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Dynamical scaling of the structure function in quenched Al-Zn and Al-Zn-Mg alloys

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The time evolution of the structure function $S(k,t)$ of quenched Al-Zn binary alloys and Al-Zn-Mg pseudobinary alloys heated to 80 and 40 °C (or 18 °C) has been measured by small-angle neutron scattering. The structure function obeys a scaling relation $S(k,t)=\alpha k_m(t)^{-3}\tilde{S}(k/k_m(t))$, in terms of a universal scaling function $\tilde{S}(x)=3x^2/(2+x^6)$, proposed by *H.* Furukawa [Physica A **123**, 497 (1984)], for a wide range of values of the aging time *t*, even for different aging temperatures and different samples.

For many systems[1-3] that undergo phase separation the time-dependent structure function $S(k,t)$ obeys a scaling relation

$$S(k,t)=\alpha k_m(t)^{-3}\tilde{S}(k/k_m(t)) \ , \tag{1}$$

where $k_m(t)$ is a time-dependent wave number the inverse of which characterizes the length scale of the system, and α is a constant. The scaling function $\tilde{S}(x)$ seems reasonably independent not only of the aging time but also of the aging temperature even for different systems.

The exact form of this scaling function has attracted the attention of many authors. Rikvold and Gunton[4] have presented an analytic form of the scaling function that is slightly dependent on the volume fraction v of the minority phase. Their theory assumes a gas of spherical droplets of the minority phase surrounded by depletion zones. Furukawa[5] has proposed a scaling function of the form

$$\tilde{S}(x)=\frac{(1+\gamma/2)x^2}{\gamma/2+x^{2+\gamma}},\quad \tilde{S}(1)=1 \ , \tag{2}$$

where $\gamma=d+1$ for off-critical concentration mixtures (cluster regime), and $\gamma=2d$ for critical concentration mixtures (percolation regime), d being the dimensionality of the system. This structure function is determined in accordance with the asymptotic forms

$$\tilde{S}(x)\propto\begin{cases}x^2 & \text{for small } x \ ,\\ x^{-\gamma} & \text{for large } x \ .\end{cases} \tag{3}$$

The first condition above arises because of the conservation of the order parameter, and the second is caused by the existence of a well-defined phase boundary and is known as Porod's law. It is to be noted that there is no dependence of $\tilde{S}(x)$ in Eq. (2) on the volume fraction v.

Here we have examined our experimental data on the structure function in quenched Al-Zn binary alloys and Al-Zn-Mg pseudobinary alloys aged at different temperatures in comparison with the proposed universal scaling function. The details of our study on the time evolution of the structure function by means of small-angle neutron scattering can be found elsewhere.[6,7] In this Rapid Communication we find that the structure function obeys the scaling relation (1) with the universal scaling function (2) for a wide range of aging times t even for different aging temperatures and different samples, provided we properly normalize the data.

A small-angle neutron scattering experiment was performed for the binary alloys Al–6.8 at.% Zn (sample B) and Al–10 at.% Zn (sample D) and pseudobinary alloys with further addition of 0.1 at.% Mg (samples A and C, respectively) homogenized at 300 °C and aged at 80 °C (A,B,C,D), 40 °C (A′, B′), or 18 °C (C′, D′) for various times ranging from 5 to 1000 min. The scattering cross section $d\Sigma/d\Omega(k,t)$, which is proportional to the structure function $S(k,t)$, was obtained for the eight different sets of data (A,B,C,D,A′,B′,C′,D′).

The data for the binary alloys (B, D, B′, D′) show the growth of a minority phase almost in accordance with those of Hennion, Ronzaud, and Guyot,[3] although there is evidence that the decomposition ceases at aging times t of more than 50 min. On the other hand, the data for the pseudobinary alloys (A,C,A′, C′) show the development of a minority phase at lower rates up to the maximum measured range of $t=1000$ min.

To calculate the scaling functions $\tilde{S}(x)$ carefully from the experimental data, we did a least-squares fit of each set of data to the function

$$\frac{d\Sigma}{d\Omega}(k,t)=A(t)\frac{3[k/k_m(t)]^2}{2+[k/k_m(t)]^6} \tag{4}$$

with the parameters $A(t)$ and $k_m(t)$ to be determined. Then we calculated the function

$$F(x,t)=\frac{k_m(t)^3\frac{d\Sigma}{d\Omega}(k,t)\int_{x_{\mathrm{min}}}^{x_{\mathrm{max}}}x^2\tilde{S}(x)\,dx}{\sum_{k=k_{\mathrm{min}}}^{k_{\mathrm{max}}}k^2\frac{d\Sigma}{d\Omega}(k,t)\Delta k} \ , \tag{5}$$

where for the cluster regime in three dimensions

$$\tilde{S}(x)=\frac{3x^2}{2+x^6},\quad \tilde{S}(1)=1 \ , \tag{6}$$

and $k_{\mathrm{min}}=x_{\mathrm{min}}k_m$ and $k_{\mathrm{max}}=x_{\mathrm{max}}k_m$ are the lower and upper ends of the measured range of k, respectively, and Δk is the

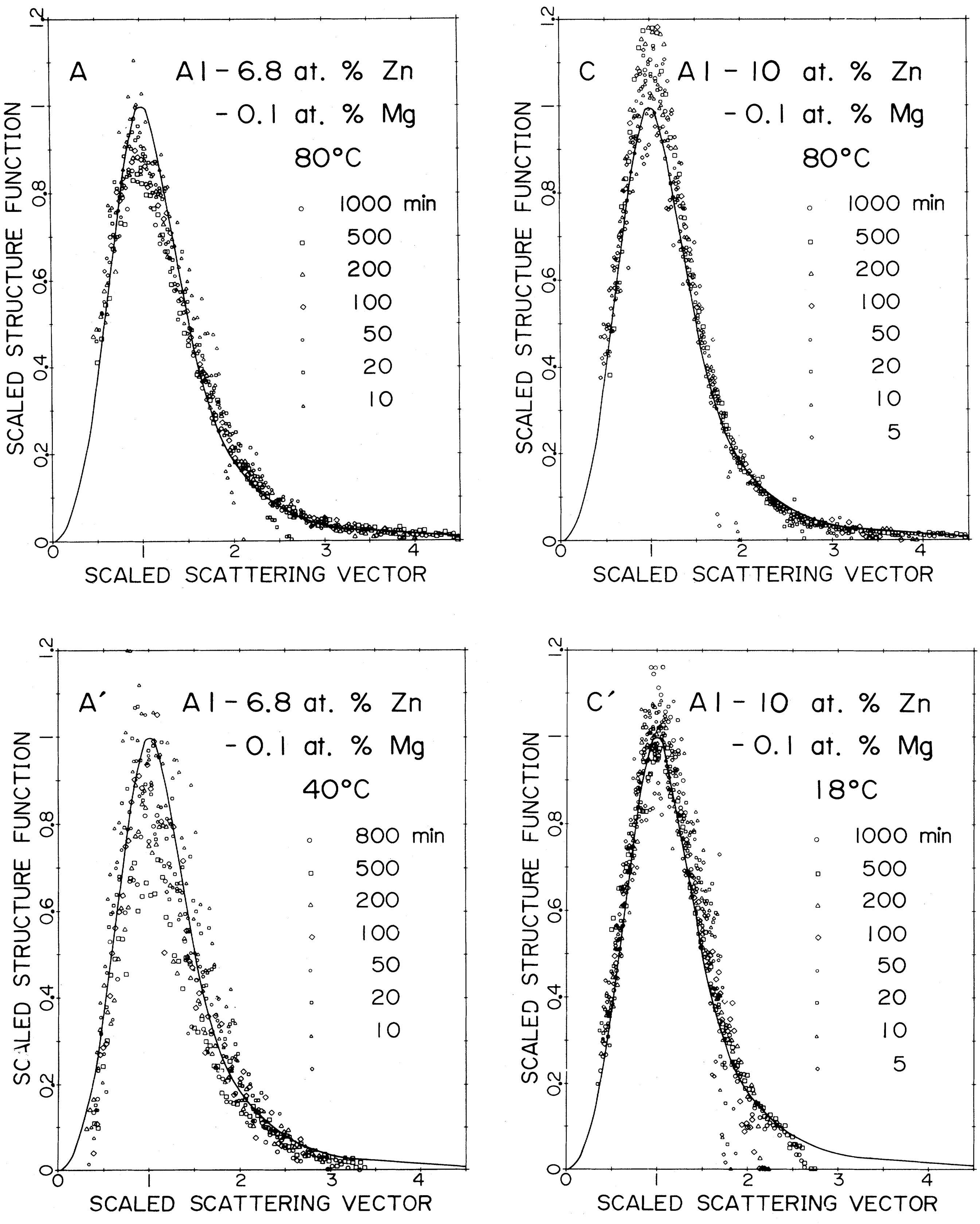

FIG. 1. Scaled structure function $F(x,t)$ defined by formula (5) as a function of scaled scattering vector $x = k/k_m(t)$ for the pseudobinary sample Al–6.8 at.% Zn–0.1 at.% Mg aged at 80 °C (A) and 40 °C (A′) and for Al–10 at.% Zn–0.1 at.% Mg aged at 80 °C (C) and 18 °C (C′) compared with $\tilde{S}(x)$ defined by (6).

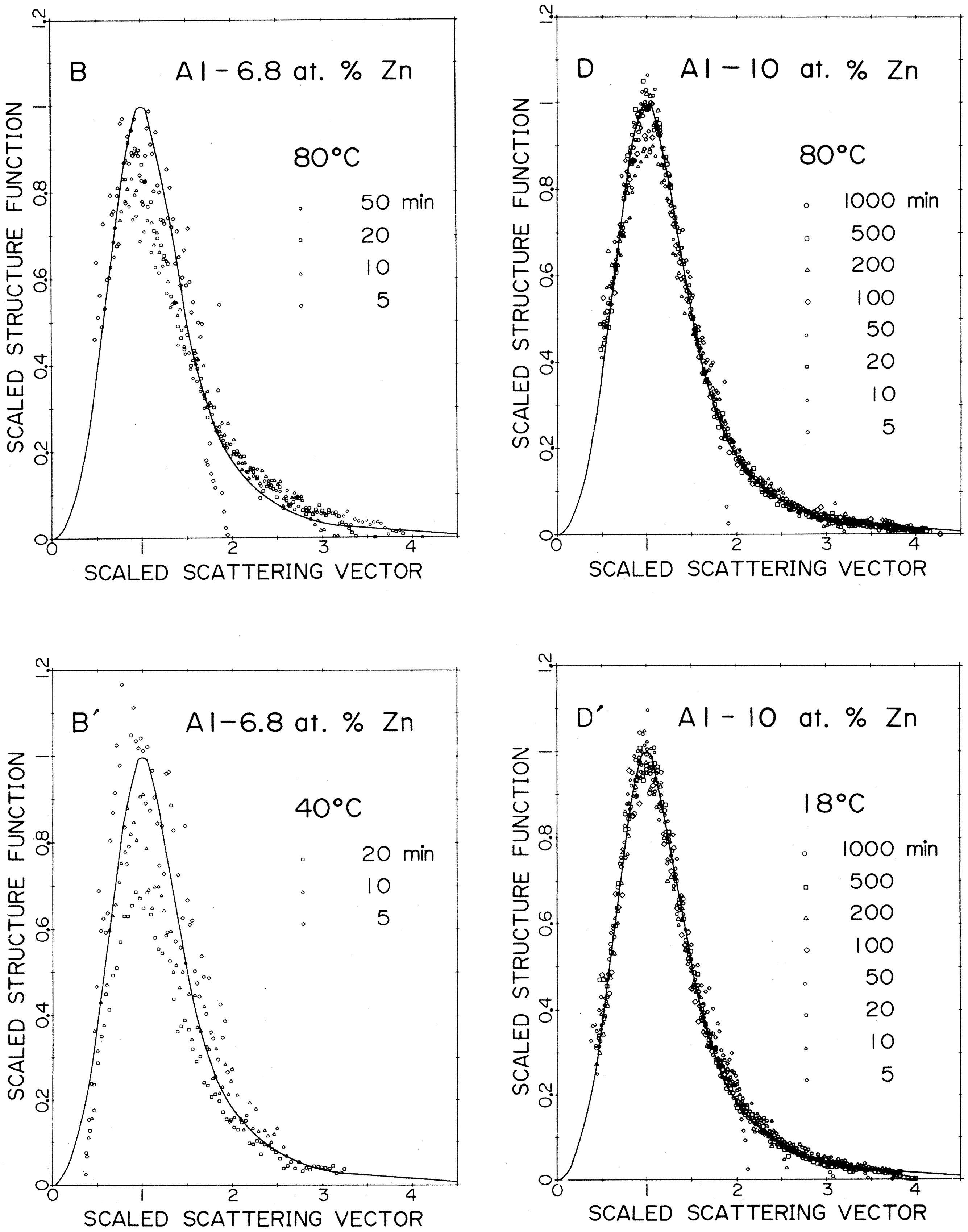

FIG. 2. Same as Fig. 1 for the binary sample Al–6.8 at.% Zn aged at 80 °C (B) and 40 °C (B′) and for Al–10 at.% Zn aged at 80 °C (D) and 18 °C (D′).

measured interval of k.

The function $F(x,t)$ should be independent of t and agree with Eq. (6), if the scaling relation (1) holds. In the definition of $F(x,t)$ in (5) it is important to include the integral $\int x^2\tilde{S}(x)dx$, because the sum in the denominator $\Sigma k^2(d\Sigma/d\Omega)\Delta k$ covers different ranges of x in $\tilde{S}(x)$ for different aging times t, which should be corrected for by this integral.

The calculated functions $F(x,t)$ for the eight different sets of data are shown in Fig. 1 (A,A′,C,C′), and Fig. 2 (B,B′,D,D′) together with Furukawa's universal scaling function. These figures clearly demonstrate that the scaling relation holds well for a wide range of values of the aging time t, for different aging temperatures, and for different samples. There is an exception in Fig. 2 (B,B′) where the range of aging times for which the scaling relation (1) holds well is rather limited with an upper value of 20 or 50 min, above which the growth of the minority phase ceases. It is surprising to see that almost all the data are consistent with a single universal function $\tilde{S}(x)$ in Eq. (6) with the normalization character $\tilde{S}(1)=1$.

In principle the scaling function should depend on the volume fraction v of the minority phase. However, the present experimental results show that the dependence of $\tilde{S}(x)$ on v is very slight, at least for the measured values of $v=0.068$ and 0.10. The universal nature of the scaled structure function demonstrated by this experiment is significant in that in the late stages of decomposition of a system, where the order parameter is conserved and a well-defined phase boundary is established, the growth process of the minority phase is characterized only by a length scale which develops with time almost irrespective of the aging temperature and the volume fraction of the minority phase of the system.

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