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Spin-glass order parameter of the random-field Ising model

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The Edwards-Anderson spin-glass order parameter Q is calculated in the critical region for the random-field Ising model. It is proportional to $h^{(d-2+\eta)/(1-\eta/2)}$ in d dimensions for a root-mean-square random field h and critical exponent η. Thus Q approaches zero as $h\rightarrow 0$, whereas simple linearized theory predicts it to diverge at the critical point of the pure system. The results are exact to order h^4 and in agreement with scaling theories. Numerical values are given both for Q and the amplitude of the "Lorentzian-squared" structure factor.

The random-field Ising model has been a subject of much interest and controversy. The controversy[1,2] is mostly connected with whether the lower critical dimension is 2 or 3, and the interest surrounds this and other theoretical questions, heightened by the physical realization of effectively random-field Ising magnets.[3,4] The Edwards-Anderson or spin-glass order parameter[5] Q is an important property of a random magnet, but it seems to have received less attention in the random field than in the random exchange problem. It would be observable as a static local field in NMR or other local-probe spectroscopies and is related to the "Lorentzian-squared" term in the neutron structure factor as noted below. In their original work[6] Imry and Ma showed that Q diverges at T_0 in $d=4$ or less dimensions to lowest order in h (the root-mean-square amplitude of the random field), although they pointed out that higher-order terms might change the result. A spherical model calculation[7] has shown Q to be finite at T_0 in two and three dimensions. (T_0 is the critical temperature of the "pure" system with $h=0$.)

In this Rapid Communication I calculated Q in a manner similar to the spherical model and by considering a series expansion in h^2. In both cases it is assumed that multispin correlation functions can be factored into products of two-spin correlation functions, which are taken from the known solutions in zero field. The result, $Q=Ch^{(d-2+\eta)/(1-\eta/2)}$, where η is the critical exponent for decay of the critical correlation function, is consistent with scaling predictions[8] and shows $Q\rightarrow 0$ as $h\rightarrow 0$ at T_0 for both three and two dimensions, the latter in disagreement with the spherical model because of the importance of η in two dimensions. When the method is applied to a uniform field the correct critical-isotherm exponent δ is obtained and the numerical coefficient is good to about 10%; so this provides a certain amount of confidence in the numerical value of C as well as the functional dependence. Further, the result for Q is exact to order h^4, which is a special simplification of the random-field problem.

The Hamiltonian is written as usual as

$$\mathscr{H}=-\tfrac{1}{2}g\mu_B\sum_i H_i s_i-\tfrac{1}{2}\sum_{ij}J_{ij}s_is_j \quad , \tag{1}$$

where H_i is the field at site i, s_i is the Ising spin which can have the values ± 1 only, and the interaction $J_{ij}=J$ for nearest neighbors and zero otherwise. The required procedure is first to compute a thermal average, denoted by $\langle\ \rangle$, for a given quenched configuration of the H_i and then do an average, denoted by [ ], over the configurations $[H_iH_j]=H^2\delta_{ij}$. The dual average is often accomplished by the replica method,[5] but we do not find that necessary here. An exact relation for the partition function Z of a given configuration is

$$Z=Z_0\left\langle \exp\left(\sum_i h_is_i\right)\right\rangle_0 \quad , \tag{2}$$

where Z_0 is the partition function of the zero-field system, $\langle\ \rangle_0$ indicates thermal average in zero field, and $h_i=g\mu_BH_i/2k_BT$. Equation (2) forms the basis for an expansion of $\langle s_i\rangle=\partial\ln Z/\partial h_i$ in powers of the field, the first term of which is the familiar

$$\langle s_i\rangle=\sum_j h_j\langle s_is_j\rangle_0 \quad , \tag{3}$$

which is the normal expression for susceptibility in terms of static correlations. The spin-glass order parameter $Q=[\langle s_i\rangle^2]$ then becomes to lowest order

$$Q=h^2\sum_j\langle s_is_j\rangle_0^2=h^2N^{-1}\sum_q\chi_q^2 \quad , \tag{4}$$

where $\chi_q=\langle s_qs_{-q}\rangle$ with s_q the Fourier component at wave vector q. For a modified Ornstein-Zericke form $\chi_q\propto(q^2+\kappa^2)^{-1+\eta/2}$, where κ is the inverse correlation length which approaches zero at T_0, this diverges for $d\leq 4$, as noted by Imry and Ma.

Consider, however, succeeding terms in the expansion. The term in h^4 is obtained from the product of (3) with the h^3 term in $\partial\ln z/\partial h_i$. This latter term contains

$$h_jh_kh_l\langle s_is_js_ks_l\rangle_0 \quad ,$$

and cannot be handled exactly for $j\neq k\neq l$ without knowledge of the four-spin correlation function. However, the configuration average requires that at least two of the indices in the four-spin function must be equal, whereby it reduces to at most a two-spin function since $s_j^2=1$. Thus the coefficient of h^4 can be obtained exactly for random fields, whereas it cannot be for uniform fields. Higher-order terms cannot, however, be so reduced. To the order of h^8 it is only necessary to handle the four-spin function which I decouple at and above T_0 as

$$\langle s_is_js_ks_l\rangle_0=\langle s_is_j\rangle_0\langle s_ks_l\rangle_0+\langle s_is_k\rangle_0\langle s_js_l\rangle_0+\langle s_is_l\rangle_0\langle s_js_k\rangle_0$$

TABLE I. Numerical results at T_0 (critical temperature of pure system) for simple-cubic (sc) and square (sq) lattices.

Quantity	sc	sq
Spin-galss order parameter Q	$0.45h$	$1.90h^{2/7}$
Amplitude of structure factor,[a] A	$4h^{-2}$	$49.0h^{-2}$
Inverse correlation length, Ka	$1.42h$	$0.34h^{8/7}$
Uniform susceptibility,[a] χ_0/χ_C (χ_C = Curie-law susceptibility)	$2h^{-2}$	$7.0h^{-2}$

[a]Depends only on dimensionality, not lattice specific. $h=[H_i^2]^{1/2}g\mu_B/2k_BT_0$.

for unequal indices. The result is

$$Q=N^{-1}\sum_q(h^2\chi_q^2-2h^4\chi_q^3+3h^6\chi_q^4-4h^8\chi_q^5+\cdots)$$
$$=N^{-1}\sum_q h^2\chi_q^2/(1+h^2\chi_q)^2\ , \quad (5)$$

where I have kept only those terms which are most divergent at T_0 (it is assumed throughout that $h \ll 1$ so that higher orders in h are negligible unless multiplied by potentially divergent terms), and have assumed the series continues indefinitely in the manner shown in order to obtain the second equality. Once again, the first two terms are exact. Similar arguments[9] have been applied to the bulk susceptibility in the random-field problem which show that it can be obtained exactly only to order h^2.

Decoupling the multispin correlation function is equivalent to assuming the probability distribution of the Fourier-transformed spin variables consists of independent Gaussians. Hence, evaluation of $\langle\ \rangle_0$ in Eq. (2) may be accomplished in this approximation by direct integration, as in the spherical model, and also, as in the spherical model, introducing a multiplier λ to satisfy the sum rule $\sum_q s_q s_{-q}=N$, while treating the q's as otherwise independent. The result

$$\langle s_q\rangle = h_q\chi_q/(1+\lambda\chi_q)\ , \quad (6)$$

where the multiplier λ is obtained from

$$\sum_q \chi_q^2(\lambda/(1+\lambda\chi_q)-h_qh_{-q}/(1+\lambda\chi_q)^2)=0 \quad (7)$$

is the same as for the spherical model, except that the known $\langle s_q s_{-q}\rangle$ is used rather than that obtained by a completely self-consistent spherical model calculation.[7] For random fields one can replace h_qh_{-q} by its configuration average h^2. If $\lambda\chi_q \ll 1$, the solution of Eq. (7) is simply $\lambda=h^2$, which leads back to Eq. (5). The solution at T_0, where the relation $\lambda\chi_q \ll 1$ breaks down for $q\rightarrow 0$, is $\lambda=(I_2/I_1)h^2$, where

$$I_n=\int_0^\infty x^{d-1}dx x^{-(4-2\eta)}/(1+x^{-(2-\eta)})^n\ ,$$

and we have assumed $\chi_q \propto q^{-2+\eta}$ at T_0. Thus we expect $\lambda\propto h^2$ to hold for all $T\geq T_0$.

A more general statement of Eq. (5) and the above argument is to assert the scaling relation

$$Q=h^2\sum_q\chi_q^2 f(h^2\chi_q)\ ,$$

where the function f decays sufficiently rapid to guarantee convergence for $q\rightarrow 0$. This alone leads to the conclusion

$$Q=Ch^{(d-2+\eta)/(1-\eta/2)} \quad (8)$$

at T_0 for $\chi_q\propto q^{-2+\eta}$, which follows from other scaling arguments[8] and which is the same as the spherical model result[7] if $\eta=0$. The small $\eta\sim 0.03$ is unimportant for $d=3$ whereby $Q\propto h$, but it is crucial for $d=2$, where the exact $\eta=\frac{1}{4}$ leads to $Q\propto h^{2/7}$. Hence the spin-glass order parameter goes to zero as h goes to zero even in two dimensions. The constant of proportionality C may be computed in terms of the integrals I_1 and I_2 and the amplitude D in $\chi_q=D/(qa)^{2-\eta}$ at T_0 (a = nearest-neighbor distance) which is given by Fisher and Burford.[10] We take $\eta=0$ for $d=3$ and $\eta=\frac{1}{4}$ for $d=2$, and consider the simple-cubic and square lattices. For convenience all numerical results are contained in Table I rather than in the text.

One can also deduce a critical isotherm for the pure system in a uniform field from Eqs. (6) and (7). For $h_q=h_0\delta_{q,0}$ and $\chi_0\rightarrow\infty$ at T_0 the solution to Eq. (7) at T_0 is $\lambda=E^{-1}h_0^{(4-2\eta)/(d+2-\eta)}$, and insertion of this into Eq. (6) for $q=0$, $\chi_0\rightarrow\infty$ gives $\langle s\rangle=Eh^{1/\delta}$ with

$$\delta=(d+2-\eta)/(d-2+\eta)$$

in agreement with scaling laws.[11] The coefficient E is the same as calculated in the spherical model[12] for $d=3$, while I find it to be $E=1.12$ for the square lattice compared with the series result[13] $E=1.01$. It seems reasonable to expect similar 10% accuracy for C in numerical estimates of the spin-glass order parameter.

The square of Eq. (6) leads to a "Lorentzian-squared" term in the neutron scattering structure factor.[14] If this is written as[15]

$$[\langle s_q\rangle^2]=A/(1+(q^2/K^2)^{1-\eta/2})^2\ ,$$

it follows that the peak amplitude at T_0 is $A=(I_1/I_2)^2/h^2$ and the inverse correlation length is $Ka=((I_2/I_1)h^2D)^{1/(2-\eta)}$. The field dependence of K is as given by Aharony and Pytte.[16] The uniform susceptibility at T_0 from Eqs. (6) and (7) is $\chi_0=\chi_C(I_2/I_1)/h^2$, where χ_C is the Curie-law susceptibility for a pure, noninteracting system at T_0. χ_0 also has the same random-field dependence as found in Ref. 16.

In conclusion numerical values have been obtained for the spin-glass order parameter Q and the related part of the neutron structure factor in the random-field Ising model at the critical point T_0 of the zero-field system. The expression for Q is exact to fourth order in the random field.

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