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Multiple scattering of second sound in superfluid He II–filled porous medium

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Observations on the multiple scattering of second sound in five fused-glass-bead porous media saturated with superfluid helium are reported. Near the λ point where attenuation is small, the measured index of refraction of second sound is in agreement with that of first sound in the same porous medium. Below 1.7 K, both the index of refraction and the attenuation of second sound increase as the temperature is lowered. The measured temperature dependence can be described by the effects of viscous drag between the normal fluid component and the walls of the porous medium.

The acoustics of fluid-filled porous solid media has been a problem of theoretical and experimental interest over many years. In a series of papers Biot[1] proposed a phenomenological theory for the acoustics of the two-component system by considering the motion of the fluid and solid separately. Biot's theory predicts that at sufficiently high frequencies there exist one shear and two (slow and fast) longitudinal propagating waves. Plona[2] recently observed the slow and fast waves in water-saturated fused-glass beads. Johnson[3] showed that the fourth-sound propagation in a superfluid He II–filled porous medium is analogous to the slow wave in the Biot theory and that the measurement of index of refraction of fourth sound[4] in a superleak is equivalent to a measurement of the tortuosity parameter for the porous medium in the Biot theory. The tortuosity parameter for a number of fused-glass beads was determined by Johnson *et al.*[5] from the measurement of the index of refraction of first sound in superfluid He II. It was found[5] that the Biot theory together with the measured tortuosity parameter and other measured parameters was in good agreement with the measurements of Plona.[2] More recently the sound propagation in a liquid-helium-filled Vycor glass has been measured by Beamish, Hikata, Tell, and Elbaum.[6] The purpose of our paper is to present measurements of the propagation of second sound (temperature wave) in a He II–filled porous (but not a superleak) medium.

Neglecting pressure changes and dissipative effects in the propagation of second sound, the equation of motion of the normal component is given by[7]

$$\frac{\partial \vec{v}_n}{\partial t} = -\frac{\rho_s}{\rho_n} S \vec{\nabla} T \ . \tag{1}$$

Here, $\vec{v}_n$ is the normal component velocity, ρ_s the superfluid component density, ρ_n the normal component density, S the entropy per unit mass, and T temperature. The conservation of entropy is expressed by

$$\frac{\partial \rho S}{\partial t} + \vec{\nabla} \cdot (\rho S \vec{v}_n) = 0 \ , \tag{2}$$

where ρ is the total fluid density. Equations (1) and (2) may be combined to give a wave equation for second sound whose speed of propagation in the absence of multiple scattering is given by

$$C_2^2 = \frac{\rho_s S^2 T}{\rho_n C_v} \ , \tag{3}$$

where C_v is the specific heat at constant volume. In the limit of long wavelength and small fluid velocity,[8] the wave equation for T becomes

$$\nabla^2 T = 0 \ . \tag{4}$$

Equation (4) must be supplemented by a boundary condition at the solid walls. Assuming that there is no heat entering He II at the walls of the porous medium, we have the boundary condition that[7]

$$\rho S T \vec{v}_{n\perp} = 0 \ . \tag{5}$$

Here $\vec{v}_{n\perp}$ is the component of the normal fluid velocity perpendicular to the solid wall. Equations (1), (4), and (5) are analogous to the corresponding equations of an ideal classical fluid motion in the long-wavelength limit in a rigid porous medium.[9] In the presence of a porous medium the sound propagation in the ideal classical fluid suffers a multiple scattering and the measured speed of sound is reduced from that in bulk. Analogously, the speed of second sound in the presence of a porous medium C_2^* is reduced from that in bulk. The reduction is characterized by an index of refraction n given by

$$n = C_2 / C_2^* \ . \tag{6}$$

In a given porous medium the index of refraction of second sound is expected to be equal to that of first sound.

The porous material used in our experiment is made by sintering closely packed glass beads whose diameter is in the range $\sim$180–210 μm at a temperature of about 700 °C.[5] The porosity (defined as the ratio of volume not occupied by glass to total volume) obtained in this manner is about 35% or less. The porosity can be decreased by increasing the time interval of sinter. A 10-cm-diam disk of 1–2.5-cm thickness is prepared. A cylindrical plug is machined out of the disk and this plug constitutes our porous medium sample. The viscous penetration depth in the frequency and the temperature range of interest is sufficiently small such that both first and second sound in He II filling these porous samples can be observed.[5] In this paper we focus on the measurements of second sound.

A sample was inserted into a cylindrical brass cavity whose diameter and length were adjusted for a snug fit. The ends of the cavity were closed by second-sound capacitive transducers utilizing Nuclepore filter (0.2 μm pore diameter and 10 μm thick) membranes as active elements.[10] The cavity was immersed in a liquid-helium bath. The cavity was filled with helium through a small leak probably at the end of the cavity when the bath temperature was reduced below the lambda point T_λ. The cavity forms a second-sound resonator. The speed of second sound in the presence of the porous sample C_2^* was determined from the measured resonant frequencies of the "length modes" from the following relation:

$$C_2^* = \frac{2f_m L}{m} , \tag{7}$$

where L is the length of the cavity and f_m the mth length mode resonant frequency. Second-sound resonances in a frequency range 1–15 kHz and at temperatures above 1.2 K and as close as 2 mK below T_λ could be observed. As many as 20 length modes could be observed. The temperature of the bath was obtained from the measured vapor pressure. To observe the propagation of first sound in the same porous sample, it was necessary only to replace the Nuclepore filter membrane with a Mylar sheet.

To test our overall measurement system, we observed second sound in bulk He II without inserting any porous sample. Q values of resonances greater than 200 were observed at $T = 1.3$ K. Our measurement of the speed of second sound in bulk He II in the absence of multiple scattering by a porous medium is indicated by open circles in Fig. 1. The data are in excellent agreement with those (shown by a solid line) of Maynard.[11]

When a porous sample was inserted into the cavity, the measured speed of second sound becomes less than the bulk value. The results for four porous samples (nominal diameter = 0.6 cm and length = 3 cm) are shown in Fig. 1. The values of C_2^* shown in Fig. 1 are taken as the average of at least 10 modes at a stabilized temperature. The symbols, crosses, diamonds, triangles, and squares refer to samples of porosities equal to 33%, 30%, 26%, and 16%, respectively. The greater the porosity, the greater the reduction in the measured speed of second sound. The dashed lines in Fig. 1 represent the temperature dependence of the bulk second-sound speed divided by an index of refraction adjusted to fit each sample. The temperature dependence of the measured speed of second sound in the porous samples is very similar to that in bulk in the temperature range shown in Fig. 1. However, more detailed measurement in another sample shows that there is a discrepancy in the temperature dependence particularly below 1.7 K. (See discussion of Fig. 2.)

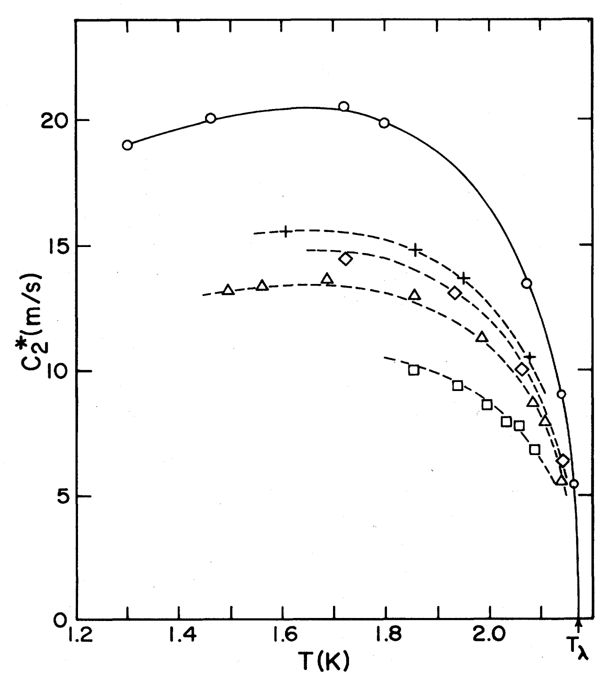

FIG. 1. Measured speed of second sound in bulk He II (circles) and in porous media saturated with He II as a function of temperature. Media porosities are 33% (crosses), 30% (diamonds), 25.7% (triangles), and 16.3% (squares). The solid line is taken from Maynard (Ref. 11).

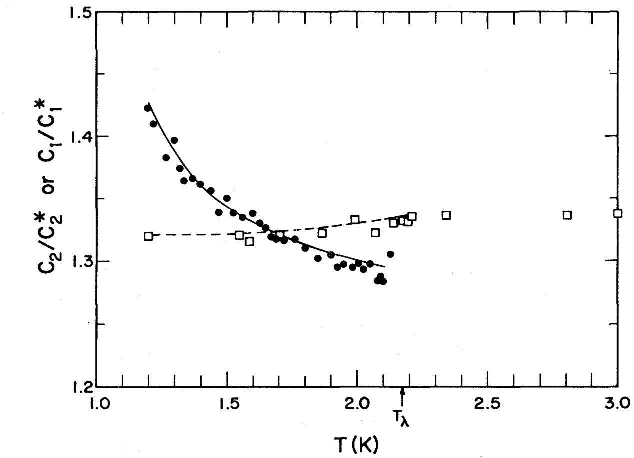

FIG. 2. Measured ratios C_2/C_2^* (dots) and C_1/C_1^* (squares) as a function of temperature for a sample of porosity 34.5%. The second-sound data is obtained using $m = 9$ mode and the first-sound data using $m = 2$ mode.

In Table I, we list the index of refraction measured by both second sound and first sound in the same porous sample. There is a fair agreement on the measured index of refraction by the two kinds of sounds. The quality factor of second-sound resonances decreases as the porosity is decreased and the accuracy of measurement also decreases. The discrepancy in the 16% sample may in part be caused by the inaccuracy. Our results indicate that the boundary condition given by Eq. (5) is valid at the He II–glass interface at $T \gtrsim 1.7$ K.

The temperature dependence of the speed of second sound was measured in more detail in a separate sample with a porosity equal to 34.5%. The length and the diameter of this sample were 1.0 and 2.0 cm, respectively. The temperature dependence of C_2^* in this sample was measured by following the ninth mode resonance. The ratio C_2/C_2^* for the ninth mode is shown by dots in Fig. 2. Between 1.7 K and T_λ, the temperature dependence of this ratio is not

TABLE I. Index of refraction of porous samples as measured by first and second sound.

Porosity (%)	Index of refraction: Second sound	Index of refraction: First sound
33	1.31	1.32
30	1.38	1.36
26	1.58	1.47
16	1.90	1.73

large as it was already seen in Fig. 1. At temperatures below 1.7 K the ratio becomes significantly larger. A similar temperature dependence of the ratio was observed for other modes between $m=2$ and 15. The measured ratio was not dependent on the drive level which was varied by a factor of 20. The measured temperature dependence of the ratio was not affected by (1) a small (~ 10 μm) gap around the perimeter of the sample, (2) a probably irreproducible acoustic radiation leakage at the ends of the resonator when the cavity was opened several times to replace Nuclepore membranes, and (3) the pore size of Nuclepore membrane.

The temperature dependence of the measured quality factor of the third mode is shown in Fig. 3. The Q value increases from about 6 near 1.4 K to about 42 at 2.10 K. The Q values for other modes behaved similarly in temperature dependence. The measured Q values for modes between $m=2$ and 7 at $T=1.90$ K are shown by circles in Fig. 4. Owing to a decreased signal level, Q could not be measured reliably at frequencies greater than 5 kHz. The error bars in Figs. 3 and 4 indicate uncertainties in the base line of the resonance curve.

As a comparison to the second-sound measurements, we measured the speed of first sound in the same 34.5% porosity sample as a function of temperature. The ratio of the measured first-sound speed to the bulk first-sound speed C_1/C_1^* is shown as squares in Fig. 2. The ratio increases only slightly ($\sim 0.8\%$) between 1.2 K and T_λ. The ratios from first-sound and second-sound measurements are close to each other near T_λ. The first-sound measurement was extended above T_λ into the normal liquid phase. The observed C_1/C_1^* in the normal phase is the same as that at T_λ. The measured quality factor of a first-sound mode with $m=2$ was about 30 at $T=3.0$ K.

We attempt below to describe the observed temperature dependence of C_2/C_2^* and the quality factor by considering the effects of the viscous drag between the normal component and the walls of the porous medium. The effects of viscous drag may be included into the equation of motion by adding a term proportional to velocity $-R\overline{v}$ to the right-hand side of Eq. (1), where R represents a dynamic flow resistance.[12] In the limit of vanishing R, one obtains the unattenuated second sound. In the opposite limit of $R \rightarrow \infty$, the second sound becomes a diffusive mode. In

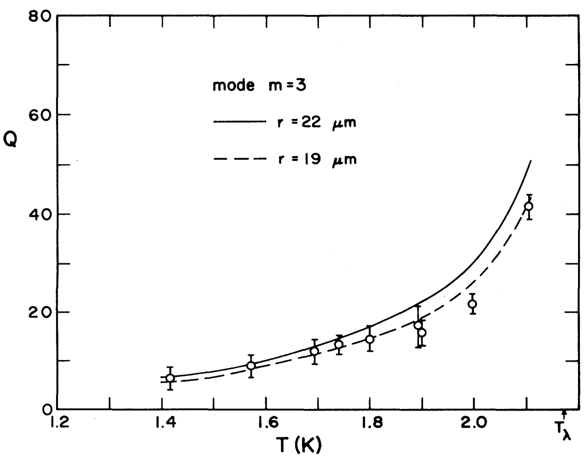

FIG. 3. Quality factor of third mode is 34.5% porosity sample as a function of temperature.

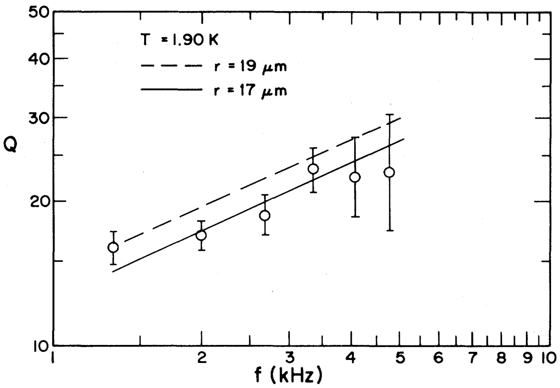

FIG. 4. Quality factor of modes 2–7 at $T=1.90$ K in 34.5% porosity sample. The solid and dashed lines show theoretical frequency dependence.

the present analysis we assume an expression for the flow resistance appropriate for a straight cylindrical pore given by[13]

$$R=\left(\frac{2\eta}{\rho_n\omega}\right)^{1/2}\frac{\omega}{r}(1-i)\ , \tag{8}$$

where η is the viscosity of the normal component and r the radius of the pore. The real part of R contributes to the attenuation of second sound. The imaginary part contributes to the acceleration term in Eq. (1) and results in a reduced speed of propagation. The linearized set of equations[7] for the conservation of mass, the motion of superfluid component, Eq. (2) and modified Eq. (1) as described above may be solved for normal modes by assuming a plane wave solution $\sim e^{i(kx-\omega t)}$ where the wave vector k is a complex quantity $k=k_r+ik_i$. In the limit of not too large R, the set of equations gives first- and second-sound modes. For each mode, the phase velocity and the attenuation coefficient may be calculated. The phase velocity is given by $V_{\rm ph}(1,2)=\omega/k_r$ and the attenuation coefficient is given by $\alpha=k_i$, where 1 and 2 refer to first- and second-sound modes, respectively. For our analysis it is more convenient to calculate the quality factor given by $Q=k_r/2k_i$.

To compare the above theory with our second-sound results in Fig. 2 we evaluated the quantity $1.29[C_2/V_{\rm ph}(2)]$ as a function of temperature and the ratio is shown as a solid line in Fig. 2. The index of refraction where viscous effects are small is assumed to be the measured C_2/C_2^* near T_λ and is taken as 1.29. The value of r was adjusted to 22 μm for a good fit. The fit is good over the temperature range of measurement. The temperature dependence of the calculated quality factor of third mode and $r=22$ μm is shown by a solid line in Fig. 3. There is qualitative agreement between the theory and the experiment. A somewhat better fit is obtained as shown by a dotted line if r is adjusted to 19 μm. The rise of Q as the temperature is raised towards T_λ results from the smaller normal component velocity (hence, smaller dissipation) required to keep the center of mass fixed.

Since the flow resistance R is frequency dependent, the calculated $V_{\rm ph}$ and Q are expected to be also frequency dependent. The mode dependence of C_2/C_2^* determined from the measured resonant frequencies between $m=2$ and $m=14$ at $T=1.90$ K is shown in Fig. 5. The expected mode dependence of C_2/C_2^* from the theory ($r=22$ μm) is

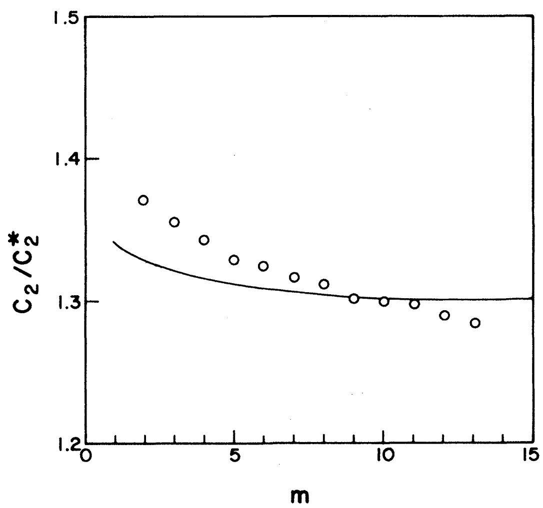

FIG. 5. Mode dependence of C_2/C_2^* in 34.5% porosity sample at $T=1.90$ K. The line represents theory for $r=22$ μm.

shown by a solid line. The measured mode dependence is larger than expected. We do not have an explanation for the discrepancy. The dashed and solid lines in Fig. 4 represent the frequency dependence of Q expected from the above theory at $T=1.90$ K with $r=19$ μ and 17 μm, respectively. The theory appears to describe the frequency dependence qualitatively.

To compare the theory with the first-sound results in Fig. 2, we plotted the ratio $1.32[C_1/V_{\text{ph}}(1)]$ as a dotted line. The value of r was 17 μm. The theory is consistent with the observed increase of C_1/C_1^*. The quality factor of the first-sound modes has not been measured systematically and will be a subject of further study.

An estimate based on the volume to surface area ratio for the 34.5% porosity sample gives a value ~ 30 μm for the pore radius. This value is fairly close to r used in fitting the theory to our data.

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