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Spin fluctuations in disordered interacting electrons

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The recent solution by Finkel'shtein of a model of two-dimensional interacting disordered electrons is reconsidered. The scaling equation requires a small modification, with the result that the conductivity remains finite down to zero temperature instead of going to infinity as envisaged by Finkel'shtein. Strong divergence in the spin susceptibility and a vanishing of the spin-diffusion constant are found at low temperatures. We also discuss the possibility that the spin fluctuation may lead to crossover to the high magnetic field or singlet-only universality classes, restoring the metal-insulator transition.

Recently it has been understood that both localization and interaction effects are extremely important in disordered electronic systems.[1] Although there now exist satisfactory scaling theories for the noninteracting localization problem,[2–4] similar theories with interactions included are still absent. Nevertheless, important progress has been made by Finkel'shtein,[5] who mapped the problem into the appropriate nonlinear σ model, and constructed the scaling theory for the interacting system, albeit with localization effects ignored. The scaling analysis is carried out to all orders in the interaction coupling constants and first order in the dimensionless resistance $t=(2\pi\epsilon_F\tau)^{-1}$. Upon scaling it is shown that the resistivity decreases with temperature and vanishes at $T=0$ in two dimensions. Some aspects of Finkel'shtein's results were recently clarified in terms of Fermi-liquid theory.[6]

In this paper we show that the perfect conductor cannot be attained, even at $T=0$. Instead, there is a dynamically generated finite length scale which serves as the infrared cutoff for the theory. It will turn out that this mechanism is closely related to a nearby ferromagnetic instability of the Fermi liquid. The strong magnetic fluctuation is responsible for the absence of a metal-insulator transition in this model. The mechanism discussed here is distinct from the effect of Zeeman splitting discussed elsewhere,[7,8] which produces insulating ground states.

Let us recall Finkel'shtein's scaling equation for the resistance in two dimensions. First-order perturbative calculations in the interactions have shown that the Hartree and Fock terms give competing logarithmic corrections to the conductivity.[9] Because of the long-ranged Coulombic interactions, the Fock term dominates to this order and the system is driven towards the insulator. Finkel'shtein[5] has generalized the Hartree-Fock calculation to all orders in the interactions by introducing the Fermi-liquid amplitudes Γ and Γ_2 for small and large angle scattering, respectively. The scaling equations for t and Γ's under a change of the cutoff are then derived. In addition, a temperature renormalization constant z has to be introduced. There is a Fermi-liquid constraint:

$$z-2\Gamma+\Gamma_2=0 \ . \tag{1}$$

The resistance scales as

$$\frac{dt}{d\xi}=t^2\left(4-3\frac{z+\Gamma_2}{\Gamma_2}\ln\frac{z+\Gamma_2}{z}\right) \ , \tag{2}$$

where $\xi=-\ln\lambda$ and λ is the energy scale. According to Finkel'shtein, Γ, Γ_2, and z scale as $(-\ln\lambda)^{2.25}$ with $\gamma_2=\Gamma_2/z\rightarrow 4$. Thus, the second term in Eq. (2) overcomes the first and $t\rightarrow 0$ as $\lambda\rightarrow 0$. As λ decreases, fluctuations are being integrated out, until they are all removed when λ reaches zT. Hence, to convert the cut-off dependence to temperature dependence one sets $\lambda=zT$. This means the resistance $t\rightarrow 0$ as $T\rightarrow 0$.

It is important in the above that Γ, Γ_2, and z only diverge as $(-\ln\lambda)^c$. In fact, we have found a slight error in Finkel'shtein's equations for Γ and Γ_2 which invalidates this conclusion. We believe the third and fourth term in Eq. (3.39) of Ref. 5 coming from Figs. 9(c) and 9(d) are too small by factors of 2. The new equations for Γ and Γ_2 now read

$$\frac{d\Gamma}{d\xi}=t\left(\Gamma_2+\frac{\Gamma_2^2}{z}\right) \ , \tag{3}$$

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and

$$\frac{d\Gamma_2}{d\xi}=t\left[\Gamma+\frac{2\Gamma_2^2}{z}\right] . \quad (4)$$

Following Finkel'shtein we define $w=\Gamma_2/\Gamma$; it scales as

$$\frac{dw}{d\xi}=t . \quad (5)$$

Thus, w increases as λ decreases. However, $w>2$ is impossible since that means $z<0$ from Eq. (1), signifying an instability in the Fermi liquid. Analyzing Eqs. (1)–(5) we find that $w\rightarrow 2$ at a finite ξ_c (or λ_c). At ξ_c, t is finite. Close to ξ_c we have $2-w\approx\xi_c-\xi$. It is instructive to introduce the coupling constant $\gamma_2=\Gamma_2/z$ which obeys the scaling equation

$$\frac{d\gamma_2}{d\xi}=\frac{t}{2}(\gamma_2+1)^2 . \quad (6)$$

We see that γ_2 scales towards strong coupling as ξ approaches ξ_c and the solution to Eq. (6) has the typical single pole structure:

$$\gamma_2\sim(\xi_c-\xi)^{-1} . \quad (7)$$

Thus, Γ_2 diverges more strongly than z, and we have

$$\Gamma,\Gamma_2\sim\left[\frac{1}{\xi-\xi_c}\right]^4 \quad (8)$$

and

$$z=\Gamma(2-w)\sim\left[\frac{1}{\xi-\xi_c}\right]^3 . \quad (9)$$

Since λ_c is finite, one might expect this instability to occur at a finite temperature. Actually this is not so; this is because the temperature scale is renormalized by z, which is now strongly divergent. Setting $\lambda=zT$ and using Eq. (9), we obtain

$$\lambda-\lambda_c\sim T^{1/3} \quad (10)$$

and so $T=0$ when $\lambda=\lambda_c$. Nevertheless, the length scale associated with λ_c is

$$L_c=[D(\lambda_c)/\lambda_c]^{1/2} \quad (11)$$

and is finite. Thus even at $T=0$ there is a dynamically generated infrared cutoff which prevents the system from becoming a perfect conductor. The physical significance of L_c will be discussed later on.

So far we have assumed the perturbative scaling equations to be valid till $\lambda=zT$. A more careful analysis of Eq. (2), for example, shows this cannot be so. In Finkel'shtein's solution, although Γ_2, z diverge, their ratio remains finite while $t\rightarrow 0$, thus making the weak-coupling assumption more and more precise under scaling. In the present case, close to λ_c, the equation becomes

$$\frac{d\ln t}{d\xi}=t\left[4-3\ln\left[\frac{1}{\lambda-\lambda_c}\right]\right] , \quad (12)$$

and since t is finite at λ_c, $t\ln[1/(\lambda-\lambda_c)]$ is no longer small near λ_c and we are in the strong-coupling regime. The perturbative renormalization procedure is no longer valid. There is an additional complication. Consider, for example, the contribution to Γ_2 coming from Fig. 10(a) of Ref. 5:

$$\delta\Gamma_2=-2t\Gamma_2^2\int_T^{\lambda}d\omega\int_0^{\lambda}\left[\frac{1}{z_2\omega+x}\right]^2dx , \quad (13)$$

where $z_2=z+\Gamma_2$, $x=Dq^2$. Initially zT, $z_2T\ll\lambda$, and Eq. (12) gives $2(T_2^2/z_2)\ln(T/\lambda)$. However, under renormalization z_2T becomes $\gg\lambda$ before $\lambda\simeq zT$ [see Eqs. (7) and (8)] and the logarithmic singularity is cut off. Thus, the scaling equations break down already when $\lambda<\lambda_s=z_2(\lambda_s)T$, before the instability is reached. Equations (7)–(9) probably overestimate the divergence of Γ, Γ_2, and z, but we are unable to calculate the correct behavior because of the scaling to strong coupling and the breakdown of the scaling equations themselves.

We next discuss the magnetic susceptibility and the spin-diffusion constant. The triplet response function contains the pole $(-iz_2\omega+Dq^2)^{-1}$, which leads to the natural interpretation that the spin-diffusion constant D_s is given by

$$D_s=D/z_2 . \quad (14)$$

Here, D is the charge diffusion constant [apart from a Fermi-liquid correction of $(1+F_0)$].[5] Thus, we see that spin diffusion scales to zero, while charge diffusion remains finite. The ratio of the magnetic susceptibility χ to the Pauli susceptibility χ_0 can be shown to obey the following scaling equation.[10]

$$\frac{d(\chi/\chi_0)}{d\xi}=2z_2(\Gamma_2/z)t \quad (15)$$

and is consistent with known perturbative results.[11,12] Combining Eq. (15) with the revised scaling equations [Eq. (3) and (4)], we obtain the simple result that

$$\chi/\chi_0=z+\Gamma_2 . \quad (16)$$

Equations (14) and (16) are reminiscent of the Fermi-liquid theory of spin fluctuations. If we use Eqs. (8) and (10) we obtain $\chi/\chi_0\sim T^{-4/3}$ and $D_s\sim T^{4/3}$. However, we must remember that Eqs. (8) and (10) are incorrect near the instability and the exponent $\frac{4}{3}$ should not be taken seriously. The vanishing of D_s suggests formation of pseudo local spin moments (pseudo because charge diffusion is finite). Furthermore, we recall that a finite length scale L_c given by Eq. (11) remains at the end of the scaling process, so that $\chi(q,\omega)$ is indepedent of q for $q\lesssim q_c=2\pi/L_c$. This suggests an interpretation of L_c as a mean size of the pseudo local spin moments.

In this paper we have reconsidered the "pure" interacting problem of Finkel'shtein in two dimensions. We have shown that in spite of the conductivity increase due to the Hartree term dominating at low T, there is a dynamically generated infrared cutoff which causes the resistivity to remain finite. The low-temperature state seems to be describable by local spin fluctuations. It is appropriate to remark here the Finkel'shtein's model is physically valid only if Zeeman splitting can be ignored and magnetic impurities, which suppress triplet fluctuations, are absent. Otherwise, it crosses over into the "high magnetic field" or "singlet-only" universality classes considered in Refs. 7 and 8. In these cases, the conductivity decreases with temperature in two dimensions. The divergence of χ at low temperature implies a crossover to the Zeeman region even for a small magnetic field. At the same time, the onset of pseudo local spin fluctuations may lead to a suppression of triplet fluctuations and the crossover to the "singlet-only" regime. Consequently, the temperature range in which Finkel'shtein's prediction of conductivity rise with decreasing temperature is expected to be extremely narrow, which may explain why such an increase has not been observed experimentally. Experimental measurements of the spin susceptibility will obviously be of great interest.

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