Electron-phonon interactions and the breakdown of the dissipationless quantum Hall effect

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The quantum Hall effect is manifested by plateaus in the Hall conductivity at which the current flows without loss. Recently it has been observed that as the current is increased to a critical value, corresponding to a carrier drift velocity of the order of the speed of sound, there is a dramatic onset of dissipation. We investigate the role in the breakdown of phonon-assisted transitions between Landau levels and calculate the steady-state power absorption. As the drift velocity of the carriers is increased there is a sudden onset of dissipation, and an upper limit for the critical current is obtained.

The normal quantum Hall effect is manifested by plateaus in the Hall resistivity ρ_H at which the current carried by the two-dimensional (2D) electron gas appears to be dissipationless.[1-3] If for a fixed filling factor ν, which is the ratio of the number of electrons present to the number of states within the lowest Landau level, the current is increased, then there is a critical current density j_{cr} at which the dissipationless transport suddenly breaks down and the longitudinal resistivity increases several orders of magnitude.[4-7] This transition does not appear to be related to temperature fluctuations, since the thermal conductivities of the GaAs heterojunctions and of the liquid-helium bath in which the heterojunction is immersed in the experiments are sufficiently large for any heat produced in the inversion layers to be rapidly removed. The question thus arises of the microscopic mechanism by which this transition occurs.

At the onset of dissipation, the Hall voltage V_H is around 2.5 V, so that eV_H is much larger than the cyclotron energy $\hbar\omega_c$. Thus, given a mechanism that would allow transitions between Landau levels, electrons can always make transitions to states of lower energy. However, the spatial translation of the electrons involves a concomitant change in wave number, and a source of crystal momentum is necessary for this to occur. This may be provided by impurities or phonons. The fact that at the critical current the drift velocity $v_d \equiv Ec/B$, with E representing the Hall field, is near[8] the speed of sound v_s suggests that phonon Cerenkov radiation may be important.[9] Related phenomena have both been observed in bulk GaAs (Ref. 10 and references therein) and modeled theoretically.[11] On the other hand, j_{cr} is found to depend on the location of the chemical potential within the gap between Landau levels, with j_{cr} being largest at the center of the Hall step. This suggests that impurities and localized states may play an important role. The purpose of this paper is to take the first step toward an understanding of this phenomenon by investigating the simplest possible model, which involves only phonon emission and neglects disorder and electron-electron interactions.

Our model is a 2D slab of dimensions L_x and L_y. The magnetic field is $\vec{B}=B\hat{z}$ and an applied electric field $\vec{E}=E\hat{x}$ forces a current to flow in the y direction. We write the electron states as

$$\psi_{k,n}=L_y^{-1/2}e^{iky}\phi_n(x-x_k)\ , \tag{1}$$

where $k=2\pi m/L_y$ with $m=0,\pm 1,\ldots,$ where $\phi_n(\xi)$ is the nth harmonic-oscillator function and where $x_k = l_B^2 k + v_d/\omega_c$. Here we have used the definitions $l_B^2=\hbar c/eB$ and $\omega_c=eB/m^*c$. The energies are then

$$\epsilon_{k,n}=(n+\tfrac{1}{2})\hbar\omega_c+\tfrac{1}{2}m^*v_d^2-\hbar v_d k\ . \tag{2}$$

We confine our attention to acoustic phonons, since the dissipation starts when v_d is around the speed of sound, and the optical phonons have energies too high to be relevant. Since GaAs is piezoelectric, we must take into account both piezoelectric phonon emission[8] and deformation-potential interactions. We will only consider transitions between the two lowest Landau levels.

In treating the deformation potential, we approximate the phonon system in the $2D$ junction by that of a 2D Bravais lattice of identical ions.[12] Since the oscillator wave functions have a width of the order of 10 nm, which is much larger than atomic dimensions, it is permissible to use a deformation-potential formalism in which the interaction between an ion and an electron is $V_0\delta(\vec{r})$, with V_0 representing a constant and $\delta(\vec{r})$ the 2D Dirac δ function. The dynamic part of the deformation-potential interaction then becomes

$$\hat{H}^{(1)}=\sum_{k,\vec{q}} N_0N_1\frac{4V_0 l_B{}^3}{(\sqrt{2}+2)L_y a}(q_y-iq_x)i\kappa\left[\frac{2\hbar\pi}{\rho_0 a\omega_{\vec{q}}}\right]^{1/2} \times\exp\left[iq_x x_k-\frac{l_B^2}{2}[q_y^2-\tfrac{1}{2}(q_y-iq_x)^2]\right] \times c^{\dagger}_{k'+q_y,n}c_{k',n'}(a_{\vec{q}}+a^{\dagger}_{-\vec{q}})s_y\ , \tag{3}$$

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where a is the GaAs lattice constant, ρ_0 the density of GaAs, V the area of the slab, $\vec{s}$ the phonon polarization vector, N_i the normalization constant for $\phi_i(\xi)$ and $\kappa \equiv k'-k$. The operators $a^\dagger$ and $c^\dagger$ create phonons and electrons, respectively.

Following the experimental geometry of Refs. 4 and 7 we take the current to flow in the [110] direction, which maximizes the strongly anisotropic piezoelectric coupling. We can then write for the piezoelectric interaction[13]

$$\hat{H}^{(2)}=\sum_{k,\vec{q}} N_0 N_1 \frac{2\pi e e_{14}}{\epsilon_0} l_B^3 \left[\frac{\hbar\pi}{\rho_0 a \omega_{\vec{q}}}\right]^{1/2} \times (q_y^2+2q_xq_y-q_x^2)\frac{q_y-iq_x}{q^2} \times \exp\left[iq_x x_k-\frac{l_B^2}{2}[q_y^2-\tfrac{1}{2}(q_y-iq_x)^2]\right] \times c^\dagger_{k'+q_y,n}c_{k',n'}(a_{\vec{q}}+a^\dagger_{-\vec{q}})s_z \; . \tag{4}$$

Here, e_{14} is the piezoelectric constant and ϵ_0 is the static dielectric constant.

An electron in the lowest Landau level may make a transition to the $n=1$ level by emission or absorption of a phonon, as illustrated in Fig. 1. The probability of a transition from $n=0$ to 1 with absorption of a longitudinal phonon will be denoted as $W(q_1,l)f(k,0)[1-f(k+q_1,1)]n_{q_1}(l)$, where f and n are the average electron and phonon occupancies, respectively. Similar expressions apply for other transitions and for transverse (t)

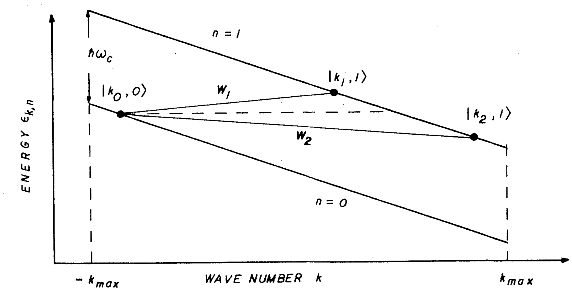

FIG. 1. Two lower Landau levels of a noninteracting electron system in crossed electric and magnetic fields are shown. The slope of the Landau levels is $\hbar v_d$. The state $|k_0,0\rangle$ is connected to the states $|k_1,1\rangle$ and $|k_2,1\rangle$ with transition rates W_1 and W_2 by phonon emission or absorption.

phonons. Energy conservation demands that

$$q_x^2=\left[\frac{v_d}{v_s}q_y-\frac{\omega_c}{v_s}\right]^2-q_y^2 \tag{5}$$

for all transitions and polarizations. By inspecting Eq. (5) and Fig. 1 we see that the upper limit for q_{1l} is $\omega_c/(v_d+v_{sl})\equiv q_{ul}$ [while $\omega_c/(v_d+v_{st})\equiv q_{ut}$] and that the lower limit for q_{2t} is $\omega_c/(v_d-v_{st})\equiv q_{Lt}$ [while $\omega_c/(v_d-v_{sl})\equiv q_{Ll}$], where we have dropped the subscript y on the phonon wave vectors. We can now, in the Born approximation, write the transition rates $W_1(q_1)\equiv W(q_1,l)+W(q_1,t)$ with

$$W_1(q_1)=\frac{2L_x l_B^2}{a\rho_0 V\hbar}\left\{\left[\frac{\sqrt{2}\pi e e_{14}}{\epsilon_0 v_{st}}\right]^2\left[\frac{\omega_c}{v_{st}}-\frac{v_d}{v_{st}}q_1\right]\exp\left[-\frac{l_B^2}{2}\left[\frac{\omega_c}{v_{st}}-\frac{v_d}{v_{st}}q_1\right]^2\right] + \left[\frac{4V_0}{(\sqrt{2}+2)aL_y v_{sl}}\right]^2\left[\frac{\omega_c}{v_{sl}}-\frac{v_d}{v_{sl}}q_1\right]q_1^2\exp\left[-\frac{l_B^2}{2}\left[\frac{\omega_c}{v_{sl}}-\frac{v_d}{v_{sl}}q_1\right]^2\right]\right\} \tag{6}$$

and $W_2(q_2)\equiv W(q_2,l)+W(q_2,t)$ given by a similar equation upon interchanging $\omega_c/v_s-v_dq_1/v_s$ with $v_dq_2/v_s-\omega_c/v_s$, where $q_2>0$. We have used the Debye model, for which $\omega_{\vec{q}}=v_s|\vec{q}|$. In the integrations over q that follow, we must ensure that $|q|$ is sufficiently small so that the electrons will not cross the boundary of the system in the x direction. On the other hand, the exponentials in the transition rates will cause the integrals to be rapidly attenuated for increasing $|q|$, and we can extend the lower limit of q_1 to $-k_{\max}$ and the upper limit of q_2 to $+k_{\max}$, where $k_{\max}=L_x/2l_B^2$, independent of the k value of the electron making the transition. The cutoff for large $|q|$ also eliminates the need to consider Umklapp processes.

To calculate the dissipated power we study the rate equations in the steady state. Since the temperature at which the experiments are done is very low we can neglect thermally excited phonons, and we find for the rate of change of $f(k,0)$,

$$\frac{df(k,0)}{dt}=\frac{L_y}{2\pi}\left[\int_{-k_{\max}}^{q_{ut}} f(k+q_1,1)[1-f(k,0)][1+n_{q_1}(t)]W(q_1,t)dq_1 + \int_{q_{Lt}}^{k_{\max}} f(k+q_2,1)[1-f(k,0)]n_{q_2}(t)W(q_2,t)dq_2 - \int_{-k_{\max}}^{q_{ut}} f(k,0)[1-f(k+q_1,1)]n_{q_1}(t)W(q_1,t)dq_1 - \int_{q_{Lt}}^{k_{\max}} f(k,0)[1-f(k+q_2,1)][1+n_{q_2}(t)]W(q_2,t)dq_2+(t\to l)\right] . \tag{7}$$

For the phonon occupancies we have

$$\frac{dn_{q_1}}{dt}=\frac{L_y}{2\pi}\left[\int_{-k_{\max}}^{k_{\max}} f(k+q_1,1)[1-f(k,0)](1+n_{q_1})W(q_1)dk - \int_{-k_{\max}}^{k_{\max}} f(k,0)[1-f(k+q_1,1)]n_{q_1}W(q_1)dk\right]-\frac{n_{q_1}}{\tau_1} \quad (8)$$

for either polarization, and similar equations for the q_{2t} and q_{2l} modes. Here, τ_i is the lifetime of the ith mode.

If we start with the $n=0$ Landau level precisely filled, then in the steady state the electron and phonon occupancies are constant in time, and $f(k,1)\equiv f_1$ and $f(k,0)\equiv f_0$ for all k. The integrals in (8) over k must then equal N, the number of electron states in a Landau level. The power P dissipated is then

$$P=\frac{L_y}{2\pi}\left[\int_{-k_{\max}}^{q_{ut}}\frac{n_{q_1}(t)\epsilon(q_1)}{\tau_t(q_1)}dq_1 + \int_{q_{Lt}}^{k_{\max}}\frac{n_{q_2}(t)\epsilon(q_2)}{\tau_t(q_2)}dq_2+(t\to l)\right], \quad (9)$$

where $\epsilon(\vec{q})=\hbar v_s|\vec{q}|$ for all modes.

The set of equations (7)–(9) was solved for the parameters given in Table I. The value for M is the average atomic mass of GaAs.

The lifetime was taken to be the same constant for all phonon modes since at these low temperatures and for the ultraclean GaAs heterojunctions used in the experiments, the limiting factor for the phonon lifetime should be boundary scattering.[14] The transit time of the phonons then gives a lifetime of 1 μs, which is the same as the time scale for fluctuations in the dissipative voltage V_x noted by Cage *et al.*[4] This assumption is not critical to the results we obtain: This was verified by repeating the calculation with $\tau=10^{-11}$ s, which surely represents a lower limit for the phonon lifetime, and noting that this only slightly increased the threshold current. The magnetic fields used were $B=5.5$ and 4.5 T, to facilitate comparison with Refs. 4 and 5.

The results of the calculations are presented in Fig. 2. We see that the model used here is in qualitative agreement with experiment in that the onset of dissipation starts dramatically, growing as $\exp[-B/(v_d-v_s)^2]$. The drift velocity is proportional to the Hall field E, and thus this expression differs from that predicted for Zener breakdown, in which dissipation grows as $\exp(-E_0/E)$. The mechanisms are, of course, totally different as in the present case the electric field alone cannot cause the electron tunneling, and the presence of phonons or impurities is essential. The value j_{cr} of the threshold current scales with the magnetic field as $B^{1/2}$. The calculations were terminated as f_0 approached 0.5 asymptotically and numerical instabilities arose.

A discrepancy between experiment and the present theory lies in the fact that the predicted breakdown current is about a factor of 20 larger than the observed[4–7] breakdown current per Landau level. This high current comes from the large separation of the Landau levels, which is due to the small effective electron mass, and so the wave-function overlap is negligible until the Hall field is so large that the electrons can make a transition for which $\Delta k\sim 1/l_B$. In this model we have not considered intra-Landau-level transitions; in the Born approximation for an impurity-free system at an integer filling factor, intra-Landau-level transitions cannot start until inter-Landau-level transitions have emptied electron states in the lower Landau level, and this is when we observe the onset of dissipation anyhow. However, if intra-Landau-level scattering were permitted, for example, by reducing the filling factor below unity, the breakdown current would be observed exactly at $v_d=v_s$ as a consequence of intra-Landau-level scattering by phonons with any wave vector ($\vec{q}\,||\,\vec{j}$). This breakdown current would then be independent of B. We have not included tunneling among higher Landau levels, but that would affect j_{cr} only to the extent that these wave functions have a somewhat larger

TABLE I. Input parameters for calculating the dissipated power.

L_x (μm)	L_y (mm)	a (Å)	M (kg)
380[a]	4[a]	5.65	1.2×10^{-25}
v_{sl} (km s^{-1})	v_{st} (km s^{-1})	m^*/m	V_0 (eV Å^2)
5.29[b]	2.48[b]	0.072[c]	5.6[d]
e_{14} (GV m^{-1})	τ (μs)		
1.4[e]	1.0		

[a]Reference 4.
[b]Reference 18.
[c]Reference 19.
[d]Range assumed to be 1 Å. Depth taken from Ref. 20.
[e]Reference 21.

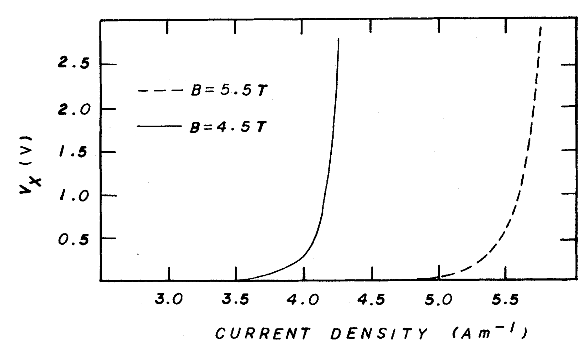

FIG. 2. Predicted longitudinal voltage is plotted as a function of current density per quantum level for $\nu=1$ and magnetic fields $B=5.5$ and 4.5 T.

width. Levels for which $n \sim 5$ would be rapidly depopulated by optical-phonon emission. In our model, we used a two-dimensional phonon system. One could have proceeded differently[15] by taking the phonon system to be the same as in bulk GaAs, and allowing the phonon wave vectors to have a q_z component. However, the net effect of that would only be to multiply the transition rates by a/L_z, where L_z would be the relevant thickness of the phonon system, which in the final result would only slightly increase the threshold current. For example, multiplying the transition rates by $\frac{1}{100}$ only increased the breakdown currents by $\sim 10\%$.

In conclusion, the predicted j_{cr} would necessarily be an upper bound for the critical current since the experiments by Cage *et al.*[4] show that the breakdown is highly inhomogeneous spatially, so that the observed j_{cr} is an *average* current density, whereas the current density might locally be much higher. Since the current density is extremely high in corners of the Hall bridge,[16] the breakdown probably always occurs in corners, even at low total current. Whether intra-Landau-level transitions do cause the breakdown should in principle be determined by experiments measuring j_{cr} as a function of B. Alternatively, exchange of virtual phonons between electrons within a Landau level could broaden it, and since the spacing between Landau levels is only of the order of 10 meV, a very small broadening could, by reducing the gap between levels, increase the transition rates for tunneling between levels significantly.

Since for long wavelengths the piezoelectric coupling is vastly more important than deformation-potential coupling, one could possibly detect tunneling involving phonon emission by measurements of Brillouin scattering,[10] or electrical detection of ultrasonic vibrations from piezoelectricity.[17] Another interesting experiment would be to drive the current in other than the [110] direction, since the piezoelectric coupling is strongly anisotropic and vanishes in the [100] direction. The significance of impurities in the breakdown mechanism could be determined by the manufacturing of several Hall bridges, differing only in their mobility, and measurement of the variation of breakdown current with width of the plateaus in ρ_{xy}.

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