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Channeling radiation in a periodically distorted crystal

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X rays emitted by electrons channeling through a periodically distorted lattice are studied. The lattice distortion works as a wiggler. The intensity of the radiation due to the resonance scattering of the wiggler field can be made comparable with that of the channeling radiation.

X rays emitted by electrons and positrons channeling through crystal lattices constitute an attractive radiation source.[1,2] They also give us important information on the properties of crystals. The purpose of this paper is to discuss the character of x rays produced by electrons or positrons channeling through a distorted lattice. Either static strain in superlattices[3] or ultrasonic waves can provide the periodic distortions.

The channeling electrons (or positrons) propagate freely along the crystal axis or planes, but they are bound in the perpendicular directions by the potential of the atomic array[2]

$$\Phi = \Phi(\vec{r}_\perp, z') \quad . \tag{1}$$

We choose z' along the beam propagation, and $\vec{r}_\perp$ is the perpendicular position vector. An attractive potential traps the bound states. The period in z' is the lattice constant in the undistorted lattice.

We first consider long-wavelength (compared to the lattice constant) phonons of wave number $\vec{k}_p = k_\perp \hat{r}_\perp + k_z \hat{z}'$ and frequency ω_p. The phonons distort the lattice, and the atomic sites deviate from their equilibrium positions by $\vec{r}_0 \cos(k_z z' - \omega_p t')$. Radiation from electrons channeling through this distorted crystal is the focus of this paper. Although we describe only the case of axial electron channeling here, the theory will work for positrons and planer cases as well.

The distorted potential will be represented by Eq. (1) if $\vec{r}_\perp$ is replaced by $\vec{r}_\perp - \vec{x}_0 \cos(k_z z' - \omega_p t')$, where $\vec{x}_0$ is a component of $\vec{r}_0$ perpendicular to $\hat{z}'$. The axial potential, then, consists of a rapidly oscillating component determined by the lattice constant and a slowly oscillating component determined by the phonon wavelength. The rapid oscillations induce bremsstrahlung upon the passage of electrons. Since we are interested in the slow oscillations, we will average the potential over a lattice constant.[2] The electron velocity v is comparable to the velocity of light c and is much greater than ω_p/k_z so that the lattice distortions appear to be almost static. The periodical distortion may also be produced by a strained-layer superlattice, in which case the distortion is entirely static, i.e., $\omega_p = 0$.

We discuss the problem in the rest frame of the electrons. After the Lorentz transformation, we obtain the scalar potential

$$\Phi(\vec{r}_\perp, z) = \gamma\phi(\vec{r}_\perp) - \gamma \vec{x}_0 \cdot \nabla\phi(\vec{r}_\perp)\cos\xi \quad , \tag{2}$$

and the axial component of the vector potential

$$A_z(\vec{r}_\perp, z) = \beta\Phi(\vec{r}_\perp, z) \quad , \tag{3}$$

where $\gamma = (1-\beta^2)^{-1/2}$,

$$\beta = v/c \quad ,$$

and

$$\xi = \gamma k_z (z - vt) \quad .$$

The channeling electrons are bound in the potential well $\gamma\phi$ and are perturbed by an oscillating potential $\gamma\vec{x}_0 \cdot \nabla\phi \cos\xi$ at the frequency of $\omega_w = \gamma k_z v$. We should note here that $-\nabla\gamma\vec{x}_0 \cdot \nabla\phi\cos\xi$ is the electric field seen by the channeling electrons having a straight trajectory. This field is entirely different from the crystal field appearing in solids, and we call it the distorted lattice field.

The Hamiltonian for our system in the electron rest frame is written as

$$H = H_0 + H_1 + H_2 \quad , \tag{4a}$$

where

$$H_0 = \frac{\vec{p}\cdot\vec{p}}{2m} + e\gamma\phi(\vec{r}_\perp) + H_{\rm rad} = H_0^e + H_{\rm rad} \quad , \tag{4b}$$

$$H_1 = -\frac{e}{mc}\vec{p}\cdot\vec{a} - e\gamma\vec{x}_0\cdot\nabla\phi(\vec{r}_\perp)\cos\xi \quad , \tag{4c}$$

$$H_2 = \frac{e^2}{2mc^2}\vec{a}\cdot\vec{a} \quad , \tag{4d}$$

and $\vec{a}$ and $H_{\rm rad}$ are the vector potential and the Hamiltonian of the radiation, respectively.

The time-independent energy,

$$H_0|i,n_{k\alpha}\rangle = [E_i + \hbar\omega_k(n_{k\alpha} + \tfrac{1}{2})]|i,n_{k\alpha}\rangle \quad , \tag{5}$$

consists of the electron energy E_i, and the energy of photons of wave number $\vec{k}$, and the polarization $\hat{\epsilon}_\alpha$. The frequency of the photons ω_k equals ck. The perpendicular en-

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ergy levels of electrons are determined by the attractive potential $\gamma\phi(\vec{r})$. The vector potential A_z given by Eq. (3) shifts the energy levels in two ways. The azimuthal magnetic field, $\vec{B}=\partial A_z/\partial r\hat{\theta}$, causes spin-orbit coupling. The average spin-orbit energy $\langle(e/2mc)\vec{\sigma}\cdot\vec{B}\rangle$ is estimated to be $\langle e\gamma\phi\rangle/$(Compton wave number × radial scale length of ϕ). A term $(e/mc)p_zA_z$ also arises in the Hamiltonian. The average value of this term is of the order of magnitude $\langle e\gamma\phi\rangle\times$(breadth of the beam energy per mean beam energy in the laboratory frame). These two contributions arising from A_z are small and, therefore, neglected in Eq. (4).

We shall assume that initially the electron is at the state $|i\rangle$ and there is no photon present, i.e., $n_{k\alpha}=0$. Due to the perturbations H_1 and H_2, photons are emitted. The first-order interaction process is the spontaneous emission of photons, which is the channeling radiation.[1,2] The photon emission rate, the number of photons of frequency $\omega_k=\omega_{if}\equiv(E_i-E_f)/\hbar$ emitted into the solid angle of $d\Omega$ per electron per second, is given by

$$P_{fi}^{(e)}d\Omega=\frac{1}{2\pi}\alpha^3a_0^2\frac{\omega_{\rm if}}{\hbar^2}|\langle f|\hat{\epsilon}_\alpha\cdot\vec{p}|i\rangle|^2d\Omega \ , \tag{6}$$

where α is the fine-structure constant and a_0 the Bohr radius. There are two other first-order processes: the emission and absorption of the distorted-lattice energy. In these processes, no photon is created. However, the electrons are efficiently pumped into higher energy levels in the absorption process.

The scattering of the distorted-lattice field by the channeling electrons also causes emission of photons. Since the interaction energy $\gamma\vec{x}_0\cdot\nabla\phi$ is very large compared to $(e/mc)\vec{p}\cdot\vec{a}$, it may be as important as the spontaneous emission of photons. The processes we consider are the absorption of distorted-lattice field energy followed by the emission of a photon and the reversed process. The contribution of H_2 in Eq. (4) can be neglected because it is the higher-order process. The transition probability for this scattering is given by

$$P_{fi}^{(s)}d\Omega=\frac{2\pi\omega_k^2}{\hbar^2(2\pi c)^3}|U_{fi}|^2d\Omega \ , \tag{7}$$

with the matrix element

$$U_{fi}=\frac{e}{mc}\left(\frac{2\pi\hbar}{\omega_k}\right)^{1/2}cx_0\sum_n\left(\frac{(\vec{p}\cdot\hat{\epsilon}_\alpha)_{fn}(\nabla\gamma\phi)_{ni}}{E_n-E_i-\hbar(\omega_w+i\delta\omega_{ni})}+\frac{(\nabla\gamma\phi)_{fn}(\vec{p}\cdot\hat{\epsilon}_\alpha)_{ni}}{E_n-E_i+\hbar(\omega_k+i\delta\omega_{ni})}\right) \ , \tag{8}$$

where $(f)_{ij}$ stands for $\langle i|f|_j\rangle$. The frequency breadth $\delta\omega_{ni}$ represents the sum of the inverse lifetime of the nth $(1/\tau_n)$ and ith $(1/\tau_i)$ level and the inverse transit time of the electrons through the crystal $(1/\tau_t)$. We have assumed that the breadth of the levels due to the finite lifetime is narrower than the separation of the levels, i.e., $\delta\omega_{ni}\ll|\omega_{ni}|$. In the case of coherent scattering that both initial and final states are in the same energy level, the transition probability has a form

$$P_{ii}^{(s)}d\Omega=\frac{1}{2\pi}\alpha^3a_0^2x_0^2\omega_k\hbar^{-4}\left|\sum_{n,\alpha}\left(\frac{(\vec{p}\cdot\hat{\epsilon}_\alpha)_{in}(p_x)_{ni}}{1-(\omega_k+i\delta\omega_{ni})/\omega_{ni}}-\frac{(p_x)_{in}(\vec{p}\cdot\hat{\epsilon}_\alpha)_{ni}}{1+(\omega_k+i\delta\omega_{ni})/\omega_{ni}}\right)\right|^2d\Omega \ . \tag{9}$$

We have used a relation $(\gamma\nabla_x\phi)_{ij}=(E_i-E_j)(p_x)_{ij}/i\hbar$ which is derived from the commutation relation $[H_e^0,\vec{p}]=i\hbar\gamma\nabla\phi$, where subscript x stands for the component of the vector in the direction of $\vec{x}_0$. The spectrum of the photon frequency has a breadth of $\delta\omega_{ni}$ around the center frequency of ω_w because of the lifetime effect.

At the low-frequency limit, where $\omega_k\ll\omega_{ni}$, as is the case in Rayleigh scattering, we find

$$P_{ii}^{(s)}d\Omega=\frac{1}{2\pi}\alpha^3a_0^2x_0^2\omega_k^3\hbar^{-4}\left|\sum_{n,\alpha}\omega_{ni}^{-1}[(\vec{p}\cdot\hat{\epsilon}_\alpha)_{in}(p_x)_{ni}+(p_x)_{in}(\vec{p}\cdot\hat{\epsilon}_\alpha)_{ni}]\right|^2d\Omega \ . \tag{10}$$

Note that this photon emission rate is proportional to ω_k^3 instead of the ω_k^4 found in the conventional Rayleigh-scattering formula.[4] This is simply because of the difference in definitions. At the high-frequency limit, where $\omega_k\gg\omega_{ni}$, we obtain the photon emission rate of Thomson scattering,

$$P_{ii}^{(s)}=\frac{1}{2\pi}\alpha^3a_0^2x_0^2\omega_k^{-1}\hbar^{-4}\left|\sum_{n,\alpha}[(\vec{p}\cdot\hat{\epsilon}_\alpha)_{in}(p_x)_{ni}+(p_x)_{in}(\vec{p}\cdot\hat{\epsilon}_\alpha)_{ni}]\right|^2 \ . \tag{11}$$

This case is analogous to the emission from the free electrons propagating through the wiggler. It is interesting to note that the Thomson scattering arises from H_2 given by Eq. (4d) in the conventional photon scattering theory.[4] In our case of distorted-lattice field scattering, the second term of the expression of H_1 in Eq. (4c) yields the Thomson scattering.

When the frequency of the distorted-lattice field ω_w matches the energy difference between two levels ω_{ni} one can expect resonance scattering. The photon emission rate for this case is

$$P_{ii}^{(s)}d\Omega=\frac{1}{2\pi}\alpha^3a_0^2\omega_k\hbar^{-2}\left(\frac{\omega_{ni}}{\delta\omega_{ni}}\right)^2|(\vec{p}\cdot\hat{\epsilon}_\alpha)_{in}x_0(p_x)_{ni}/\hbar|^2d\Omega \ . \tag{12}$$

It is interesting to compare this transition probability to the probability of spontaneous emission. Equations (6) and (11) yield

$$P_{ii}^{(s)}/P_{ni}^{(e)}\simeq(\omega_{ni}/\delta\omega_{ni})^2(x_0/\Delta_{in})^2 \ , \tag{13}$$

where $\Delta_{ni}^{-1}=(p_x)_{ni}/\hbar$ is approximately the scale length of the electron wave function in the perpendicular direction. If one takes typical values,[1,2] e.g., $\Delta_{ni}=0.3$ Å and $\omega_{ni}/\delta\omega_{ni}=10$, the intensity of emission due to scattering becomes comparable to the intensity of spontaneous emission when $x_0=0.03$ Å. The frequency-matching condition $\omega_w=\omega_{ni}$ requires the wave number of the lattice distortion:

$$k_z=\omega_{ni}/\gamma v \ . \tag{14}$$

The wavelength of the distortion satisfying this relation depends on the shape of the potential distribution and the electron beam energy. For values of $\gamma = 10–100$, the wavelength of the distortion falls in a range between a few hundred and a few thousand angstroms. This value of the wavelength is a typical period of the superlattice. The amount of displacement in so-called strained layer superlattice[3] is much larger than the displacement we need, and the electrons would be dechanneled. Ge_xSi_{1-x}-Si superlattice with x equals a few times 10^{-3} gives us a right amount of displacement. GaAlAs-GaAs is also suitable for our purpose provided that the positron channeling is employed. The monochromaticity of the photon is determined by both the breadth of the energy levels and the finite transit time of charged particles in the crystal. If the number of periods of the superlattice exceeds 100, then the breadth due to the transit time can be neglected. The photons are polarized in the direction of lattice displacement and are emitted in the forward direction within a solid angle of γ^{-2}. Their energy is $\hbar 2\gamma^2\omega_k$ in the laboratory frame.

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[1] R. H. Pantell, H. Park, R. L. Swent, J. O. Kephart, R. K. Klein, S. Datz, and B. L. Berman, IEEE Trans. Nucl. Sci. **NS-30**, 3150 (1983), and papers referenced.

[2] J. U. Andersen, E. B. Bonderup, and R. H. Pantell, Annu. Rev. Nucl. Sci. **33**, 453 (1983).

[3] S. T. Picraux, L. R. Dawson, G. C. Osborn, and W. K. Chu, Appl. Phys. Lett. **43**, 930 (1983); W. K. Chu, J. A. Ellison, S. T. Picraux, R. M. Biefeld, and G. C. Osbourn, Phys. Rev. Lett. **52**, 125 (1984).

[4] See, for instance, J. J. Sakurai, *Advanced Quantum Mechanics* (Addison-Wesley, Reading, MA, 1982), p. 47.