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Hot electrons in heterolayers

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The theory of the dependence of electron temperature on Joule energy input, for degenerate carriers in a heterolayer, is outlined. Numerical results for GaAs electrons are computed and displayed.

The numerous experimental investigations that have been made on two-dimensional electron transport parallel to the interfaces, in inversion layers of the Si/oxide type and in heterolayers of the GaAs/(Ga,Al)As type, have included hot-electron studies.[1,2] Recently Shah *et al.*[3] have used luminescence techniques to determine $f(E)$ the electron energy distribution function (in terms of an electron-hole convolution). They found a well-defined Maxwell-Boltzmann tail for $f(E)$, with an electron temperature T_e that increased with field $\mathscr{E}$. In these essentially steady-state experiments, one may assume that the Joule input $e\mu\mathscr{E}^2$ is to be equated with an energy dissipation rate per electron, $w(T_e)$. The present Rapid Communication is on the calculation of this dissipation function when the energy loss is predominantly by net emission of optical-mode phonons, as may be assumed to be the case for most of the data of Shah *et al.*. The dissipation by acoustic-mode phonon scattering has been treated elsewhere.[4]

We assume that $f(E)$ is a Fermi function $1/\{\exp[(E-\zeta)/KT_e]+1\}$, which is probably justified (see below), that the mode occupation number N for the phonons is the Planck function $1/[\exp(\hbar\omega/KT)-1]$ for the lattice temperature T, which is not necessarily valid (but see below), and that only the lowest electron subband is involved, which will be correct in some cases but not in others. Then, for a single phonon energy $\hbar\omega_0$, the dissipation per electron is

$$w=(\hbar\omega_0/E_F)\int_0 \nu_+(E)[(f_+-f)N+f_+(1-f)]\,dE \quad , \quad (1)$$

where $f_+(E)=f(E+\hbar\omega_0)$, and $\nu_+(E)$ is the spontaneous (angle averaged) scattering rate from $E+\hbar\omega_0$ to E, and $E_F=\int_0 f\,dE$ is the degeneracy energy KT_F for the particular electron sheet density n. The density of states has appropriately been taken as independent of E. The integrand of (1) is simplified by the identity

$$f_+(1-f)=(f-f_+)N_e \quad , \quad (2)$$

where N_e is the Planck function (like N, for a phonon energy $\hbar\omega_0$) with T replaced by T_e. We may remove ν_+ from the integrand of (1), equating it to $\nu_0\equiv\nu_+(E=0)$, which should be a reasonable approximation (see below) except at highest T_e values. (For nonpolar coupling to the optical modes, as in Si, it is correct without approximation, for a single "parabolic" subband, so long as we are neglecting screening effects.) Then, by (2), Eq. (1) becomes

$$w=\nu_0\hbar\omega_0(N_e-N)F(T_e) \quad , \quad (3)$$

where

$$F=\frac{1}{E_F}\int_0(f-f_+)\,dE=\frac{T_e}{T_F}\ln\left[\frac{1-f_+(0)}{1-f(0)}\right] \quad . \quad (4)$$

For fixed electron sheet density (fixed E_F), the limit of (4) for T_e small is $\min(1,\hbar\omega_0/E_F)$, and the limit for T_e large is $1/(N_e+1)$. On simplifying the argument of ln( ) on the right of (4), we obtain a more useful form:

$$F(T_e)=(T_0/T_F)-(T_e/T_F)\ln[1+(1/N_e)\times\exp(-T_F/T_e)] \quad , \quad (5)$$

where $KT_0\equiv\hbar\omega_0$. Figure 1 shows this function of T_e, with T_0 equal to 420 K (i.e., for GaAs). The curves are, respectively, from top to bottom, for degeneracy temperature (T_F) values 100, 200, 300, and 400 K. The value of $\nu_0\hbar\omega_0$ depends on the effective width of the heterolayer. It is equal to $w_0H(k_0)$ where $w_0=4.38\times10^{11}$ eV/s for GaAs, $H(Q)$ is a previously defined coupling function for polar optical-mode scattering,[5] and $(\hbar k_0)^2/2m^*\equiv\hbar\omega_0$, giving $k_0=2.5\times10^6\ \mathrm{cm}^{-1}$ for GaAs. [In practice, $H(k_0)$ will be between 0.1 and 1.] With this value of w_0, and neglecting N compared to N_e in (3), we obtain for GaAs the dependence of T_e on w given by $w/H(k_0)=w_0N_eF$, which is displayed

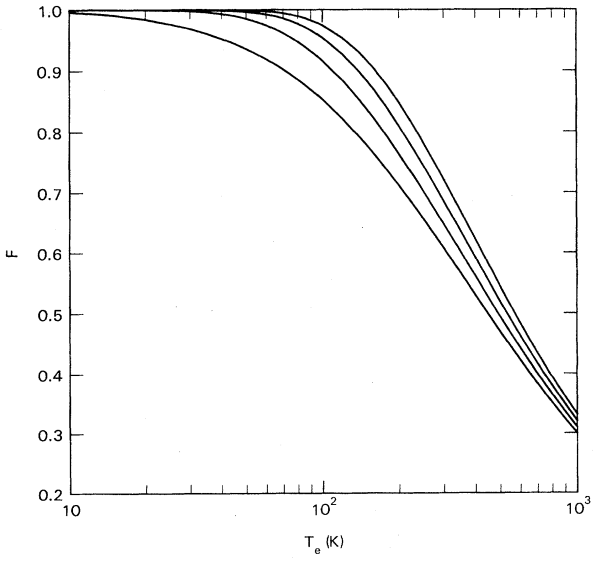

FIG. 1. The function $F(T_e)$, for GaAs, at degeneracy temperatures 100, 200, 300, and 400 K.

in Fig. 2 for the same four degeneracy temperatures as in Fig. 1 (the higher w/H value again being for the lower T_F).

To estimate the effect of replacing ν_+ in (1) by its value at $E=0$, we may take

$$\nu_+(E)=\nu_0[1-\alpha(E/\hbar\omega_0)]+O(E^2) \quad . \tag{6}$$

where it can be shown that

$$\alpha=1+[(Q/H)\,dH/dQ]_{Q=k_0} \quad . \tag{7}$$

Approximating the correction term $\int E(f-f_+)\,dE$ for $T_0>T_F>T_e$, we arrive at the correction to (4) and (5):

$$F\rightarrow F-\frac{\alpha}{2}\frac{T_F}{T_0}\left[1+\frac{\pi^2}{3}\left(\frac{T_e}{T_F}\right)^2\right] \quad . \tag{8}$$

For higher electron temperatures, the second term of (8) can be taken as $-\alpha T_e/T_0$. In practice, the value of α is somewhat less than $H(k_0)$, so that the second term on the right of (8) is $\sim 10^{-1}\times(T_e$ factor). The neglect of the second term of N_e-N in (3) is, of course, justified at lattice temperatures of a few degrees—unless the "hot phonon" effect, which can be estimated to be significant for heterolayers in the conditions of interest,[6] results in effective N values not $\ll N_e$. At electron temperatures $>T_0$, the distribution $f(E)$ should be a "hot Fermi function" even without the effect of the electron-electron scattering.[4] At lower T_e values, the powerful effect of the e-e scattering in the heterolayers can be expected to maintain this form of $f(E)$. Eventually as T_e decreases, however, we arrive at a regime where the Fermi form does not apply in general; this needs further investigation.

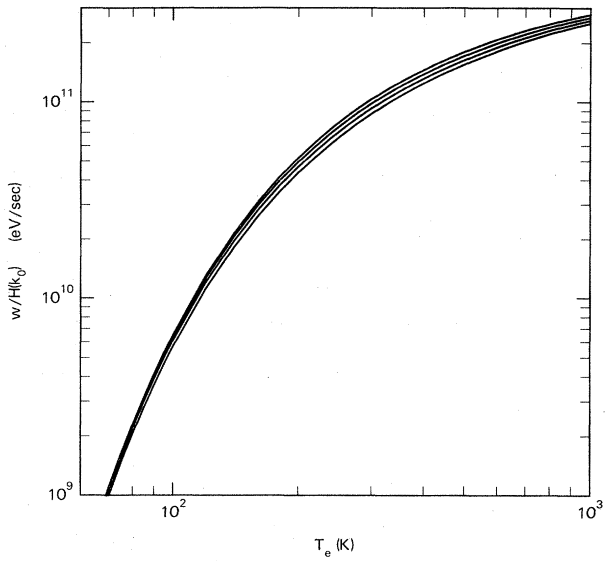

FIG. 2. Dissipation w per electron, according to (3) and (5), plotted as $w/H(k_0)$ vs T_e, for GaAs at degeneracy temperatures 100, 200, 300, and 400 K.

An estimate of the time for energy relaxation is the ratio of $\delta E\equiv\langle E-E_F\rangle$ to w. For $T_e\sim T_F\sim 200$ K, from Fig. 2 one may estimate a value $\delta E/w$ of $\sim$ a picosecond. Obviously, a range from much smaller than to much larger than this representative figure can be expected.

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