Effect of Newtonian gravitational potential on a superfluid Josephson interferometer

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It is known that when a toroidal tube, with a Josephson junction, containing superfluid helium rotates about its axis, there should be a phase shift across the junction, due to the rotation. It is shown that there is an additional phase shift due to the variation of the gravitational potential in the superfluid.

I. INTRODUCTION

The effect of the gravitational field on the quantum interference of a massive particle was detected for the first time in neutron interferometry by means of an experiment proposed by Overhauser and Colella[1] and performed by Colella *et al.*[2] An experiment to detect the effect of Earth's rotation in neutron interferometry was also proposed by Anandan[3] and subsequently performed by Staudenmann *et al.*[4]

The purpose of this paper is to point out that a novel effect should occur in a superfluid-helium Josephson interferometer when gravity and rotation are *simultaneously* present. While there is plenty of experimental evidence to establish superfluid helium as a quantum fluid analogous to the Cooper pairs in a superconductor, there seems to be some difficulty in observing the analog of the superconducting Josephson effect[5] for superfluid helium. An interference effect in superfluid helium was observed by Gamota,[6] but it was never adequately confirmed. The difficulty in observing the Josephson effect in superfluid ^{4}He seems to be related to its small coherence length ($\lesssim 10$ Å) which makes it difficult to construct an adequate Josephson junction. For superfluid ^{3}He, on the other hand, the coherence length ~ 400 Å. This may be comparable to the coherence length of superconductors, which vary from a few hundred to several thousand angstroms. But there are experimental difficulties associated with the very low temperatures needed to produce superfluid ^{3}He. The recent observation[7] of persistent currents in superfluid ^{3}He-*B*, however, perhaps gives some encouragement to the expectation that the Josephson effect in superfluid ^{3}He may be observed soon.

II. SUPERFLUID HELIUM IN A GRAVITATIONAL FIELD

Superfluid helium, by which we mean the superfluid phase of either ^{4}He atoms or "Cooper pairs" of ^{3}He atoms, is an example of a quantum-mechanical system on a macroscopic scale. It is therefore tempting to investigate possible effects of the gravitational field on it. A general relativistic theory of the effect of the gravitational field on superfluid helium has accordingly been proposed previously.[8,9] We present this theory, with more details, in this section and, in the next section, apply it to the specific case of superfluid helium in a toroidal tube with a Josephson junction.

We let $\psi(\vec{r},t)=\alpha(\vec{r},t)e^{i\phi(\vec{r},t)}$, where α and ϕ are real scalar functions, denote the order parameter of superfluid helium. This ψ may be thought of as a wave function representing the quantum-mechanical state of each superfluid-^{4}He atom or a Cooper pair of ^{3}He atoms which is assumed to have mass m. Nonrelativistically, ψ may be assumed to satisfy the Gross-Pitaevskii equation[10]

$$i\hbar\frac{\partial\psi}{\partial t}=-\frac{\hbar^2}{2m}\nabla^2\psi+K\,|\,\psi\,|^2\psi\ ,$$

where the nonlinear term represents collective interactions, K being a constant. A general relativistic generalization of this equation is[8,9]

$$g^{\mu\nu}\nabla_\mu\nabla_\nu\psi+\frac{m^2c^2}{\hbar^2}\psi=-\frac{2mK}{\hbar^2}\,|\,\psi\,|^2\psi\ , \tag{2.1}$$

where $g_{\mu\nu}$ is the pseudo-Riemannian metric, of signature $(+,-,-,-)$, on space-time and ∇_μ denotes covariant derivative.[11] Now we define the superfluid "four velocity" v^μ by $v_\mu=-\lambda_c\partial_\mu\phi$, where $\lambda_c=\hbar/mc$ is the Compton wavelength. Then (2.1) is equivalent to

$$v^\mu v_\mu=1+f(\alpha)\ , \tag{2.2}$$

where $f(\alpha)=\lambda_c^2 g^{\mu\nu}(\nabla_\mu\nabla_\nu\alpha)/\alpha+2K\alpha^2/mc^2$, and

$$\nabla_\mu(\alpha^2v^\mu)=0\ , \tag{2.3}$$

which is the continuity equation. It is shown below that usually $|f(\alpha)|\ll 1$. Hence, from (2.2), v^μ is a timelike vector field. We shall also place the restriction that v^μ is in the forward null cone, everywhere, for the physically relevant solutions of (2.1). Owing to the smallness of $f(\alpha)$, v^μ cannot then undergo a transition to a vector in the backward null cone. This restriction, therefore, eliminates the "negative energy" solutions of (2.1). Otherwise, it would be necessary to treat ψ in (2.1) as an operator field, similar to the Klein-Gordon field when it is quantized.

It follows from (2.2) that $v^\nu\nabla_\nu v^\mu=\frac{1}{2}\nabla^\mu f(\alpha)$ This leads to the identification $f(\alpha)=2P/\rho c^2$, where P is the pressure and ρ is the density, when ρ is constant. Hence, (2.2) is a generalization of the eikonal equation in relativistic quantum mechanics, which takes into consideration "pressure effects" in the superfluid, contained in the $f(\alpha)$

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term. We let $\alpha=\tilde{\alpha}$ at some fixed point in the superfluid. Then

$$f(\alpha)=\lambda_c^2 g^{\mu\nu}(\nabla_\mu \nabla_\nu \alpha)/\alpha+(\lambda_c \alpha/\xi\tilde{\alpha})^2 \tag{2.4}$$

where $\xi=(\hbar^2/2m\tilde{\alpha}^2 K)^{1/2}$ is the coherence length. For superfluid ^{4}He, $\lambda_c=5.2\times10^{-17}$ m and $\xi\sim10^{-10}$ m, whereas for superfluid ^{3}He, $\lambda_c=3.5\times10^{-17}$ m and $\xi\sim10^{-8}$ m. Now, $g^{\mu\nu}\nabla_\mu\nabla_\nu\alpha\sim\alpha/L^2$, where L is the distance scale over which α varies appreciably. We assume $L\sim1$ m for the apparatus considered in this paper. Then the first term in (2.4) is $\lambda_c^2/L^2\sim10^{-33}$ for superfluid ^{4}He and ^{3}He. But the second term in (2.4) is $\sim\lambda_c^2/\xi^2\sim10^{-14}$ for superfluid ^{4}He and $\sim10^{-18}$ for superfluid ^{3}He. Therefore, the first term in (2.4) is negligible when compared to the second term, in the present case. Hence,

$$f(\alpha)\simeq(\lambda_c\alpha/\xi\tilde{\alpha})^2=2K\alpha^2/mc^2 . \tag{2.5}$$

Since the flow velocity of the superfluid relative to the container is much smaller than the velocity of light, all special relativistic corrections are negligible. We shall also assume that a coordinate system can be chosen such that the apparatus (container) is at rest in this coordinate system, $g_{ij}=-\delta_{ij}$ $(i,j=1,2,3)$, and $g_{\mu\nu}$ is independent of time. This can always be done if the size of the container is small compared to the radius of curvature of space-time and the container is rigid. Then $h_{00}=g_{00}-1=2V/c^2$, where V is the Newtonian potential for the gravitational and centrifugal fields in the frame of the apparatus, i.e., $g^i=-(\partial V/\partial x^i)$ is the acceleration relative to the frame of the container of a particle released from rest. If the apparatus is rotating relative to a local inertial frame, then this coordinate system can be chosen such that at any point on the apparatus $g_{0i}=-u_a^i/c$, where u_a^i is the velocity of the apparatus relative to this inertial frame. We shall neglect terms that are second order in the small quantities h_{00} and g_{0i}. Then (2.2) may be written as

$$(1-h_{00})v_0^2-\sum_i v_i^2+2\sum_i g_{0i}v_0v_i=1+f(\alpha) .$$

Therefore,

$$v_0=1+\tfrac{1}{2}f(\alpha)+\tfrac{1}{2}\sum_i(v_i-g_{0i})(v_i-g_{0i})+\tfrac{1}{2}h_{00} . \tag{2.6}$$

We now remove the rest mass energy by defining $\theta=\phi+mc^2t/\hbar$. Then

$$v_0=-\frac{\hbar}{mc^2}\frac{\partial\phi}{\partial t}=1-\frac{\hbar}{mc^2}\frac{\partial\theta}{\partial t} .$$

On defining $u^i=-cv_i+cg_{0i}=(\hbar/m)(\partial\theta/\partial x^i)+cg_{0i}$, which has the interpretation of the velocity of superfluid relative to the apparatus, and on using (2.5), we finally have

$$-\hbar\frac{\partial\theta}{\partial t}=K\alpha^2+mV+\tfrac{1}{2}m\vec{u}^2 . \tag{2.7}$$

Equation (2.7) is the nonrelativistic limit of (2.2). With the identification above that $\frac{1}{2}mc^2f(\alpha)\simeq K\alpha^2=mP/\rho$, where P is the pressure and ρ is the density, (2.7) can be recognized as the generalization of a known equation[12] to include the effects of rotation represented by g_{0i}. The results that will be obtained in the next section depend only on (2.3) and (2.7) of this section.

III. PHASE SHIFT DUE TO GRAVITY AND ROTATION IN A SUPERFLUID-HELIUM JOSEPHSON INTERFEROMETER

Consider two regions containing superfluid helium, separated by a Josephson junction, by which we mean a link that allows the superfluid from either side to tunnel through it. It was shown by considering the interference between the corresponding wave functions, for a particular geometry of the junction, that there should then be a Josephson current

$$I=I_0\sin\Delta\phi , \tag{3.1}$$

relative to the junction, where $\Delta\phi$ is the phase difference across the junction.[13] More generally, the current I would be a periodic function of $\Delta\phi$ with $I=0$ for $\Delta\phi=0$. Then (3.1) would be the lowest-order term in the Fourier transform of I. We shall then assume this to be the dominant term and neglect the higher-order Fourier components.

We consider now the special case of a hollow circular toroidal tube containing superfluid helium and a Josephson junction inside the tube. The thickness of the tube is assumed to be small compared to the radius R of the circle formed by the tube. Suppose that the tube rotates about the axis of symmetry which is normal to the plane of the tube with angular velocity $\vec{\Omega}$ relative to a local inertial frame. We assume also that the Newtonian gravitational potential V is different at different points on the tube and the situation is assumed to be stationary in the frame of the tube, i.e., $\vec{\Omega}$ and the value of V at any given point on the tube are assumed to be independent of time. Then the velocity of the superfluid relative to the tube would also be constant. For example, that tube may be in a vertical plane, at rest with respect to the earth, at the equator. Then $\vec{\Omega}$ is the earth's angular velocity and we may take $V=gz$, where g is the acceleration due to gravity and z is the height above the lowest point on the tube.

We shall assume that the effects of space-time curvature are negligible and that the apparatus is at rest in the coordinate system described in the last section. Moreover, the metric coefficients $g_{\mu\nu}$, in this coordinate system, are independent of time. On defining $h_{\mu\nu}=g_{\mu\nu}-\eta_{\mu\nu}$, where $\eta_{\mu\nu}=\text{diag}(1,-1,-1,-1)$, $h_{ij}=0$. Also terms that are second order in h_{00}, h_{0i} will be neglected. Equation (2.3) can be written as

$$\partial_\mu[(-||g||)^{1/2}\alpha^2v^\mu]=0 , \tag{3.2}$$

where $||g||\equiv\det(g_{\mu\nu})\simeq-g_{00}$. Therefore, for the present stationary situation, $\partial_i[(g_{00})^{1/2}\alpha^2v^i]=0$. By assuming v^i to be constant across each cross section of the tube, it follows that $(g_{00})^{1/2}\alpha^2v=$const, where $v=(\delta_{ij}v^iv^j)^{1/2}$. But $v^i=g^{i\mu}v_\mu\simeq g^{i0}-v_i=g_{i0}-v_i=u^i/c$, where we have used (2.6) and neglected terms that are second order is small quantities so that $g^{i0}v_0\simeq g^{i0}$. Therefore, $v=u/c$, where $u=(\delta_{ij}u^iu^j)^{1/2}$ is the speed of the superfluid relative to the tube, as measured by a local observer. We choose the coordinate system so that $g_{00}=1+2gz/c^2$, where z is the height from the lowest point 0 of the tube and g is the acceleration due to gravity. If $u=\tilde{u}$ and $\alpha=\tilde{\alpha}$ at 0, then

$$\left[1+\frac{gz}{c^2}\right]\alpha^2 u=\tilde{\alpha}^2\tilde{u} \; . \tag{3.3}$$

Equation (3.3) is a consequence of the general relativistic time dilation in a gravitational field and the continuity equation. Since the current density relative to the tube is proportional to $\alpha^2 u$, the continuity equation implies that $\alpha^2 u$ is constant along the tube in the absence of the gravitational field. This condition is modified to (3.3) in the presence of the gravitational field, because clocks at different heights run at different rates, according to general relativity. But as will be seen shortly, this effect is negligible.

We now consider a stationary solution of (2.7) for which $\partial\theta/\partial t=-\omega_0$, where ω_0 is a constant. Then from (2.7) and (3.3),

$$\hbar\omega_0=\frac{K\tilde{\alpha}^2\tilde{u}}{(1+gz/c^2)u}+mgz+\tfrac{1}{2}mu^2 \; . \tag{3.4}$$

It is clear from (3.4) that u is a function of the height z and $\tilde{u}$. Indeed, $\delta u(z)\equiv u-\tilde{u}$ satisfies, from (3.4), to lowest order,

$$\left[1-\frac{K\tilde{\alpha}^2}{m\tilde{u}^2}\right]\tilde{u}\,\delta u+\left[1-\frac{K\tilde{\alpha}^2}{mc^2}\right]gz=0 \; . \tag{3.5}$$

But $\tilde{u}$ depends on the angular velocity of the interferometer relative to a local inertial frame, on account of (3.1). We assume, for the present that the Josephson junction is at the lowest point 0 of the tube. It was shown,[13] using (3.1), that in the absence of hysteresis the velocity of the superfluid relative to the Josephson junction should not exceed $v_q\equiv\hbar/2\pi Rm$, where R is the radius of the toroidal tube. Therefore, $\tilde{u}\lesssim v_q$. Hence

$$K\tilde{\alpha}^2/m\tilde{u}^2=\tfrac{1}{2}(\lambda_c/\xi)^2(c/\tilde{u})^2\gtrsim\tfrac{1}{2}(\lambda_c/\xi)^2(c/v_q)^2 \; .$$

This is $\sim 10^{16}$ for superfluid ^{4}He and $\sim 10^{12}$ for superfluid ^{3}He. But $K\tilde{\alpha}^2/mc^2=\frac{1}{2}(\lambda_c/\xi)^2$ is $\sim 10^{-14}$ for superfluid ^{4}He and $\sim 10^{-18}$ for superfluid ^{3}He. It follows that in (3.5) $K\tilde{\alpha}^2/m\tilde{u}^2\gg 1$, whereas $K\tilde{\alpha}^2/mc^2\ll 1$. Hence, from (3.5),

$$\delta u=\frac{mgz}{K\tilde{\alpha}^2}\tilde{u} \; . \tag{3.6}$$

The phase difference across the Josephson junction is

$$\begin{aligned}\Delta\phi&=\int_c\frac{\partial\phi}{\partial x^i}dx^i=\frac{mc}{\hbar}\int_C v_i dx^i\\&=\frac{mc}{\hbar}\int_C g_{0i}dx^i-\frac{m}{\hbar}\int_c \vec{u}\cdot d\vec{x}\\&=-\frac{m}{\hbar}\int_C(\vec{u}_a+\vec{u})\cdot d\vec{r} \; ,\end{aligned}$$

where $\vec{u}_a$ is the velocity of the apparatus relative to a local inertial frame with respect to which it is rotating, and C is a curve that joins the opposite sides of the Josephson junction by the longer route around the tube through the superfluid.[14] Suppose that the length of the junction is much smaller than R. Then, using (3.6) to a good approximation,

$$\Delta\phi\simeq\frac{m}{\hbar}\oint_{C'}\vec{u}_a\cdot d\vec{x}+\frac{m\tilde{u}2\pi R}{\hbar}+\frac{m^2 g\tilde{u}}{\hbar K\tilde{\alpha}^2}\oint_{C'}\frac{z\vec{u}}{u}\cdot d\vec{x} \; ,$$

where C' is a closed curve, through the tube. If Ω_n is the component of the angular velocity of the apparatus normal to the torus it follows that

$$\Delta\phi\simeq\frac{2\pi m\Omega_n R^2}{\hbar}+\frac{m\tilde{u}2\pi R}{\hbar}\left[1+\frac{mgR}{K\tilde{\alpha}^2}\right] . \tag{3.7}$$

Also, (3.1) can be written as[13,15]

$$\tilde{u}=v_0\sin(\Delta\phi) \; , \tag{3.8}$$

where the constant $v_0=I_0/\rho_0 A_0$, with ρ_0 and A_0 being the density of the superfluid and the area of cross section of the tube just outside the Josephson junction at 0. The coupled equations (3.7) and (3.8) can be solved for $\Delta\phi$ and $\tilde{u}$.

It is easy to obtain this solution for a small perturbation. Suppose that Ω_n is changed by $\delta\Omega_n$. For example, the interferometer may be turned about a vertical axis so as to change Ω_n, the component of the earth's angular velocity normal to the plane of the interferometer. Or $\delta\Omega_n$ may be the change in Ω_n due to the local precession of inertial frames due to the Lense-Thirring field of a rotating body.[13] Then, from (3.7), the change in the phase shift $\delta\phi$ satisfies

$$\delta\phi=\frac{2\pi mR^2\delta\Omega_n}{\hbar}+\frac{2\pi mR\,\delta\tilde{u}}{\hbar}\left[1+\frac{mgR}{K\tilde{\alpha}^2}\right] . \tag{3.9}$$

But from (3.8), assuming $\delta\phi\ll 2\pi$,

$$\delta\tilde{u}=\tilde{u}\cot(\Delta\phi)\delta\phi \; . \tag{3.10}$$

On substituting (3.10) into (3.9), the change in the phase shift due to the change $\delta\Omega_n$ in Ω_n is

$$\delta\phi=\left[1-\frac{2\pi Rm\tilde{u}\cot(\Delta\phi)}{\hbar}\left[1+\frac{mgR}{K\tilde{\alpha}^2}\right]\right]^{-1}\frac{2\pi R^2 m\,\delta\Omega_n}{\hbar} \; . \tag{3.11}$$

Equation (3.11), for the special case when $g=0$, has been obtained previously.[13,15]

Alternatively, Ω_n in (3.7) may be kept constant, but the effect of the gravitational field may be varied. This can be accomplished, for instance, by keeping the tube in the *same vertical plane* and turning it about its axis so that the Josephson junction is at a variable height z_J from the lowest point. Then, on using (3.6), the velocity at the Josephson junction relative to the junction is $\tilde{u}$ $[1+(mgz_J)/(K\tilde{\alpha}^2)]$. Hence (3.8) must be replaced by

$$\tilde{u}\left[1+\frac{mgz_J}{K\tilde{\alpha}^2}\right]=v_0\sin(\Delta\phi) \; . \tag{3.12}$$

On substituting for $\tilde{u}$ in (3.7) from (3.12),

$$\Delta\phi = \frac{2\pi R^2 m\Omega_n}{\hbar} + \frac{2\pi R m}{\hbar}\left[1+\frac{mg(R-z_J)}{K\tilde{\alpha}^2}\right] v_0 \sin(\Delta\phi) , \quad (3.13)$$

neglecting $O(g^2)$ terms. Therefore, $\Delta\phi$ can be obtained by solving (3.13) in this case. Hence, the change $\delta\phi$ in the phase shift as $R-z_J$ is varied from zero satisfies

$$\delta\phi = \frac{2\pi R m^2 g(R-z_J)}{\hbar K\tilde{\alpha}^2} v_0 \sin(\Delta\phi) + \frac{2\pi R m}{\hbar} v_0 \cos(\Delta\phi)\, \delta\phi .$$

Hence,

$$\delta\phi = \left[1 - \frac{2\pi R m v_0}{\hbar}\cos(\Delta\phi)\right]^{-1} \times \frac{\xi^2 m^3 g 4\pi R(R-z_J) v_0 \sin(\Delta\phi)}{\hbar^3} , \quad (3.14)$$

where $\xi = \hbar/(2m\tilde{\alpha}^2 K)^{1/2}$ is the coherence length. Equation (3.14) can be tested by measuring $\delta\phi$ for different values of z_J, the height of the Josephson junction. The experiment can then be repeated with the interferometer in different planes that are inclined at a variable angle θ to the vertical. Then g in (3.14) must be replaced by $g\cos\theta$, the component of the acceleration due to gravity in this plane, to obtain the phase shift.

IV. DISCUSSION AND CONCLUSIONS

We have obtained, above, a new effect in superfluid Josephson interferometry that arises due to rotation and the variation of the Newtonian gravitational potential. This is unlike the effects discussed previously[13,15,16] for a superfluid-helium interferometer, which can be understood as being due to the rotation of the interferometer relative to local inertial frames. This new effect is also unlike the Overhauser-Colella effect[1] or the effect of rotation on neutron interference[3,17] which, in the nonrelativistic limit,[8] can be regarded as two separate unrelated effects.

This difference is basically due to the fact that the neutron beam is freely falling in between reflections. Superfluid helium in a toroidal tube, on the other hand, is not freely falling and is being supported by its own pressure. So, the nonlinear term in (2.1) plays an essential role in obtaining the present effect. Since we expect the pressure to decrease with height, α also would be expected to decrease with height. Then from the continuity equation, the velocity u *increases* with height as seen from (3.6). This is another fundamental difference between the present effect and the Overhauser-Colella effect[1] due to gravity in neutron interferometry. In the later case, the velocity of the neutron beam *decreases* with height, because of conservation of energy, so that the phase shift due to gravity between the upper and lower beams is *negative*. But in the present case, the phase shift due to gravity in the direction of flow of superfluid, given by (3.14), is *positive* for small $\Delta\phi$.

To obtain an order of magnitude estimate of the phase shift (3.14),

$$\delta\phi \sim m^3 g \xi^2 v_0 R^2/\hbar^3 \sim m^2 g \xi^2 R/\hbar^2 ,$$

since we must have $v_0 \lesssim \hbar/m 2\pi R$ to avoid hysteresis.[13,16] If $R \sim 1$ m, then $\delta\phi \sim 10^{-1}$ rad for superfluid He^3. A phase shift of this order of magnitude would be large and easily measurable in neutron interferometry. But for superfluid-helium Josephson interferometry, such a fractional phase shift is at present very difficult to measure. Hence, it appears that the confirmation of the effect proposed in this paper has to await further advances in the detection of the Josephson effect in superfluid helium. But if this experiment is performed, then it would be the only experiment, apart from the experiment proposed by Overhauser and Colella,[2,3] which would test the effect of gravity on a quantum-mechanical system. It would also test the nonlinear term in the equation that ψ is assumed to satisfy in this paper. Since (2.7) can be derived from the nonrelativistic Gross-Pitaevskii equation modified by adding the term $mV\psi$, the main results of this paper can be obtained from the latter equation and (2.1) is needed only to compute the relativistic corrections.

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where the path P must be a world line of the superfluid. But it is not possible to choose such paths to form the closed curve to define the former integral in a superconducting or superfluid interferometer.

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