RAPID COMMUNICATIONS

PHYSICAL REVIEW B VOLUME 30, NUMBER 3 1 AUGUST 1984

Renormalizability of the density of states of interacting disordered electron system

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(Received 23 April 1984)

A perturbative analysis of the single-particle density of states for an interacting disordered Fermion system is carried out up to second order both in the inverse conductance and in the interaction when the singularity in the particle-particle channel is suppressed. By imposing the renormalizability condition in two dimensions on our perturbative expression of the single-particle density of states, we obtain the group equation for a particular combination of the effective couplings due to the interaction at first order in the inverse conductance. This result agrees with the one obtained from the one-loop analysis of the effective-field theoretic Lagrangian obtained by Finkel'stein for the same system. The present calculation is the only available check of that theory at second order in the inverse conductance.

Despite many attempts a general scaling theory of the interacting disordered electron systems does not yet exist. Two mechanisms are at the origin of this problem: the well-established localization of a single electron in a random potential and the interaction among electrons in the presence of disorder. They are already hard when treated separately and become extremely difficult when combined together.

For the single-electron localization[1,2] the two-dimensional case turned out to be of special importance because of logarithmic singularities in the conductivity. This singularity is due to the well-known summation of the impurity ladder in the particle-particle channel.[3] This summed up ladder is called the Cooper propagator. The corresponding summation in the particle-hole channel leads to the impurity scattering amplitude which is called diffuson. Both the Cooper propagator and the diffusion propagator have diffusive poles (if no magnetic field is present) and these appear also in the density-density response function.[4,5]

When interactions between the electrons are present, together with a weak impurity scattering, the diffusive form of the density-density response function implies drastic corrections to the normal Fermi liquid theory. In two dimensions the interaction $V(q)$ leads to additional logarithmic singularities in the conductivity. The single-particle density of states, which is finite in the one electron problem, also becomes logarithmically singular. In the case of short-range forces between the electrons, to lowest order in the effective coupling t_0 for the system with pure disorder ($t_0 = 1/(2\pi)^2\sigma_0$, σ_0 is the Drude conductivity) and in the interaction, the conductivity σ and the single-particle density of states N, as a function of frequency Ω at zero temperature are given by[6,7]

$$\sigma = \sigma_0 \left(1 + t_0 [1 + V_1 + V_3 - s(V_2 + V_4)] \ln \frac{\Omega}{\Lambda^2} \right) , \quad (1)$$

$$N = N_0 \left(1 + t_0 [V_1 + V_3 - s(V_2 + V_4)] \ln \frac{\Omega}{\Lambda^2} \right) , \quad (2)$$

where Λ^2 is a suitable cutoff of the order of the inverse of the elastic scattering time τ and s is the spin multiplicity. The different V_i's in Eqs. (1) and (2) are the effective interactions generated by $V(q)$. $V_1 = N_0 V(0)$ and $V_3 = F V_1$ (F being defined in Ref. 6) are associated with the Fock terms with diffusion propagators and Cooper propagators dressing at the vertices, respectively. V_2 and V_4 are the corresponding interactions for the Hartree diagrams. They coincide with V_3 when its frequency dependence is not considered. For their detailed definition see Refs. 7 or 8. In the case of long-range forces a $\ln^2$ term[6] appears in Eq. (2) instead of the simple ln term. This makes it questionable if N is going to be a scaling quantity even in the short-range case.

For the interpretation of the experiments a complete scaling theory is essential which sums up all the logarithmic singularities. This could be achieved either by matching the perturbative expressions of some physical quantities to a scaling behavior (thus assuming that they are scaling quantities) or by discovering the underlying field theoretical model.

A first attempt to produce a scaling theory[9] turned out to be based on oversimplified assumptions.[8,10] On the hypothesis that the V_3 and V_4 contributions could be summed to zero, a scaling theory was subsequently proposed[11] by matching to a scaling behavior a second-order (both in t_0 and in the interaction) expression of the three physical quantities σ, N, and the spin susceptibility. The effective coupling associated to V_2 was shown to scale to zero. However, certain diagrams in their perturbative expressions of the physical quantities were omitted. In fact, a gauge invariant calculation of the single particles density of state[8] showed that at first order in V and at second order in t_0 there are two contributions associated with V_1 and V_2:

$$\delta N = -N_0 t^2 [(V_1 - sV_2) + (V_2 - sV_1)] \ln^2 \frac{\Omega}{\Lambda^2} \quad (3)$$

and similarly for $V_3 = V_4$. The first term in the square bracket of (3) is in line with the first-order result given by Eq. (2). In the second term instead the role of V_1 and V_2 is interchanged. As we shall see due to this last term, which was missing in the analysis of Ref. 11, even if V_2 would have been zero to start with, it would be generated by V_1 and vice versa. It was also observed that infinite resummations in the V's could be carried out without changing the order in t_0.

At the same time substantial progress was achieved by Finkel'stein[10] who constructed, at least under some restrictions, an effective Hamiltonian and its one loop renormalization group analysis for the interacting disordered electron system with long-range forces. In order to simplify the problem he introduces a magnetic field to suppress the singular behavior associated with the Cooper propagators. One has then to deal with V_1 and V_2 only. This magnetic field is small enough, however, not to produce any Zeeman splitting in the spin channels of the diffusion propagators. Because of these assumptions, in reality this model could be valid only in a narrow region of temperature. Even so it is a very feasible model, which, not dealing with Cooper propagators, can be considered as the ideal case for the "pure" interaction problem. The interpretation of the Finkel'stein analysis in terms of perturbative results starting from the weak disorder limit was carried out by Castellani, DiCastro, Lee, and Ma,[12] together with its extension to the short-range forces case and to the physically relevant situations which arise when either the full effect of the magnetic field or the spin flip due to magnetic impurities are considered.[13,14]

The renormalization-group analysis of Finkel'stein is carried out at first order in t_0 and at any order in V. Under the same assumptions of neglecting Cooper propagators, we present here a perturbative calculation of the single-particle density of states up to second order both in t_0 and V for the short-range case at zero temperature. We show that the renormalizability of the perturbative expression of N is implied by the renormalization group equations for the couplings given by Finkel'stein. Vice versa we can follow the idea of obtaining the group equations for the couplings by a matching procedure of the perturbative expressions of physical quantities. We therefore impose the renormalizability condition on our second-order perturbative expression for N and obtain, at first order in t_0 and second order in V, the group equation for a suitable linear combination of the couplings, which is in agreement with the Finkel'stein result.

Let us recall that the infinite resummation in V has been carried out[10] by summing the static part first, with no diffusion propagator present. V_1 and V_2 are then simply substituted by the corresponding static amplitude U_{10} and U_{20}. Instead, by performing the summations associated with the diffusion propagators one obtains the dynamic amplitudes

$$\tilde{U}_{10}=\frac{U_{10}(-i\Omega z_0+D_0k^2)^2}{[-i\Omega(z_0-2U_{10}+U_{20})+D_0k^2][-i\Omega(z_0+U_{20})+D_0k^2]}\ ,\quad \tilde{U}_{20}=U_{20}\frac{-i\Omega z_0+D_0k^2}{-i\Omega(z_0+U_{20})+D_0k^2}\ , \tag{4}$$

where D_0 is the diffusion constant and z_0, whose bare value is unity, is the coupling associated with the term of the Finkel'stein's effective Hamiltonian where the frequency acts as a source. In perturbation theory z_0 starts deviating from unity signaling the existence of a renormalization parameter of the frequency in the diffusion propagators. The amplitude associated with the long-range part of the potential is not considered in the present analysis.

$\tilde{U}_{10}$ and $\tilde{U}_{20}$ contain the relevant "hydrodynamical modes" of the theory. The perturbative analysis in t_0 of the physical quantities like N and σ can now be carried out at any order in V_1 and V_2 by substituting V_1 and V_2 with $\tilde{U}_{10}$ and $\tilde{U}_{20}$ in the corresponding diagrams.[10,12,15]

For N at first order in t_0, instead of the V_1 and V_2 terms of Eq. (2), one obtains

$$\Gamma_N=\frac{N}{N_0}=1+t_0\left(\frac{1}{2}\ln\frac{1+\gamma_{20}}{1-2\gamma_{10}+\gamma_{20}}-2\ln(1+\gamma_{20})\right)\ln\frac{\Omega}{\Lambda^2}\ , \tag{5}$$

where $\gamma_{10}=U_{10}/z_0$ and $\gamma_{20}=U_{20}/z_0$.

For our future purpose we need to expand Γ_N up to second order in γ_{10} and γ_{20}:

$$\Gamma_N=1+a\ln\frac{\Omega}{\Lambda^2}\ , \tag{6}$$

where

$$a=t_0(\gamma_{10}-2\gamma_{20}+\gamma_{10}^2+\gamma_{20}^2-\gamma_{10}\gamma_{20})\ . \tag{7}$$

The linear terms in a correspond to the original perturbative results given by Eq. (2) when Cooper propagators are suppressed. The quadratic terms in a come from the dynamical resummation in the amplitudes.

We now calculate the perturbative terms of the single-particle density of states N at second order in t_0, as well as in γ_{10} and γ_{20}. In Ref. 8 part of this calculation has already been presented. In that work, as already pointed out, N has been evaluated up to second order in t_0 and first order in V when both Cooper propagators and diffusion propagators are present. In the present approximation we ignore Cooper propagators and the first term in the square bracket of Eq. (3) vanishes, whereas the second term is reduced by a factor of $\frac{1}{2}$. The second-order term both in t_0 and V was shown in (8) to add up to zero, provided no crossing of any impurity line is allowed (i.e., no Cooper propagators and no crossing of diffusion propagators). Within the present approximation we allow crossing of diffusion propagators and, in addition to the relevant diagrams of Appendix B of Ref. 8, many more diagrams contribute to the term $t_0^2\gamma_{\dots}^2\ln^2\Omega/\Lambda^2$. Again their sum is zero. Finally in the present model the perturbative analysis of N up to second order in t_0, γ_{10}, and γ_{20} leads to

$$\Gamma_N=1+a\ln\frac{\Omega}{\Lambda^2}+b\ln^2\frac{\Omega}{\Lambda^2}\ , \tag{8}$$

where a is given by Eq. (7) and b is equal to

$$b=-\tfrac{1}{2}t_0^2(\gamma_{20}-s\gamma_{10}) \tag{9}$$

We are now in the position to impose a multiplicative renormalization[16] on Γ_N in two dimensions. If we define in general

$$\bar{\Gamma}_N=z_N\Gamma_N\left(\frac{\Omega}{\Lambda^2}\right) \tag{10}$$

and introduce a normalization point λ^2 such that

$$\bar{\Gamma}_N(\Omega=\lambda^2)=1\ ,$$

then from Eq. (8) at the order we are considering, we have

$$z_N^{-1}=1+a\ln\frac{\lambda^2}{\Lambda^2}+b\ln^2\frac{\lambda^2}{\Lambda^2} \quad . \tag{11}$$

Substituting Eqs. (8) and (11) in Eq. (10), leads to

$$\overline{\Gamma}_N=1+a\ln\frac{\Omega}{\Lambda^2}+b\ln^2\frac{\Omega}{\Lambda^2}-a^2\ln\frac{\Omega}{\Lambda^2}\ln\frac{\lambda^2}{\Lambda^2} -a\ln\frac{\lambda^2}{\Lambda^2}+(a^2-b)\ln^2\frac{\lambda^2}{\Lambda^2} \quad . \tag{12}$$

In building up the renormalized $\overline{\Gamma}_N$, the couplings t_0, γ_{10}, and γ_{20} appearing in a and b have to be rewritten in terms of their renormalized expressions t, γ_1, and γ_2, which at the leading ln term are given by

$$t_0=t\left(1+tC_t\ln\frac{\lambda^2}{\Lambda^2}\right),\quad \gamma_{i0}=\gamma_i+tC_i\ln\frac{\lambda^2}{\Lambda^2} \quad , \tag{13}$$

where i stands for 1 and 2, respectively, and C_t and C_i depend on γ_1 and γ_2.

Since we are working up to second order in the couplings, their renormalization has to be considered only in the a terms of Eq. (12). In the b terms t_0, γ_{10}, and γ_{20} are simply replaced by t, γ_1, γ_2. Let us then make the dependence on λ^2 in a explicit by substituting Eq. (13) in Eq. (7). At leading orders we obtain

$$a=a_0+a_1\ln\frac{\lambda^2}{\Lambda^2} \quad , \tag{14}$$

where a_0 and a_1 are now expressed in terms of the renormalized couplings:

$$\begin{aligned} a_0&=t(\gamma_1-2\gamma_2),\\ a_1&=t^2[C_1-2C_2+2(\gamma_1-\gamma_2)(C_1-C_2)\\ &\quad +C_1\gamma_2+\gamma_1C_2+C_t(\gamma_1-2\gamma_2)] \quad . \end{aligned} \tag{15}$$

Renormalizability of N means that when Eq. (14) is used in Eq. (12), $\overline{\Gamma}_N(\Omega,\Lambda^2,\lambda^2)$ should depend only on Ω/λ^2. We obtain the condition

$$a_1-a_0^2=-2b \quad . \tag{16}$$

It is clear from Eq. (15) that we need to know C_t only up to linear terms in γ_1 and γ_2. According to Eq. (1) C_t reads

$$C_t=\gamma_1-2\gamma_2 \quad . \tag{17}$$

As far as the C_i's are concerned, we need their expressions up to the second order in the γ's.

We can now use Eq. (16) in two different ways. Either we assume the Finkel'stein expressions for C_1 and C_2,[10] or we try to obtain information on C_1 and C_2 from Eq. (16).

In the first case, recalling that from Finkel'stein C_1 and C_2 are given by[10,17]

$$C_1=\gamma_2+(\gamma_1-\gamma_2)^2,\quad C_2=\gamma_1+\gamma_1\gamma_2 \quad . \tag{18}$$

It is easy to verify that the expression (15) for a_0 and a_1 satisfies the renormalizability condition (16) with b given by our perturbative expression of Eq. (9).

Vice versa without using Eq. (18) we can assume that within our perturbative analysis only the linear terms γ_2 for C_1 and γ_1 for C_2 are known via the terms in N which invert the role of γ_1 and γ_2 and write

$$C_1-2C_2=\gamma_2-2\gamma_1+f(\gamma_1^2,\gamma_2^2,\gamma_1\gamma_2) \quad , \tag{19}$$

where f is unknown.

Substituting Eqs. (17) and (19) in Eq. (15) for a_1, up to second order in γ_1 and γ_2 we have

$$a_1=t^2[C_1-2C_2-2(\gamma_1-\gamma_2)^2+\gamma_2^2+\gamma_1^2+(\gamma_1-2\gamma_2)^2] \quad . \tag{20}$$

The renormalizability condition (16) for N determines f to be

$$f(\gamma_1^2,\gamma_2^2,\gamma_1\gamma_2)=\gamma_1^2+\gamma_2^2-4\gamma_1\gamma_2 \quad . \tag{21}$$

The result given by Eq. (21) when inserted in Eq. (19) leads to $C_1-2C_2=\gamma_2-2\gamma_1+\gamma_1^2+\gamma_2^2-4\gamma_1\gamma_2$ in agreement with Finkel'stein expressions (18).

In the short-range case by imposing that our perturbative expression for N up to a second order in t and in the interaction is multiplicative renormalizable, it is therefore possible to obtain a linear combination of the Finkel'stein group equations for the couplings at first order in t.

In conclusion, the renormalizability of N up to second order in t is compatible with the Finkel'stein group equations for the couplings at first order in t. This result can also be considered as the only available check of that theory at second order in t.

This achievement is of particular importance because the appearance of the $t\ \ln^2$ term in the single-particle density of states for the long-range case made it questionable if this last quantity would scale at all even in the short-range case. As it was shown by Finkel'stein and further analyzed in Refs. 12 and 15, the single-particle density of states can be completely eliminated via a wave function renormalization from the group equations for the couplings. Here we have shown further that in the short-range forces case, N is in itself renormalizable at least at the order considered, and it is therefore a good candidate to be a scaling quantity.

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