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Anisotropy in weakly localized electronic transport: A parameter-free test of the scaling theory of localization

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We have measured the negative magnetoresistance of an anisotropic two-dimensional electron gas at low temperatures. We report the first observation of the predicted anisotropy in the strength of the logarithmic terms in the weak localization theory for a system with effective mass and scattering time anisotropy. The results provide the first parameter-free test of the scaling theory of localization.

Experiments on disordered two-dimensional (2D) electronic systems such as metallic films[1,2] and silicon inversion layers[3,4] have shown convincing quantitative support for the scaling theory of localization by Abrahams, Anderson, Ramakrishnan, and Licciardello.[5] These results indicate that there is no true metallic behavior in two dimensions no matter how weak the disorder. These effects manifest themselves as a logarithmic dependence on length which in an isotropic system yields a temperature-dependent conductivity of the form

$$\sigma(T) \sim \sigma(T_0) - (\alpha P)\frac{e^2}{2\pi^2\hbar}\ln(T_0/T) \quad , \qquad (1)$$

where the inelastic scattering length has a power-law temperature dependence of $l_{\text{in}} \propto T^{-P}$. In this Rapid Communication we report the first measurements of the role of effective mass and scattering time anisotropy in weakly localized transport in a two-dimensional electron gas. Our experiments consist of temperature-dependent resistance and low- and high-field magnetoresistance measurements on (110) Si metal-oxide-semiconductor field-effect transistors (MOSFET's) which have an anisotropic effective mass and elastic scattering time. We report the first observation of an anisotropy of the α parameter as well as an anisotropic Coulomb interaction parameter F. Our results are in quantitative agreement with the calculation of Wölfle and Bhatt. In the theory of Wölfle and Bhatt the ratio of the strength of the anisotropic logarithmic terms is given by σ_x/σ_y independent of all other parameters of the system. Our measurement of $\alpha_{[100]}/\alpha_{[110]}$ therefore represents the first *parameter-free test* of the scaling theory of localization.

Our samples were N channel MOSFET's fabricated on a (110) surface of p-type silicon with a mobility of ~ 1000 cm^2/V sec at 4.2 K. The measurements were performed in a He^3-He^4 dilution refrigerator at temperatures from 10 mK to 2 K in parallel and perpendicular magnetic fields to 12 kG. Our samples had a van der Pauw geometry with the current and voltage leads oriented along the $[1\bar{1}0]$ and [100] directions (see Fig. 1). The resistances were measured using a four-terminal bridge and the anisotropic resistances per square, calculated using the algorithm of Montgomery and Logan, Rice, and Wick.[6]

At low temperatures the samples had an anisotropic conductivity ratio less than the ratio of effective masses. For example, for the data shown in Fig. 1 $R_{\square}^{[1\bar{1}0]} = 1903\ \Omega/\square$ and $R_{\square}^{[100]} = 1041\ \Omega/\square$. Because $m_{[1\bar{1}0]} = 0.585$ and $m_{[100]} = 0.190$ we conclude that on the (110) surfaces there exists anisotropic elastic scattering as has been shown by previous workers.[7]

In Fig. 1. we show typical low-field perpendicular magnetoresistance curves for the two orthogonal directions ([100] and $[1\bar{1}0]$) of the silicon MOSFET.

Fukuyama,[8] Hikami, Larkin, and Nagaoka,[9] and Lee and Ramakrishnan[10] have studied the effects of a magnetic field on transport in the weakly localized regime. For a perpendicular magnetic field these workers have shown that the localization effects are diminished when the size of the first Landau orbit is comparable to the inelastic diffusion length $(\frac{1}{2}l_{\text{in}}l_{\text{el}})^{1/2}$. Thus in the weakly localized regime one obtains a negative magnetoresistance which depends on the magnitude of the perpendicular component of field. Lee and Ramakrishnan[10] show that the change in conductivity with field at a fixed temperature should be given by

$$\Delta\sigma(H,T) = \frac{\alpha e^2}{2\pi^2\hbar}\left[\psi\left(\frac{1}{2}+\frac{\hbar c}{2eHl_{\text{el}}l_{\text{in}}}\right) - \psi\left(\frac{1}{2}+\frac{\hbar c}{2eHl_{\text{el}}l_{\text{el}}}\right) + \ln\frac{l_{\text{in}}}{l_{\text{el}}}\right] \quad , \qquad (2)$$

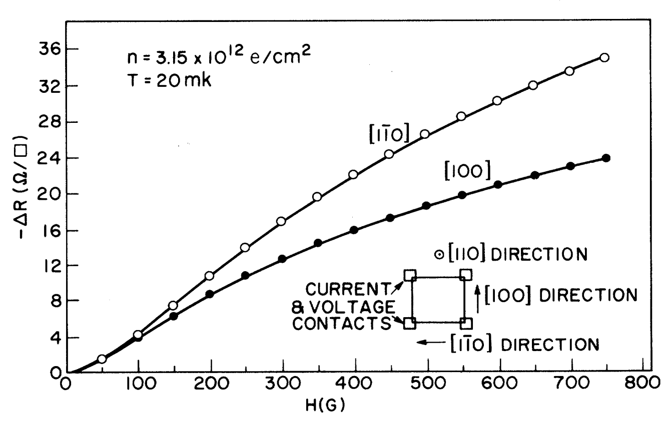

FIG. 1. Shown is the change in resistance with field for the two orientations on the (110) surface of a Si MOSFET. The solid line is a fit to the data.

where l_{in} and l_{el} are the elastic and inelastic scattering lengths.

In our analysis l_{el} is calculated from the value of the zero-field conductivity at 4.2 K and then l_{in} and α are obtained through a fit to data such as is shown in Fig. 1. For the data shown we obtain $\alpha_{[1\bar{1}0]}=0.693$ and $\alpha_{[100]}=1.256$. In Fig. 2 we show the results of fits to different temperatures for α and in Fig. 3 we show the inelastic scattering lengths obtained as a function of temperature. In measurements on the isotropic (100) and (111) surfaces we obtained $\alpha \cong 1.0$ and an inelastic scattering time varying as $1/T$ implying $P=1$.[3] This was consistent with our measurements of the temperature dependence of the conductivity which showed $\alpha P=1$ and indicated electron-electron scattering as the dominant inelastic process. From the data in Fig. 3 we see $P_{[100]}=P_{[1\bar{1}0]}=1$. We have also measured the temperature dependence of the zero-field conductivity and obtain $(\Delta R/R)_{[100]}=4.55\%$/decade and $(\Delta R/R)_{[1\bar{1}0]}=4.45\%$/decade for the same density. As for the isotropic case this is consistent with our determination of $\alpha_{[100]}$ and $\alpha_{[1\bar{1}0]}$ from the magnetoresistance fits. In Fig. 4 we show the dependence of $\alpha_{[110]}$ and $\alpha_{[100]}$ on electron density. Our measurements indicate that as for the isotropic case α is independent of both temperature and electron density.

Wölfle and Bhatt[11] extending the previous calculation of Ref. 12 have calculated the effect of mass anisotropy on the low-temperature conductivity of disordered two-dimensional systems. They have considered a model with an anisotropic mass and an anisotropic elastic scattering time of relevance to the present experiment. In their results they obtain a logarithmic divergence of $\sigma(T)$ analogous to (1) indicating localization of all states at $T=0$ in two dimensions. However they find that the anisotropy changes the universal coefficient $(e^2/2\pi^2\hbar)$ in Eq. (1). They obtain

$$\sigma(T) \sim \sigma(T_0) - \frac{\sigma_{\mu\mu}}{\bar{\sigma}}(\alpha P)\frac{e^2}{2\pi^2\hbar}\ln(T_0/T) \; , \qquad (3)$$

where $\bar{\sigma}=(\sigma_{xx}\sigma_{yy})^{1/2}$.

Therefore they find that the effects of anisotropy can be completely absorbed into an anisotropic diffusion coefficient. This result can be compared to our experimental observations. By taking the ratio of $(\alpha_{[1\bar{1}0]}/\alpha_{[100]})_{\rm expt}$ and

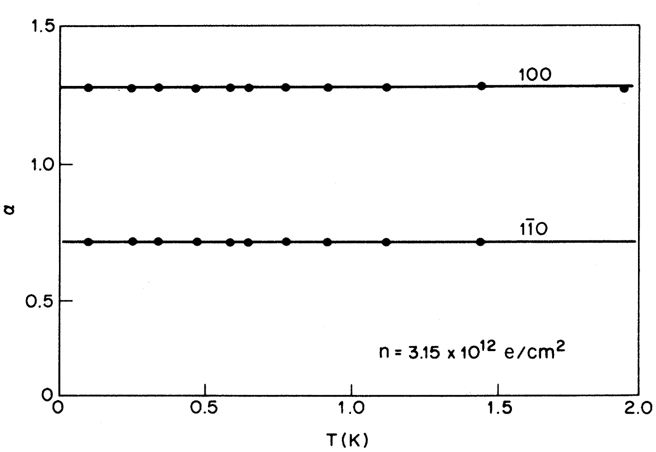

FIG. 2. The α parameters for the $[1\bar{1}0]$ and [100] directions are shown as a function of temperature.

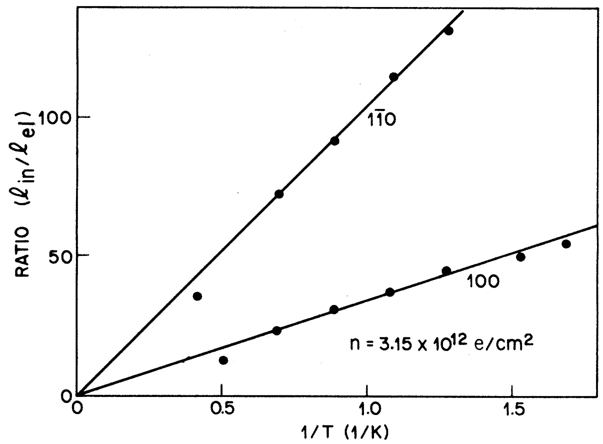

FIG. 3. The ratio l_{in}/l_{el} is shown as a function of temperature.

comparing it to $(\alpha_{[1\bar{1}0]}/\alpha_{[100]})_{\rm theory}=\sigma_{[100]}/\sigma_{[1\bar{1}0]}$ we can obtain a parameter-free test of the scaling theory used to calculate (1) and (3).

As an example for the data shown in Fig. 1 we obtained $\alpha_{[100]}=1.256$ and $(\alpha_{[1\bar{1}0]}=0.693$. Using $R_{\Box}^{[100]}=1041\,\Omega/\Box$ and $R_{\Box}^{[1\bar{1}0]}=1903\,\Omega/\Box$, and the *assumption* that $\alpha=1.0$ we obtain $(\alpha_{[100]})_{\rm theory}=1.352$ and $(\alpha_{[1\bar{1}0]})_{\rm theory}=0.740$ to be compared with $(\alpha_{[100]})_{\rm expt}=1.256$ and $(\alpha_{[1\bar{1}0]})_{\rm expt}=0.693$ in good quantitative agreement. However, the ratio $(\alpha_{[100]}/\alpha_{[1\bar{1}0]})_{\rm theory}$ contains no assumptions concerning α and one expects $(\alpha_{[100]}/\alpha_{[1\bar{1}0]})_{\rm theory}=1.828$ in excellent

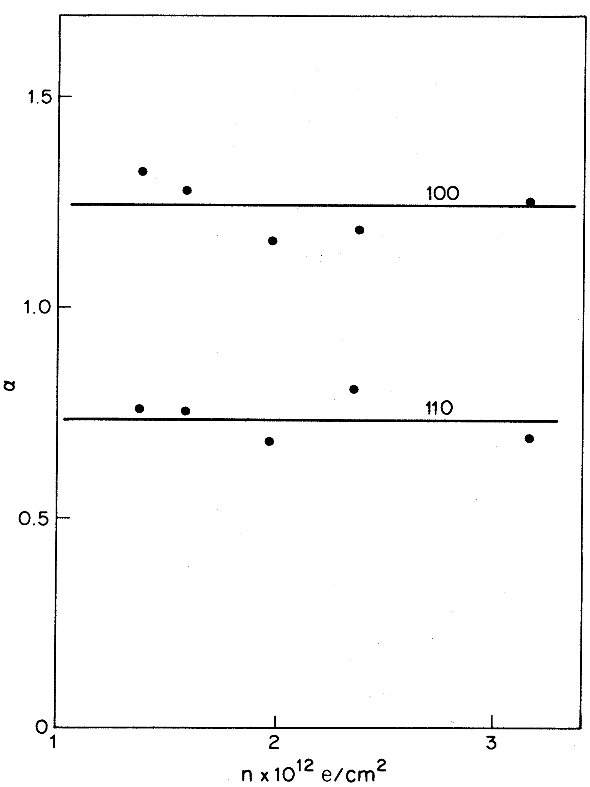

FIG. 4. The α parameters are shown as a function of electron density.

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quantitative agreement with the experimental value of $(\alpha_{[100]}/\alpha_{[1\bar{1}0]})_{\text{expt}} = 1.812$. Our measurement, therefore, represents the first quantitative parameter-free test of the scaling theory of localization and we find excellent numerical agreement. Therefore the theory correctly obtains both the magnitudes of $\alpha_{[100]}$ and $\alpha_{[1\bar{1}0]}$ as well as their ratios.

Finally we have explored the impact of anisotropy on the strength of the Coulomb interaction terms. Altshuler, Aronov, and Lee[13] have shown that these many-body effects are also important. Lee and Ramakrishnan[10] have shown that these effects give a high-field positive contribution to the magnetoresistance of the form

$$\Delta\sigma(H,T) = \frac{-e^2}{2\pi^2\hbar}\frac{F}{2}G(h) \ ,$$

where $h = \mu g H/kT$ and

$$G(h) = 0.084h^2 \quad \text{for } h >> 1 \ ,$$

$$G(h) = \ln(h/1.3) \quad \text{for } h << 1 \ .$$

We have measured the high-field parallel magnetoresistance and have observed an anisotropy of the F parameter. For example, at a density of $1.18\times10^{12}\ e/\text{cm}^2$ and a temperature of 1.0 K we find $F_{[100]} = 9.92$ and $F_{[110]} = 6.24$. Our measured value of $F_{[100]}/F_{[1\bar{1}0]}$ is 1.60 which is also close to the conductivity ratio.

In conclusion, we have reported the first observation of the predicted anisotropy in the strength of the logarithmic terms in the weak localization theory for a system with effective mass and scattering time anisotropy. The observations are in good quantitative agreement with the theory of Wölfle and Bhatt and provides the first parameter-free test of the scaling theory of localization.

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