 ± 1.1[j]	24 ± 2.4[j]
$\eta T^2(\mu\text{P}\,\text{K}^2)$	$0.034\pm.003$[k]	$0.28\pm.02$[k,l]

[a]Reference 13.
[b]Reference 14.
[c]Reference 15.
[d]This work; data from Ref. 7.
[e]This work; data from Ref. 8.
[f]Reference 22.
[g]Reference 23.
[h]Reference 24.
[i]Reference 21.
[j]Reference 25.
[k]Reference 26.
[l]Reference 27.

B. Temperature dependence of u_2

In analyzing the second-sound data of Sec. II A, it is striking that all the velocities, for all concentrations and pressures, can be accurately fit to a finite-temperature generalization of the Khalatnikov formula (2). This was deduced as follows. Equation (2) can be rewritten in the form

$$u_2^2(T=0)=\frac{10}{9}\frac{U_0}{N_3m_i}(1+F_0^s)\times\left[1-x\frac{m_4}{m_i}\left[1+\alpha+\frac{m_i-m_3}{m_4}\right]^2\right] , \tag{8}$$

where N_3 is the number of ^{3}He quasiparticles, and $U_0=\frac{3}{5}N_3\epsilon_F$ is the internal energy at $T=0$. This may be compared with Eq. (3), the result for first sound as a function of temperature in an ideal Fermi gas. The correspondence suggests that for an ideal gas of ^{3}He *dissolved in* ^{4}*He*, the correct expression for $u_2(T)$ is

$$u_{\text{ideal}}^2(T)=\frac{10}{9}\frac{U(T)}{N_3m_i}\left[1-x\frac{m_4}{m_i}\left[1+\alpha+\frac{m_i-m_3}{m_4}\right]^2\right] . \tag{9}$$

The least-squares slopes of the experimental u_2^2 as a function of U were compared with the ideal gas slope predicted by Eq. (9), and the ratios were unexpectedly found to equal one within experimental error. The numerical average for 29 values of x at three pressures, from five experimental groups, is

$$\frac{\left.\dfrac{\partial(u_2^2)}{\partial U}\right|_{\text{expt}}}{\left.\dfrac{\partial(u_2^2)}{\partial U}\right|_{\text{ideal}}}=0.9986\pm0.02 .$$

The scatter in this ratio is plotted in Fig. 3. This remarkable result implies that, within experimental error,

$$u_2^2(T)=u_{\text{ideal}}^2(T)+b , \tag{10}$$

where b is a constant term containing all the Fermi-liquid corrections. From Eqs. (8) and (9),

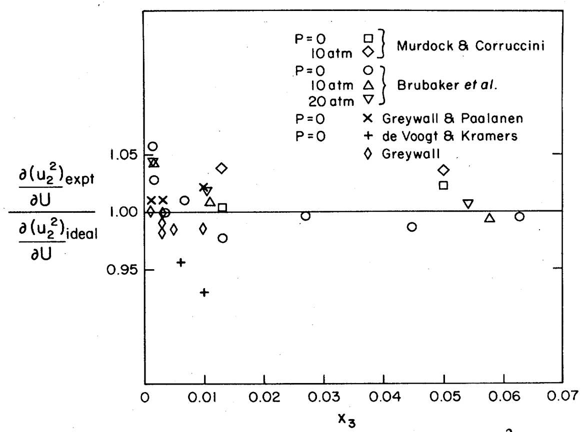

FIG. 3. Experimental least-squares slopes of u_2^2 vs U (as plotted in Fig. 1) normalized by the ideal Fermi-gas prediction of Eq. (9). The average of all the data shown is 0.9986±0.02.

$$b=F_0^s u_{\mathrm{ideal}}^2(T=0)\ . \tag{11}$$

As discussed previously, all the experimentally determined F_0^s are negative within experimental error. This is exhibited as a negative intercept, according to Eq. (11), when u_2^2 is extrapolated back to the zero of internal energy. This is shown in Fig. 4. In particular, at least for the larger concentrations, these curves show that $u_2^2(T)$ does not differ from the ideal prediction by a simple multiplicative constant, a model which has been used to fit data for $x<0.01$.[5] Equation (10) agrees with the observation of Greywall[9] that, for concentrations below 1%, the effects of quasiparticle interaction show up only in the concentration dependence of u_2, not in the temperature dependence.

It should be emphasized that the conclusions above depend on the value of inertial effective mass m_i used to

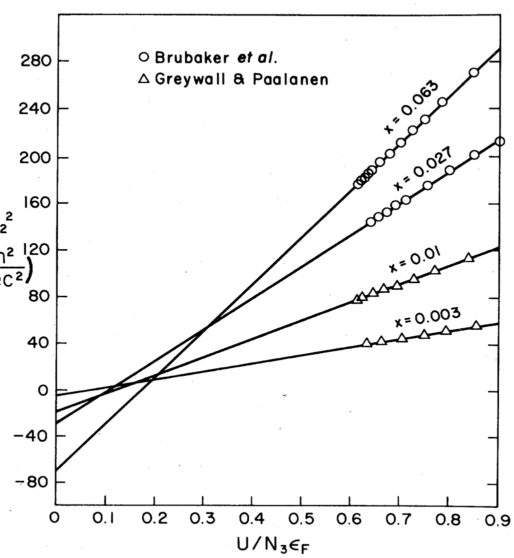

FIG. 4. Some values of the second-sound velocity, squared, extrapolated to the zero of internal energy U. The straight lines are linear least-squares fits. The negative intercepts reflect the negative values of F_0^s in Eq. (10), $u_2^2(T)=u_{\mathrm{ideal}}^2(T)+F_0^s u_{\mathrm{ideal}}^2(T=0)$.

analyze the data. The simple result of Eqs. (10) and (11) is obtained only for $m_i=m_0^*$, where m_0^* is the specific-heat effective mass extrapolated to $x=0$. The use of other published values[9,16,17] of m_i lead to slope ratios

$$\left.\frac{\partial(u_2^2)}{\partial U}\right|_{\mathrm{expt}} \Big/ \frac{\partial(u_{\mathrm{ideal}}^2)}{\partial U}$$

which change with concentration and temperature, and differ from one by as much as 12%; they therefore predict an interaction term b which depends on temperature. At present there exist no theoretical expressions for the velocity of second sound over the temperature and concentration range studied here. Thus there is no fundamental reason why this ratio of slopes should equal one, or why the interaction term (11) should be so independent of temperature. The fact that Eq. (10) holds so well may place new constraints on the form of the quasiparticle interaction in ^{3}He-^{4}He solutions.

Recent measurements by Greywall of the specific heat,[31] and second-sound determinations of ρ_n,[9,19] appear to indicate that above about 0.25 K these properties cannot be reconciled with predictions based on the quadratic Landau-Pomeranchuk excitation spectrum for the dissolved ^{3}He quasiparticles. Neutron scattering measurements[32] also appear to indicate negative deviations from $\epsilon(k)=\hbar^2k^2/2m^*$ for $k \gtrsim 1$ Å^{-1}. Therefore it seems surprising that the square of the second-sound velocity should produce such linear curves when plotted against the internal energy of an ideal Fermi gas with a purely quadratic spectrum. Closer examination shows that the deviations to be expected from linearity (from this cause) are not very large, at least below 0.5 K where phonon contributions can be ignored. Greywall has provided a convenient empirical form for his measured specific-heat data:

$$C_v=C_v^{\mathrm{LP}}\quad (T<0.245\ \mathrm{K})\ ,$$

$$C_v=C_v^{\mathrm{LP}}+\tfrac{3}{2}N_3k(0.20\ \mathrm{K}^{-1})(T-0.245\ \mathrm{K})\quad (T>0.245\ \mathrm{K})\ ,$$

where C_v^{LP} is the (Landau-Pomeranchuk) specific heat of an ideal Fermi gas with mass m^*. An integration of this result to yield the internal energy U_{expt} of the solution shows that U_{expt} deviates from the ideal Fermi gas $U(T)$ by only about 1% at $T=0.5$ K, and less at lower temperatures, over the entire concentration range of interest. This deviation is comparable to the experimental uncertainty in most of the measurements. Therefore it appears that the temperature dependence of u_2 in solutions is a much less sensitive measure of deviations from the Landau-Pomeranchuk spectrum than the specific heat.

There is a close parallel between Eqs. (10) and (11) and the behavior observed previously in the magnetic susceptibility of dilute solutions. Over an extended range of temperature and concentration, Anderson *et al.*[13] and Husa *et al.*[33] both found that the experimentally measured inverse susceptibility χ^{-1} was a linear function of the inverse susceptibility $\chi_{\mathrm{ideal}}^{-1}$ of an ideal Fermi gas, calculated with a mass equal to the specific-heat effective mass m^*. The following empirical relation was closely obeyed:

$$\frac{1}{\chi(T)}=\text{const}\times\left[\frac{1}{\chi_{\text{ideal}}(T)}+\frac{F_0^a}{\chi_{\text{ideal}}(T=0)}\right]. \quad (12)$$

This may be compared with Eq. (10):

$$u_2^2(T)=u_{\text{ideal}}^2(T)+F_0^s u_{\text{ideal}}^2(T=0)\ .$$

To date, only relative measurements of susceptibility have been obtained, so the constant appearing in Eq. (12) was *assumed* equal to one. Values of F_0^a have been obtained only by extrapolating $1/\chi$ to zero. In the case of second-sound velocity, this ambiguity is removed because absolute measurements of u_2 are available.

The second-sound velocity is related to the osmotic pressure π by a derivative:[16,19]

$$u_2^2=\frac{v_4}{m_4}\frac{\partial\pi}{\partial\ln\xi}\bigg|_{S,P}\frac{1-f\xi}{\rho_n/\rho_s+f^2\xi^2}$$
$$\cong\frac{1}{m_i}\frac{\partial\pi}{\partial n_3}\bigg|_{S,P}\left[1-x\frac{m_4}{m_i}\left[1+\alpha+\frac{m_i-m_3}{m_4}\right]^2\right] \quad (13)$$

to first order in x, where $f=1+\alpha-m_3/m_4$, $\xi=n_3v_4$, and ρ_s is the density of the superfluid component. In the limit $x\ll 1$, this reduces to

$$u_2^2=\frac{\partial\pi}{\partial\rho_n}\bigg|_{S,P}\ ,$$

analogous to the relation

$$u_1^2=\frac{\partial P}{\partial\rho}\bigg|_{S}$$

for first sound. It is interesting to look for an explanation of the simple temperature dependence of u_2, described by Eq. (10), in the behavior of $\pi(T)$ with temperature. This has been analyzed previously by Emery,[34] by Disatnik and Brucker,[35] and by Bashkin.[4] As first observed by Emery, the temperature dependence of the osmotic pressures measured by Landau *et al.* can be attributed completely to π_{ideal} in the relation

$$\pi=\pi_{\text{ideal}}+\pi_{\text{int}}\ . \quad (4)$$

That is, the least-squares slope of π as a function of U is the same as that predicted by Eq. (6). Numerically, the average slope for 13 values of x is found to be

$$\frac{\partial\pi_{\text{expt}}/\partial U}{\partial\pi_{\text{ideal}}/\partial U}=0.9971\pm0.016\ .$$

The experimental scatter in these ratios is shown in Fig. 5. The implication of this result is that π_{int} is a constant with no temperature dependence at all. This provides a natural explanation for the separation of the second-sound velocity into an ideal and a constant interactive part, in

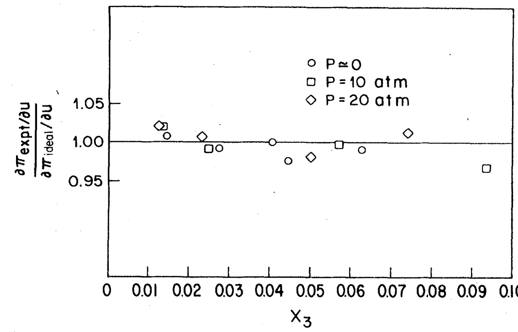

FIG. 5. Experimental least-squares slopes of the osmotic-pressure data of Landau *et al.* (Ref. 18) vs the internal energy U, normalized by the ideal Fermi-gas prediction of Eq. (6). The average of these data is 0.9971 ± 0.016.

Eq. (10). A straightforward calculation shows that an equation of the form of (10) follows directly from Eqs. (13) and (6) using the relation

$$\pi_{\text{ideal}}=-\frac{\partial U}{\partial V}\bigg|_{S,P,N_3}=n_3^2\frac{\partial u}{\partial n_3}\bigg|_{S,P}\ ,$$

where $u(T)$ is the internal energy per quasiparticle.

These conclusions differ somewhat from those of Landau *et al.*, who found a weak temperature dependence to π_{int}. This may be related to the different (non-specific-heat) effective masses which they employed, or to the fact that they chose a slightly different form for π_{ideal}:

$$\pi_{\text{ideal}}=\frac{2}{3}\frac{xU}{N_3v_4}+\frac{\mu_\alpha}{v_4}\ ,$$

where μ_α varies between $\frac{1}{4}x^2kT_F(1-\frac{3}{5}\alpha)$ at $T=0$ and $\frac{1}{2}x^2kT(1-\alpha)$ at $T\gg T_F$.

III. SUMMARY

Values of the Landau parameter F_0^s have been obtained from both second-sound velocity and osmotic-pressure data. They agree qualitatively with theory but appear systematically smaller in magnitude than theory predicts, at least for most concentrations.

Previous investigators[5] have noted the close similarity between second-sound velocity in ^{3}He-^{4}He mixtures and that expected for sound in a perfect Fermi gas. For inertial effective masses equal to m_0^*, this connection has been shown to be very simple and suggestive, with $u_2^2(T)$ given by

$$u_2^2(T)=u_{\text{ideal}}^2(T)+F_0^s u_{\text{ideal}}^2(T=0)\ ,$$

where $u_{\text{ideal}}(T)$ is the velocity of second sound in an ideal Fermi gas of ^{3}He dissolved in ^{4}He.

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