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Temperature dependence of the quantum Hall resistance

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We report high-precision measurements of the temperature dependence of the quantum Hall resistance for two GaAs heterostructures. The Hall resistivity $\rho_{xy}(T)$ is found to vary *linearly* with the minimum resistivity along the device $\rho_{xx}^{\min}(T)$ and to depend upon the sample, Hall probe set, and magnetic field direction, but to approach a sample-independent value as $T \rightarrow 0$. The temperature-dependent shift of $\rho_{xy}(T)$ from $\rho_{xy}(0)$ can be significant even for very flat Hall steps and is inconsistent with standard mechanisms.

The quantum Hall effect[1] shows great promise as a means for establishing a new resistance standard and for determining the fine-structure constant. These possibilities have already been demonstrated by several measurements[2-4] of the quantum Hall resistance $R_H(i) = h/(e^2 i)$, where i is an integer quantum number, h is the Planck constant, and e is the elementary charge. The measured Hall resistivity $\rho_{xy}(T)$ is expected to approach R_H in the limit of vanishing dissipation $\rho_{xx}(T) \rightarrow 0$. However, one may ask what is the value of ρ_{xy} when ρ_{xx} is finite? This question was first addressed by Yoshihiro *et al.*[5] who empirically found that $\Delta\rho_{xy} \equiv \rho_{xy}(T) - \rho_{xy}(0) \approx -0.1\rho_{xx}^{\min}(T)$ for silicon metal-oxide-semiconductor field-effect transistor (MOSFET) devices. They could ascribe no mechanism to explain this linear relationship.

In this Rapid Communication we report temperature-dependence measurements of ρ_{xy} and ρ_{xx} for GaAs heterostructure devices. The measurements have a greatly improved precision over those of Ref. 5 and provide additional unexpected results. We find that the value of ρ_{xy} is strongly temperature dependent and show that $\Delta\rho_{xy}$ satisfies the equation $\Delta\rho_{xy} = -s\rho_{xx}^{\min}$ down to at least the 0.01-ppm level of accuracy and is very reproducible, but that the value of s is device dependent and Hall probe set dependent. The value of ρ_{xy} is found to approach a sample-independent constant (presumably equal to R_H) as T and $\rho_{xx}^{\min}$ approach zero. We present evidence that the linear relationship between $\Delta\rho_{xy}$ and ρ_{xx} cannot be explained by standard mechanisms.[6-9]

We also report the surprising result that Hall steps can be very flat, but yet have values of ρ_{xy} that deviate significantly from the zero temperature value. This observation should not be confused with that of Ebert, Herzog, Obloh, and Tausendfreund[10] and Tausendfreund and v. Klitzing[11] who have reported a temperature dependence of the *slopes* of the Hall steps for both silicon MOSFET and GaAs heterostructure devices.

Two high-quality GaAs-Al$_x$Ga$_{1-x}$As ($x = 0.29$) heterostructure devices[12] were used in this study. They will hereafter be referred to as GaAs(7) and GaAs(8). Figure 1(a) shows the device geometry and probe notation. Two Hall voltage probe sets were used to measure V_H and V_H'

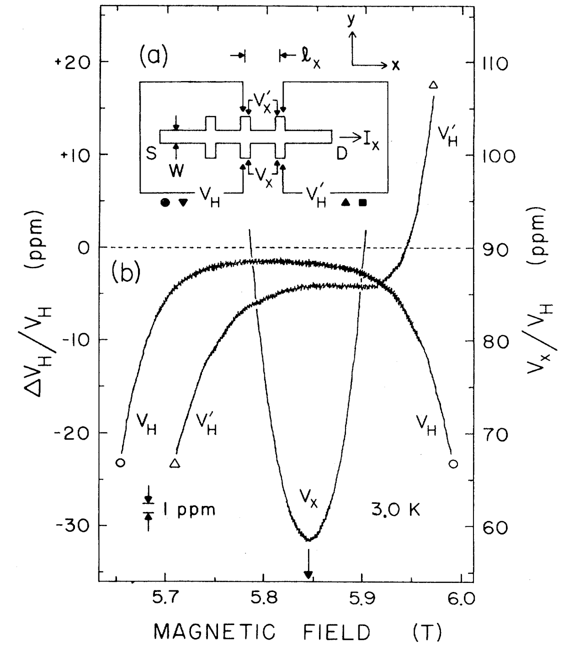

FIG. 1. (a) The device geometry, where $l_x = 1.0$ mm, $W = 0.38$ mm, and the total device length is 4.6 mm. (b) High sensitivity chart recordings of the two $i = 4$ Hall steps and the V_x curve for GaAs(7) at 3.0 K, with $I_x = I_{SD} = 25.5$ μA and a magnetic field pointing down (into figure in $-z$ direction). $V_x^{\min} = 9.63$ μV occurs at 5.845 T. The dashed line at $\Delta V_H/V_H = 0$ corresponds to the value of $\rho_{xy}(0)$.

for each device, as well as the intervening voltages V_x and V_x' at temperatures between 1.2 and 4.2 K for the $i=4$ Hall step. Since the external Hall probe current I_y is negligible ($<10^{-15}$ A) in our measurement system,[2] $V_x = R_x I_x = (l_x/W)\rho_{xx} I_x$ and $V_H = V_y = \rho_{xy} I_x$.

Plots of V_H, V_H', and V_x are shown in Fig. 1(b) for GaAs(7) at 3.0 K and $I_x = 25.5$ μA for a magnetic field pointing down. The GaAs(7) V_H' (down) Hall steps were the only ones that had "ideal" shapes, all the others had inverted U shapes. On some cool downs the inverted U steps of V_H (down) and V_H' (up) of GaAs(7) were not symmetric about $V_x^{\min}$. Such a case is shown in Fig. 1(b) for V_H (down).

Figure 2 shows high resolution, digitally integrated data for the V_H' (down) Hall step of GaAs(7) at 1.2 K and 25.5 μA. This step is flat to within our 0.01-ppm resolution over a magnetic field range that is 2% of the central value. All eight Hall steps were equally as wide and flat at 1.2 K. Notice that the GaAs(7) V_H' (down) step no longer has an ideal shape at this higher resolution.

The values of $\rho_{xx}^{\min}$ and $\Delta\rho_{xy}$ increased rapidly with temperature for both devices. Excellent empirical fits to the data could be obtained by assuming equations of the form $\rho_{xx}^{\min} = aT^7$ and $\Delta\rho_{xy} = -bT^7$, where T is the temperature and the values of a and b are device and cool-down dependent. However, very simple linear relationships are found between the temperature-dependent quantities $\Delta\rho_{xy}$ and $\rho_{xx}^{\min}$ that are device dependent but cool-down *independent.* They are shown fitted to straight lines A, B, C, and E in Fig. 3 for temperatures between 1.2 and 3.0 K. The solid lines have been obtained from least-squares fits to the data assuming equations of the form $\Delta\rho_{xy} = -s\rho_{xx}^{\min}$, where the values of s for lines A, B, C, and E are, respectively, 0.507, 0.452, 0.190, and 0.015. The dashed line D results from the aforementioned cool downs in which GaAs(7) Hall steps were not symmetric about $V_x^{\min}$. All the data shown in Fig. 3 were obtained at 25.5 μA, but additional data at 9.4 μA were consistent with the appropriate curves A to E, and thus $\Delta\rho_{xy}$ and $\rho_{xx}^{\min}$ appear to be current independent at these low current levels.

Figure 4 shows that lines A, B, C, and E satisfy the $\Delta\rho_{xy} = -s\rho_{xx}^{\min}$ equations over at least four orders of magnitude change in $\rho_{xx}^{\min}$. All five lines (A to E) yield extrapolated values of ρ_{xy} that are the *same* within our 0.01 ppm resolution.

An important observation is that $\Delta\rho_{xy}$ can be much larger than variations in the value of ρ_{xy} across the step. One example is shown in Fig. 1(b) for the V_H' (down) Hall step at 3.0 K which is flat to within at least 1 ppm but $\Delta\rho_{xy}$ is -4.2 ppm because $\rho_{xx}^{\min}$ is not sufficiently small (0.144 Ω). Another example for the same Hall step is shown in Fig. 2 which, for this particular cool down to 1.2 K, is flat to within at least 0.01 ppm but $\Delta\rho_{xy}$ is $-(0.017 \pm 0.001)$ ppm.

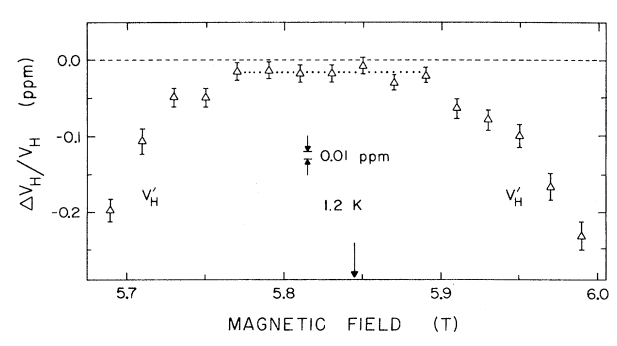

FIG. 2. A digital mapping of the V_H' (down) Hall step of GaAs(7) at 1.2 K and 25.5 μA. The 0.011-ppm one-standard-deviation uncertainties of most points were obtained in 55 min. The dashed line at $\Delta V_H/V_H = 0$ corresponds to the value of $\rho_{xy}(0)$.

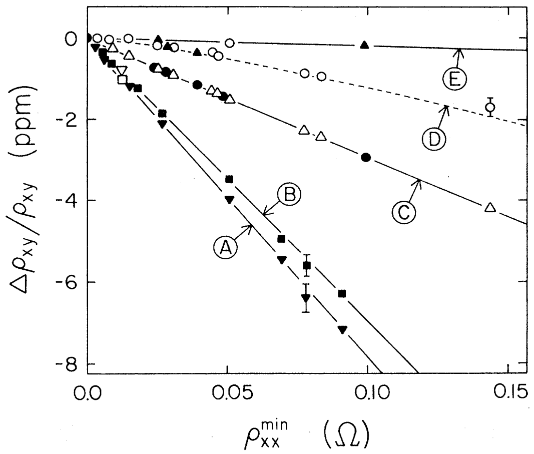

FIG. 3. The error $\Delta\rho_{xy}/\rho_{xy}$ vs $\rho_{xx}^{\min}$ for temperatures between 1.2 and 3.0 K. Circles and triangles are V_H and V_H', respectively [see Fig. 1(a)] for GaAs(7). Inverted triangles and squares are for the corresponding Hall probe sets of GaAs(8). Solid symbols are for magnetic fields pointing up and open symbols for fields pointing down. For all but three points the error bars are smaller than the symbols. Lines A and B are for GaAs(8) and C, D, and E are for GaAs(7).

There are a number of possible mechanisms for $\Delta\rho_{xy}$ being linearly dependent upon $\rho_{xx}^{\min}$, but none seem completely satisfactory. One is the finite size of the devices, but the predicted errors[6] are two orders of magnitude smaller than those observed. A second possibility is that the Hall probe sets are not perfectly aligned across the devices and thus any finite amount of V_x that occurs within the probe misalignment region will enter *directly* into the V_H measurements as an error. This possibility can be discounted for two reasons: (1) zero-magnetic-field measurements give estimates of the Hall probe set electrical misalignments that

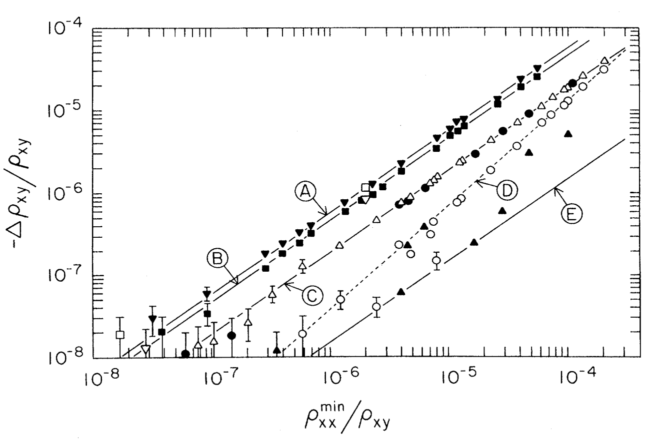

FIG. 4. Log-log plots of $\Delta\rho_{xy}/\rho_{xy}$ vs $\rho_{xx}^{\min}/\rho_{xy}$ over the temperature range 1.2–4.2 K for GaAs(7) and 1.2–3.6 K for GaAs(8).

are an order of magnitude too small; and (2) the sign of $\Delta\rho_{xy}$ does not reverse with magnetic field reversal.

A third possibility is that the $\Delta\rho_{xy} = -s\rho_{xx}^{\min}$ dependence arises from two-dimensional variable range hopping conduction.[7-9] Due to a limited temperature range, our $\rho_{xx}^{\min}$ data are equally consistent with the $\rho_{xx}^{\min} \propto T^{-2/3}\exp(-T_0/T)^{1/3}$ model[7,8] or the $\rho_{xx}^{\min} \propto T^{-1}\exp(-T_0/T)^{1/2}$ model.[8,9] However, Wysokinski and Brenig[9] predict that $\Delta\rho_{xy}$ and $\rho_{xx}^{\min}$ have different temperature dependences and therefore are not linearly related.

A fourth possibility is thermal activation across a constant energy gap. Our data (and the data of Refs. 7, 8, and 11) do not fit this $\rho_{xx}^{\min} \propto \exp(-T_0/T)$ model. Tausendfreund and v. Klitzing[11] report that *slopes* of Hall steps with inflection points at $V_x^{\min}$ fit the $\exp(-T_0/T)$ thermal activation model. Although the Hall step *slope* may perhaps be thermally activated, its *value* is certainly not.

A fifth possibility for the linear dependence of $\Delta\rho_{xy}$ is bulk leakage currents [such as three-dimensional variable range hopping conduction for which $\rho_{xx}^{\min} \propto \exp(-T_0/T)^{1/4}$ is also consistent with our $\rho_{xx}^{\min}$ data]. Bulk leakage currents proportional to V_x are a good possibility because the measured values of ρ_{xy} would always be too small. Also, when the $\Delta\rho_{xy} = -s\rho_{xx}$ corrections are applied to the inverted U steps they tend to have ideal shapes, lending support to this mechanism. Nevertheless, this possibility must be seriously questioned for these two devices because Figs. 3 and 4 show that the values of $\Delta\rho_{xy}$ observed on one Hall probe set *interchange* with those observed on the *other* probe set on magnetic field reversal. No simple leakage current mechanism can account for this interchange when the external I_y current is negligibly small. The interchange on field reversal is quite conclusive for the GaAs(7) data (lines *C* to *E*), but less so for GaAs(8) because of the smaller difference between lines *A* and *B*.

The temperature dependence of ρ_{xy} can be quite large and must be investigated for every device if it is to be used as a high-precision quantum Hall resistor or to determine the fine-structure constant. The mechanism for this dependence is not understood, but whatever the cause, there is strong evidence that the value of $\rho_{xy}(T)$ approaches a universal constant as T and $\rho_{xx}^{\min}$ approach zero.

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