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Rapid Communications

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Polar-optical-mode scattering for an ideal quantum-well heterostructure

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It is pointed out that approximations, such as have been proposed, in the normal-direction form factor which enters into the basic lattice scattering rates for two-dimensional heterolayer transport are not necessitated by the singularities of this form factor in the ideal "quantum-well" case. The $H(Q)$ function which actually governs the polar-optical-mode scattering rates in this case is not singular. It is given as an analytical function of the two-dimensional transfer wave vector and is displayed in a numerical plot.

The purpose of this Rapid Communication is to elucidate the theoretical analysis of polar-optical-mode scattering in the "quantum-well" type of semiconductor heterostructure.[1-4] For the two-dimensionally itinerant electrons (holes) of a semiconductor heterolayer, there is no selection rule for the normal-direction component, q, of the wave vector of an absorbed or emitted phonon, as there would be in the homogeneous bulk solid. Instead, the scattering matrix element for the intraband process is proportional to the form factor[2]

$$I(q)=\int \rho(z)\exp(iqz)\,dz \ , \tag{1}$$

where $\rho(z)$ is the square of the (envelope) wave function $\Psi(z)$ for the normal-direction quantization of the electron state, with $\int \rho\,dz=1$. In the case of a quantum well, as opposed to the single interface type of heterolayer, the sinusoidal form of the $\Psi(z)$ entails singularities in $I(q)$ at multiples of aq/π, where a is the well width.[2,4] It has been proposed[4] to smooth these out by substituting a simpler approximate form for $I(q)$. This substitution is unnecessary, however, since the actual carrier scattering rate is proportional to

$$\int |M(Q,q)|^2 |I(q)|^2 dq \ , \tag{2}$$

where M is the corresponding three-dimensional scattering matrix element with $\vec{Q}$ the phonon wave-vector component in the heterolayer plane, which is a well behaved functional of $\rho(z)$ without singularities. In particular, for polar-optical-mode phonons, $|M|^2$ is proportional to $1/(Q^2+q^2)$ and consequently the scattering rate at transfer wave vector $\vec{Q}=\vec{k}_2-\vec{k}_1$ is[2,3] proportional to $H(Q)/Q$, where

$$H(Q)=\int\int \rho(z_1)\rho(z_2)\exp(-Q|z_1-z_2|)\,dz_1\,dz_2 \ . \tag{3}$$

For the limiting case of a deep quantum well, we have

$$\rho(z)=(2/a)\sin^2(\pi z/a) \ , \tag{4}$$

and hence

$$H(Q)=\frac{3}{u^2+4\pi^2}(1-e^{-u})+\frac{u}{u^2+4\pi^2}-\left\{\frac{u^2-4\pi^2}{u^2+4\pi^2}\frac{1}{u^2+4\pi^2}(1-e^{-u})\right\}+\frac{2}{u}\left\{1-\frac{1}{u}(1-e^{-u})\right\} \ , \tag{5}$$

where $u\equiv aQ$. This somewhat complicated, but quite tractable, function is plotted in Fig. 1. It would be of interest to modify (4) and hence (5) by some suitable model function

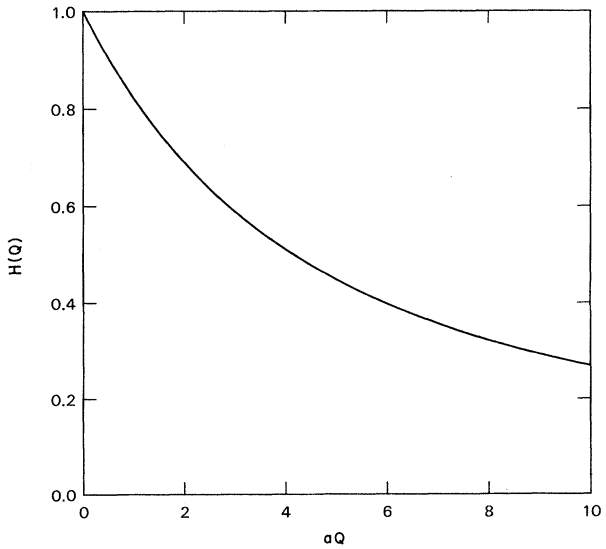

FIG. 1. The $H(Q)$ function for the ground level of an ideal "quantum well" of width a.

to allow for space-charge effects, as well as making some allowance where appropriate for penetration of $\Psi(z)$ beyond the "well" interfaces.

The transport effects are, in general, given by averages of scattering rates $\nu(\theta)=\text{const}[H(Q)/Q]$ over scattering angle θ, where

$$Q^2=k_1^2+k_2^2-k_1k_2\cos\theta \ , \tag{6}$$

and we have $\pm(k_2^2-k_1^2)=k_0^2\equiv 2m^*\omega_0/\hbar$, where the optical-mode quantum is $\hbar\omega_0$. Here we examine the limiting case which is not complicated by averaging over scattering angle: between the bottom of the subband, $E=0$, and the states of energy $E=\hbar\omega_0$ the scattering rate, apart from any screening, is just $N\nu_0H(k_0)$ (absorption) or $(N+1)\times\nu_0H(k_0)$ (emission) where N is the Planck function $1/[\exp(\hbar\omega_0/KT)-1]$, and where

$$\nu_0=\frac{\pi}{2}\frac{e^2}{\hbar}\left(\frac{1}{\epsilon_0}-\frac{1}{\epsilon_\infty}\right)k_0 \ . \tag{7}$$

For electrons in GaAs, $k_0=2.5\times10^6$ cm^{-1} and $\nu_0=1.21\times10^{13}$ sec^{-1}. [Thus, for a quantum-well width of $a=200$ Å, the value of u in Eq. (5) and Fig. 1 is 5.0, and $\nu_0H(k_0)=5.4\times10^{12}$ sec^{-1}.] The $H(Q)$ function also occurs as a coefficient of the screening constant, in the screening function for these two-dimensional systems.[5,6] For the optical-mode phonons, however, the screening constant is significantly reduced because it expresses the response to a high-frequency force.[6]

For intersubband scattering, $\rho(z)=\Psi^2$ is replaced in Eq. (3) by the product of the two $\Psi(z)$, giving an $H(Q)$ function[2] which is initially proportional to Q, has a maximum, and finally is $\sim 1/Q$ like the intraband function. For acoustic-mode scattering, the deformation-coupled process has M a constant and hence (2) is a constant inversely proportional to the heterolayer width.[1] The piezoelectric-coupled process is given[2,3] by a more complicated integral in place of (3); but still it is a smooth, well behaved function of Q.

One may compare (5) with the result for the single interface case, using the Fang-Howard-Stern wave function,[5] for which

$$\rho(z)=\tfrac{1}{2}\beta^3z^2\exp(-\beta z) \ , \tag{8}$$

the coefficient β being given by

$$\beta^3=(33\pi/2)(m^*e^2/\epsilon\hbar^2)n_{\text{eff}} \ , \tag{9}$$

where $n_{\text{eff}}=n+(\frac{32}{11})n_{\text{depl}}$ with en_{depl} the depletion charge per unit area in the accumulation layer. (In GaAs, β is normally $2\sim3\times10^6$ cm^{-1}.) Then (3) gives

$$H(Q)=\frac{\beta}{8}\frac{8\beta^2+9\beta Q+3Q^2}{(\beta+Q)^3} \ . \tag{10}$$

This function is qualitatively similar to the quantum-well one (5), and likewise for the intraband-scattering function.[3]

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