where n_i is the impurity density. The self-energies are always of second order in impurity-scattering potentials. In terms of diagrams, this equation is shown in Fig. 1. Similar to the self-energy diagrams, vertex corrections are also associated with corresponding terms of second order in the impurity-scattering potential (see p. 332 of AGD). Thus, his arguments in terms of linearity in $V_s(\vec{x} - \vec{R}_a)$ and second order in $V_{sd}(\vec{x} - \vec{R}_a)$ are completely meaningless. Along with this, Sung makes an incorrect assertion in footnote 16 of his paper, a rebuttal against which is published separately [W. S. Chow, Phys. Rev. B (to be published)].

[8]S. Skalski, O. Betboder-Matibet, and P. R. Weiss, Phys. Rev. 136, A1500 (1964).

[9]V. Ambegaokar, in *Superconductivity*, edited by R. D. Parks (M. Dekkar, New York, 1969).

[10]I. M. Tang, Phys. Rev. B 2, 1299 (1970). Tang's paper involves a serious error. His Eqs. (4.3) and (4.4) are valid only when temperature is very close to T_c (or T_{cd}). His conclusion, Eq. (4.16), that the change in specific heat due to impurity scattering is *negative*, is not correct in this temperature region. Leupold and Boorse [H. A. Leupold and H. A. Boorse, Phys. Rev. 134, A1322 (1964)] noticed that, at $T \lesssim T_c$, the niobium specific-heat data exhibit a small upward deviation from the specific-heat values predicted by the BCS theory for pure one-band superconductors; that is to say, the change in specific heat due to impurity scattering should be *positive* in this temperature region. The correct theory concerning the specific heat in this temperature region is given by Chow [W. S. Chow, J. Phys. F (to be published)].

PHYSICAL REVIEW B VOLUME 3, NUMBER 3 1 FEBRUARY 1971

Electronic Structure of the Intermediate State in Type-I Superconductors

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(Received 12 August 1970)

The quasiparticle wave functions and the energy-eigenvalue equations of the intermediate state in extreme type-I superconductors are calculated for the full range of the excitation spectrum. A WKBJ method of solving the Bogoliubov equations at any temperature below T_c is used. The periodicity of the pair potential leads to Bloch-type wave functions and a band structure of the energy spectrum for fixed momenta parallel to the phase boundaries. The magnetic field has an effect on the quasiparticle energies only by its influence on the structural and thermodynamic properties of the system. The width of the normal regions, and an effective variation length summing up the space dependence of the pair potential, are the variational parameters of the theory. From the general eigenvalue equations explicit energy spectra are obtained for simplified models of the pair potential.

I. INTRODUCTION

The excitation spectrum of a sequence of superconducting and normal or nearly normal regions has been discussed in a number of recent investigations.

Bound states with quantized energy levels have been found (a) in the isolated normal regions of the intermediate state of type-I superconductors,[1] (b) in the core of a single vortex line in the mixed state of type-II superconductors,[2-4] and (c) in the normal regions of normal-superconducting contacts.[5,6] The spectrum of these states with energies less than the maximum value Δ of the pair potential determines the low-temperature properties of the respective samples.

For the mixed state of type-II superconductors, the scattering states with $E > \Delta$ have also been analyzed,[3] and the periodic structure of the intermediate state has been considered by van Gelder, who, using a Kronig model of a periodic steplike pair potential, obtained a band structure of the energy spectrum in the one-dimensional case.[7]

Common to all these investigations is the use of the Bogoliubov equations,[8] the Schrödinger-like equations for electrons and holes, coupled by the pair potential $\Delta(\vec{r})$ of the superconductor. Whereas often it has been found necessary to assume simple forms such as step functions for the pair potential, or to limit the discussion to rather low temperatures where only the lowest-lying bound states are important, we intend to look into the periodic intermediate-state structure using the WKBJ approximation of solving the Bogoliubov equations, developed in Ref. 3. This allows us to treat the full energy spectrum at any temperature below T_c without introducing a simplified pair potential.

In Ref. 3 the problem of self-consistency of the pair potential $\Delta(\vec{r})$ and of the local magnetic field $\vec{h}(\vec{r})$ of a vortex line has been dealt with by using variational forms for $\Delta(\vec{r})$ and $\vec{h}(\vec{r})$ and minimizing the free energy with respect to the variational parameters. In the present problem of the intermediate state, as in the case of a normal-superconducting contact,[6] the approximation in which we solve the Bogoliubov equations uses the pair potential $\Delta(\vec{r}) = \Delta(z)$ only in a spatially integrated form so that no knowledge of the details of its spatial variation is required for the determination of the energy eigenvalues; we obtain them as functions of the effective spatial variation length of the pair potential,

$$d = \int_a^D (1 - \Delta(z)/\Delta)\, dz ,$$

with $2a$ being the width of a normal region and $2D$ being the periodicity of the laminar structure of the intermediate state. We will assume an extreme type-I superconductor which practically expels the magnetic field completely out of the superconducting regions, and in the normal regions $\vec{h}(z)$ will be treated as spatially constant.[9]

In Sec. II we transform the Bogoliubov equations into two nonlinear first-order differential equations for the exponents of the WKBJ-type ansatz for the quasiparticle wave functions. Section III deals with the asymptotic behavior of the wave functions and their periodic properties; the energy-eigenvalue equations for the bound and scattering states result from the matching and periodicity conditions and include the special case of the Kronig model. In Sec. IV the quasiparticle wave functions are calculated as second-order solutions of the transformed Bogoliubov equations written as integral equations. We finally obtain explicit energy-eigenvalue equations in terms of the specific sample parameters so that the problem of the energy spectrum of the intermediate state is reduced to one of numerical analysis.

At this point the present paper terminates. The author will try to work out the remaining numerical part as soon as access to computing facilities will be easier than at the present time. What should be done after the computation of the energy eigenvalues is the following.

One can calculate the free energy G_s of the superconductor in the external magnetic field $\vec{H}$ at temperature T from[3]

$$G_s = -2k_B T \sum_n \ln\left(2\cosh\frac{1}{2}\frac{E_n}{k_B T}\right) + \int\left(\frac{|\Delta(z)|^2}{V} + \frac{1}{8\pi}[\vec{h}(z) - \vec{H}]^2\right) d^3 r, \qquad (1.1)$$

where V is the attractive BCS-interaction constant, $\vec{h}(z)$ is the locally varying magnetic field, and the E_n are the solutions of the energy-eigenvalue equations. The variational parameters of the theory are the spacing $2a$ of the normal regions and the effective variation length d of the pair potential. They correspond to the parameters s and d of Ref. 3 and likewise they can be determined by minimizing the free energy (1.1) with respect to them. The periodicity interval $2D$ is related to $2a$ by the condition of flux conservation.

II. WKBJ APPROACH TO BOGOLIUBOV EQUATIONS

Let us consider a superconductor with alternating normal and superconducting layers. This structure is produced by an external magnetic field H of magnitude $(1 - D^*)\, H_c < H < H_c$, where D^* is the geometry- and orientation-dependent demagnetizing factor of the sample and H_c is the critical field at which superconductivity is completely destroyed. The magnetic field in the normal regions is equal to H_c. This state is called the intermediate state. It occurs in superconductors of type I, characterized by a positive surface energy between the normal and the superconducting phases which results in a Ginzburg-Landau parameter $\kappa < 2^{-1/2}$. For detailed discussions on the intermediate state and information on the previous work done on it see Refs. 8, 10, and 11.

One obtains a simple laminar structure of the intermediate state as shown in Fig. 1 by applying a magnetic field $0 < H < H_c$ in direction y perpendicular to the plane of a thin superconducting slab with $D^* = 1$. (A laminar structure may also be obtained in a disk specimen by applying a slanting field.[12,13]) We assume that the plate is sufficiently thick so that broadening of the normal layers at the surface[11] may be neglected.

A material with $\kappa \ll 1$ will confine the magnetic field $\vec{h}(z) = \vec{e}_y h(z)$ to the normal regions so that

$$\vec{h}(z) = \vec{e}_y H_c \theta(|a| - z), \text{ mod } 2D \qquad (2.1)$$

where

$$\theta(x) = \begin{cases} 1 \text{ for } x > 0 \\ 0 \text{ for } x < 0 . \end{cases}$$

The vector potential $\vec{A}$ related to the magnetic field by

$$\vec{h} = \overrightarrow{\text{rot}}\, \vec{A}$$

is

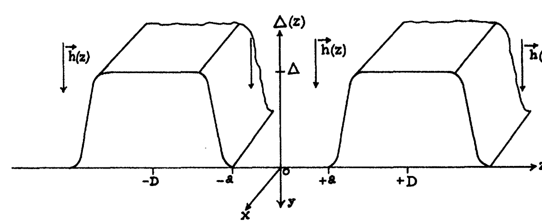

FIG. 1. Spatial variation of the pair potential $\Delta(z)$ and the local magnetic field in the laminar intermediate state.

$$\vec{A} = \vec{e}_x H_c z\theta(|a| - z), \text{ mod } 2D \tag{2.2}$$

apart from a gauge function $\overrightarrow{\text{grad}}\,\chi(\vec{r})$. The superconducting pair potential is constant along the x and y axes and varies periodically in z direction.

The width $2a$ of the normal layers will have to be determined by minimization of the free energy. The periodicity $2D$ of the laminar structure is related to $2a$ by the condition of conservation of magnetic flux[8] which yields

$$D/a = H_c/H. \tag{2.3}$$

The Bogoliubov equations for the electron – and hole – wave functions $u(\vec{r})$ and $v(\vec{r})$ in the intermediate state are

$$Eu(\vec{r}) = \left[\frac{1}{2m}\left(\frac{\vec{\nabla}}{i} - \frac{e}{c}\vec{A}\right)^2 - \epsilon_F\right] u(\vec{r}) + \Delta(z)v(\vec{r}), \tag{2.4a}$$

$$Ev(\vec{r}) = -\left[\frac{1}{2m}\left(\frac{\vec{\nabla}}{i} + \frac{e}{c}\vec{A}\right)^2 - \epsilon_F\right] v(\vec{r}) + \Delta^*(z)u(\vec{r}), \tag{2.4b}$$

where the Hartree-Fock potential already has been included in the Fermi energy ϵ_F; we use units so that $\hbar = 1$. By a suitable gauge of $\vec{A}$ the pair potential can be made real. We assume that $\vec{A}$ of Eq. (2.2) is compatible with a real pair potential of an extreme type-I superconductor.

In the directions x and y parallel to the phase boundaries, the quasiparticle wave functions u and v are plane waves, and in spinor notation we may write them as

$$\begin{pmatrix} u(\vec{r}) \\ v(\vec{r}) \end{pmatrix} = e^{i\vec{k}_{\parallel}\cdot\vec{\rho}}\,\hat{g}(z), \tag{2.5a}$$

where

$$\hat{g}(z) = \begin{pmatrix} g_+(z) \\ g_-(z) \end{pmatrix}, \tag{2.5b}$$

$$\vec{k}_{\parallel} = \vec{e}_x k_x + \vec{e}_y k_y, \quad \vec{\rho} = \vec{e}_x x + \vec{e}_y y. \tag{2.5c}$$

Inserting the wave functions of Eqs. (2.5) in Eq. (2.4) and defining

$$k_z^2/2m \equiv \epsilon_F - k_{\parallel}^2/2m, \tag{2.6}$$

we obtain, with the Pauli matrices

$$\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad \sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix},$$

the Bogoliubov equations in spinor notation as

$$E\hat{g} = \sigma_z \frac{1}{2m}\left[-\nabla_z^2 - \sigma_z\frac{2e}{c}k_x A_x + \left(\frac{e}{c}A_x\right)^2 - k_z^2\right]\hat{g} + \Delta(z)\sigma_x\hat{g}. \tag{2.7}$$

The ratio of the quadratic term in the vector potential to the linear one is

$$\frac{[(e/c)A_x]^2}{2(e/c)k_x A_x} = \frac{(e/c)A_x}{2k_x} = \frac{\pi A_x(z)}{2\phi_0 k_x},$$

where $\phi_0 = \pi(e/c)$ is the magnetic flux quantum in units where $\hbar = 1$. The flux quantum can be expressed by the critical magnetic field at zero temperature $H_c(0) = [4\pi N(0)\Delta^2]^{1/2}$, the London penetration depth $\lambda_L = (mc^2/4\pi n e^2)^{1/2}$, and the coherence length $\xi = v_F/\pi\Delta$ as

$$\phi_0 = (\tfrac{2}{3})^{1/2}\pi^2\xi\lambda_L H_c(0).$$

With that and Eq. (2.2) the ratio of the quadratic to the linear field term is less than

$$(\tfrac{8}{3}\pi^2)^{-1/2} H_c a / H_c(0)\xi k_x \lambda_L.$$

For the vast majority of states k_x is comparable to k_F, so that $k_x\lambda_L \gg 1$; in not too strong external magnetic fields, where a and ξ do not differ by more than two orders of magnitude, the quadratic term in the magnetic field is considerably smaller than the linear term and may be neglected.

An approximate WKBJ-type solution of Eq. (2.7) may be obtained by writing the z-dependent part of the wave function in the form

$$\hat{g}(z) = \begin{pmatrix} g_+(z) \\ g_-(z) \end{pmatrix} = \text{const}\begin{pmatrix} e^{-i\eta(z)/2} \\ e^{+i\eta(z)/2} \end{pmatrix} e^{i\xi(z)}\, e^{ik_z z}. \tag{2.8}$$

This leads from Eq. (2.7) to

$$2E = (1/2m)[-i\nabla_z^2\eta + 2\nabla_z\eta\cdot\nabla_z\xi + 2k_z\nabla_z\eta - (4e/c)k_x A] + 2\Delta(z)\cos\eta,$$

$$0 = (1/2m)[2\nabla_z^2\xi - 2(\tfrac{1}{2}\nabla_z\eta)^2 - 2(\nabla_z\xi)^2 - 4k_z\nabla_z\xi] + i2\Delta(z)\sin\eta.$$

We suppose that $\eta(z)$ and $\xi(z)$ vary slowly over atomic distances so that their second derivatives and products of their first derivatives may be neglected. Thus we are left with

$$\nabla_z\eta = \frac{2m}{k_z}[E - \Delta(z)\cos\eta] + \frac{2e}{c}\frac{k_x}{k_z}A(z), \tag{2.9a}$$

$$\nabla_z\xi = i(m/k_z)\Delta(z)\sin\eta. \tag{2.9b}$$

These equations have the same structure as Eqs. (4.17) and (4.18) for an isolated vortex line in Ref. 3.

III. QUASIPARTICLE WAVE FUNCTIONS

Knowing the quasiparticle wave functions in one periodicity interval $-D < z < +D$ around the origin one obtains them in the whole specimen by the Bloch periodicity condition (3.12).

A. Asymptotic Behavior

Normal region. In the normal region $-a<z<+a$, the pair potential is zero and the vector potential is $A(z)=H_c z$. We can integrate Eqs. (2.9) directly and obtain

$$\eta(z)=(2m/k_z)[Ez+(eH_c/2mc)k_x z^2],\quad \xi(z)=0. \tag{3.1}$$

With the definitions

$$q\equiv(m/k_z)E,\quad p\equiv(m/k_z)\omega_L k_x,\quad \omega_L\equiv eH_c/2mc, \tag{3.2}$$

we may write the z-dependent part of the quasiparticle wave functions in the normal region as

$$\hat{g}(z)=\left[a\begin{pmatrix}1\\0\end{pmatrix}e^{i(qz+pz^2)}+b\begin{pmatrix}0\\1\end{pmatrix}e^{-i(qz+pz^2)}\right]e^{ik_z z}. \tag{3.3}$$

Superconducting region; bound states $E<\Delta$. We assume that the superconducting region is so large that the pair potential $\Delta(z)$ assumes its constant asymptotic value Δ at some distance from D. According to the two degenerate solutions with positive momentum for zero magnetic field and spatially constant pair potential,[1,14]

$$\hat{g}_{1,2}(z\to D)=A_{1,2}\begin{pmatrix}(1\pm\alpha)^{1/2}\\(1\mp\alpha)^{1/2}\end{pmatrix}e^{i[k_z\pm\alpha(m/k_z)E]z},$$

$$\alpha\equiv i\left(\frac{\Delta^2}{E^2}-1\right)^{1/2}, \tag{3.4}$$

there are two independent solutions $\eta_{1,2}$ and $\xi_{1,2}$. One has the asymptotic behavior

$$\eta_1(z\to D)=+\arccos(E/\Delta)\equiv+\eta_D,\quad \xi_1(z\to D)=+i(m/k_z)(\Delta^2-E^2)^{1/2}z \tag{3.5a}$$

and represents an exponentially decreasing wave; the other has the asymptotic behavior

$$\eta_2(z\to D)=-\arccos(E/\Delta)=-\eta_D,\quad \xi_2(z\to D)=-i(m/k_z)(\Delta^2-E^2)^{1/2}z=-\xi_1(z\to D) \tag{3.5b}$$

and represents an exponentially increasing wave; hereby is $0<\arccos(E/\Delta)<\pi$. Note that $e^{\pm i\eta_D}=[(1\pm\alpha)/(1\mp\alpha)]^{1/2}$.

Since $\Delta(z)$ is an even function of z it follows from Eqs. (2.9) that in the superconducting regions, where $A=0$,

$$\eta_{1,2}(z)=-\eta_{1,2}(-z)\ \text{and}\ \xi_{1,2}(z)=\xi_{1,2}(-z). \tag{3.6}$$

Superconducting region; scattering states $E>\Delta$. The asymptotic solutions (3.4) demonstrate that η must be complex for $E>\Delta$ and ξ, in general, too. For the real and imaginary parts of

$$\eta=\eta_r-i\eta_i,\quad \xi=\xi_r-i\xi_i \tag{3.7}$$

we obtain from Eqs. (2.9) the following set of equations:

$$\nabla_z\eta_r=(2m/k_z)[E-\Delta(z)\cos\eta_r\cosh\eta_i], \tag{3.8a}$$

$$\nabla_z\eta_i=(2m/k_z)\Delta(z)\sin\eta_r\sinh\eta_i, \tag{3.8b}$$

$$\nabla_z\xi_r=(m/k_z)\Delta(z)\cos\eta_r\sinh\eta_i, \tag{3.8c}$$

$$\nabla_z\xi_i=-(m/k_z)\Delta(z)\sin\eta_r\cosh\eta_i. \tag{3.8d}$$

There are two linearly independent solutions of Eqs. (3.8) which differ in the sign of η_i, with ξ_r changing its sign when η_i does. The symmetry relations are

$$\eta_{r,i}(z)=-\eta_{r,i}(-z),\quad \xi_{r,i}(z)=\xi_{r,i}(-z). \tag{3.9}$$

The asymptotic conditions for constant Δ,

$$\eta_r(z\to D)=0,\quad \eta_i(z\to D)=\operatorname{arccosh}(E/\Delta),\quad \xi_i(z\to D)=0,\quad \xi_r(z\to D)=(m/k_z)(E^2-\Delta^2)^{1/2}z, \tag{3.10}$$

follow from the wave functions (3.4) for $E>\Delta$.

B. Periodic Properties

According to Bloch's (Floquet's) Theorem which holds for any set of equations with periodic coefficients,[15] we may write the wave functions for the intermediate state with periodic pair potential in the form of Bloch waves

$$\hat{g}(z)=\hat{\chi}_k(z)\,e^{ikz}, \tag{3.11}$$

where

$$\hat{\chi}_k(z)=\hat{\chi}_k(z+2D) \tag{3.12}$$

and k is the propagation vector of a plane wave modulated by the periodic function $\hat{\chi}_k$.

Adding and subtracting ikz in the exponent of $\hat{g}$ of [Eq. (2.8)] and defining

$$\kappa\equiv k_z-k,$$

we obtain the following: In the normal regions $-a<z<a$, mod $2D$,

$$\hat{\chi}_k(z)=\left[a\begin{pmatrix}1\\0\end{pmatrix}e^{i(qz+pz^2)}+b\begin{pmatrix}0\\1\end{pmatrix}e^{-i(qz+pz^2)}\right]e^{i\kappa z}; \tag{3.13}$$

in the superconducting regions $\pm a\lesseqgtr z\lesseqgtr\pm D$, mod $2D$,

for $E<\Delta$,

$$\hat{\chi}_k(z)=\left[A^{\gtrless}\begin{pmatrix}e^{(i/2)\eta_1}\\e^{-(i/2)\eta_1}\end{pmatrix}e^{i\xi_1}+B^{\gtrless}\begin{pmatrix}e^{(i/2)\eta_2}\\e^{-(i/2)\eta_2}\end{pmatrix}e^{i\xi_2}\right]e^{i\kappa z}, \tag{3.14}$$

for $E>\Delta$,

$$\hat{\chi}_k(z)=\left[C^{\gtrless}\begin{pmatrix}e^{(i/2)\eta}\\e^{-(i/2)\eta}\end{pmatrix}e^{i\xi}\right.$$

$$+ D^{\gtrless} \begin{pmatrix} e^{(i/2)\eta^*} \\ e^{-(i/2)\eta^*} \end{pmatrix} e^{-i\xi^*} \Big] e^{i\kappa z}, \quad (3.15)$$

where, according to Eq. (3.7), $\eta = \eta_r - i\eta_i$ and $\xi = \xi_r - i\xi_i$, and the asterisk marks the complex conjugate.

These quasiparticle wave functions are linear combinations of the two degenerate solutions of Eqs. (2.9) and (3.8) with z momentum in direction of the positive z axis. They represent an electron travelling in $+z$ direction and a hole moving in $-z$ direction. There is another set of solutions of equal energy representing quasiparticles of negative z momentum.

There are six undetermined integration constants in each quasiparticle wave function: two in the normal regions $|z| < a$, mod $2D$, two in the superconducting regions $a < z < D$, mod $2D$, marked by the sign >, and two in the superconducting regions $-a > z > -D$, mod $2D$, marked by the sign <.

Matching the solutions at $z = a$ gives two equations, one for g_+ and one for g_-, and so does matching at $z = -a$. (Neglecting the second derivatives and quadratic terms of η and ξ corresponds to approximating the Bogoluibov equations by first-order differential equations for $\hat{g}$ or $\hat{\chi}$. Therefore, one only has to match the wave functions and not also their derivatives.) The remaining two equations are provided by the periodicity condition (3.12) which, for $z = -D$, becomes

$$\hat{\chi}_k(-D) = \hat{\chi}_k(D). \quad (3.16)$$

Putting the determinant of the coefficients of the six integration constants equal to zero yields the energy-eigenvalue equations.

C. Eigenvalue Equation for the Bound States, $E < \Delta$

With the help of Eqs. (3.5) and (3.6) and using the abbreviations

$$\alpha \equiv e^{(i/2)\eta_1(a)}, \quad \beta \equiv e^{(i/2)\eta_D}, \quad \gamma \equiv e^{i\xi_1(D)}, \quad \delta \equiv e^{i\xi_1(a)},$$
$$\omega \equiv e^{(i/2)\eta_2(a)}, \quad \mu \equiv e^{iqa}, \quad \sigma \equiv e^{ipa^2}, \quad \nu \equiv e^{i\kappa 2D},$$
$$\tau \equiv e^{i\xi_2(a)}, \quad (3.17)$$

we find the determinant to be

$$D_{et} = \begin{vmatrix} \alpha\delta & \omega\tau & 0 & 0 & -\mu\sigma & 0 \\ \delta/\alpha & \tau/\omega & 0 & 0 & 0 & -1/\mu\sigma \\ 0 & 0 & \delta/\alpha & \tau/\omega & -\sigma/\mu & 0 \\ 0 & 0 & \alpha\delta & \omega\tau & 0 & -\mu/\sigma \\ \beta\gamma\nu & \nu/\beta\gamma & -\gamma/\beta & -\beta/\gamma & 0 & 0 \\ \gamma\nu/\beta & \beta\nu/\gamma & -\beta\gamma & -1/\beta\gamma & 0 & 0 \end{vmatrix} . \quad (3.18)$$

The expansion of D_{et} yields

$$D_{et} = \left(\beta^2 - \frac{1}{\beta^2}\right) \nu\delta\tau \left\{ \left(\frac{\omega}{\alpha} - \frac{\alpha}{\omega}\right)\left(\nu + \frac{1}{\nu}\right) + \frac{\delta}{\tau\gamma^2}\left(\frac{\alpha^2}{\mu^2} - \frac{\mu^2}{\alpha^2}\right) - \frac{\tau\gamma^2}{\delta}\left(\frac{\omega^2}{\mu^2} - \frac{\mu^2}{\omega^2}\right)\right\} . \quad (3.19)$$

The condition $D_{et} = 0$ is satisfied, if (a) $\beta^4 \equiv e^{i2\eta_D} = 1$, i.e.,

$$\eta_D \equiv \arccos(E/\Delta) = n\pi \text{ or } E = \Delta \text{ for } n = 0;$$

(b) the term in curly brackets vanishes, i.e.,

$$2\cos[(k - k_z)2D] \sin\tfrac{1}{2}[\eta_2(a) - \eta_1(a)]$$
$$= \exp i[\xi_2(a) - \xi_1(a) + 2\xi_1(D)] \sin[\eta_2(a) - 2qa]$$
$$- \exp\{-i[\xi_2(a) - \xi_1(a) + 2\xi_1(D)]\} \sin[\eta_1(a) - 2qa]. \quad (3.20)$$

It is interesting to note that the term $pa^2 = m\omega_L k_x a^2/k_z$ describing the direct influence of the magnetic field completely drops out of the eigenvalue equation. Formally one might interpret $k_x a\omega_L = k_x a\mu_B H_c$ as the magnetic energy of an electron of angular momentum $k_x a$ with respect to $z = 0$. This would be an electron localized at $z = a$ moving with momentum k_x in the x direction. However, the actual quasiparticle wave function given by Eqs. (2.5) and (3.3) represents an electron and a hole forming a standing wave spread across the normal region $-a < z < +a$, with plane-wave character parallel to the phase boundaries. The nature of such an excitation is discussed in Refs. 1, 14, and 16 and the papers on the Tomasch effect.[17,18] The angular momentum of such a quasiparticle wave with respect to $z = 0$ is zero. Thus, contrary to a vortex line[3,4] there is no magnetic moment associated with quasiparticle motion parallel to the phase boundaries, so that in the framework of our approximation the magnetic field has no direct influence on the energy eigenvalues. The situation is similar to a normal square-well potential with a magnetic field, on one hand, and the Zeeman effect in atoms, on the other hand.

Equation (3.20) simplifies considerably if one assumes that the superconducting region is so large that one may treat the normal regions as being completely isolated from each other. This is true if the applied magnetic field is very weak. Then one may put the coefficients $B^{\gtrless}$ of the exponentially increasing solutions in Eq. (3.14) equal to zero, and one obtains the eigenvalue equation

$$\eta_1(a) = (2m/k_z)Ea - n\pi \,, \tag{3.21}$$

where n is any integer. This equation is exactly the same one as for a normal-superconducting contact with a being the thickness of the normal layer.[6] Thus, for $E<\Delta$, the energy spectrum of the intermediate state in a weak magnetic field is the same as the one calculated in Ref. 6.

For a steplike pair potential with $\eta_1(a) = \arccos(E/\Delta)$, the solutions of Eq. (3.21) are readily obtained (see Fig. 2), and include Andreev's results for $E \ll \Delta$.[1]

In the general case of the intermediate state with nonisolated normal regions we must solve Eq. (3.20) after having computed the values of η and ξ at the phase boundary a. Only such energies will be allowed for which the right-hand side of Eq. (3.20) does not exceed 2. Therefore, in general, one will find intervals of allowed and forbidden energies for fixed values of k_z. The discrete bound-state levels corresponding to Eq. (3.21) split up into energy bands, when the normal regions come so close to each other that the wave functions overlap. Within an allowed band the energy varies as a function of k.

Let us illustrate this by the example of the step-

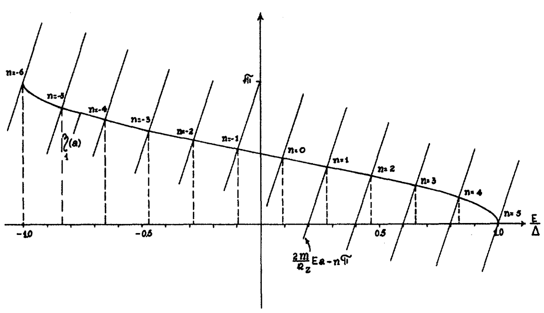

FIG. 2. Graphical solution of the eigenvalue equation for the bound states in isolated normal regions. The energy eigenvalues are given by the intersections of the straight lines $2mEa/k_z - n\pi$ with $\eta(a) = \arccos E/\Delta$ for a steplike pair potential; we have put $2m/k_z = 5\pi/\Delta$. The negative energies belong to depaired quasiparticles in the ground state of the system. Each quasiparticle state is characterized by a set of quantum numbers (k_x, k_y, n) and its spin.

like pair potential, where

$$\eta_1(a) = \eta_D = \arccos(E/\Delta) = -\eta_2(a)\,,$$

$$i[\xi_2(a) - \xi_1(a) + 2\xi_1(D)] = -(2m/k_z)[\Delta^2 - E^2]^{1/2}(D-a).$$

Equation (3.20) becomes

$$\begin{aligned}\cos(k-k_z)2D = {} & \cos[(2m/k_z)Ea] \\ & \times \cosh\{(2m/k_z)[\Delta^2 - E^2]^{1/2}(D-a)\} \\ & - E[\Delta^2 - E^2]^{-1/2} \sin[(2m/k_z)Ea] \\ & \times \sinh\{(2m/k_z)[\Delta^2 - E^2]^{1/2}(D-a)\}\,. \end{aligned} \tag{3.22}$$

This equation is very similar to that of the Kronig model for the periodic lattice potential.[19] Van Gelder[7] has numerically solved this and the corresponding equation for $E>\Delta$ (see below) for two values of the parameters a, D, and k_z. Figure 3 shows some energy bands obtained from Eqs. (3.22) and (3.25).

The energy bands for different k_z, i.e., different k_x and k_y, will overlap because of the tiny energy gaps smaller than Δ in each band sequence. Therefore, only experiments like tunneling into the intermediate state preferring electrons moving with Fermi momentum normal to the phase boundaries may be able to detect the band structure.

D. Eigenvalue Equation for the Scattering States, $E>\Delta$

Matching the wave functions (3.13) and (3.15) at $z=\pm a$ and using the periodicity condition (3.16) leads to a set of six equations, the determinant of which is formally identical with the determinant of the bound states, Eq. (3.18), if we define

$$\alpha \equiv e^{(i/2)\eta(a)}, \quad \beta \equiv e^{(i/2)\eta(D)}, \quad \gamma \equiv e^{i\xi_r(D)}, \quad \delta \equiv e^{i\xi(a)},$$

$$\omega \equiv e^{(i/2)\eta^*(a)}, \quad \mu \equiv e^{iqa}, \quad \sigma \equiv e^{ipa^2}, \quad \nu \equiv e^{i\kappa 2D},$$

$$\tau \equiv e^{-i\xi^*(a)}.$$

Rearranging the evaluated determinant of Eq. (3.19) in the appropriate way we obtain

$$\begin{aligned} D_{et} = {} & \left(\beta^2 - \frac{1}{\beta^2}\right) \nu\delta\tau \left\{ \frac{\omega}{\alpha} \left[\left(\nu + \frac{1}{\nu}\right) \right.\right. \\ & - \left.\left(\frac{\alpha\omega\tau}{\mu^2\delta}\gamma^2 + \frac{\mu^2\delta}{\alpha\omega\tau}\frac{1}{\gamma^2}\right)\right] \\ & \left. - \frac{\alpha}{\omega}\left[\left(\nu + \frac{1}{\nu}\right) - \left(\frac{\alpha\omega\delta}{\mu^2\tau}\frac{1}{\gamma^2} + \frac{\mu^2\tau}{\alpha\omega\delta}\gamma^2\right)\right]\right\}. \end{aligned} \tag{3.23}$$

We see that $D_{et}=0$, if (a) $\beta^4 = 1$, i.e.,

$\eta_i(D) \equiv \operatorname{arcosh}(E/\Delta) = 0$ so that $E = \Delta$;

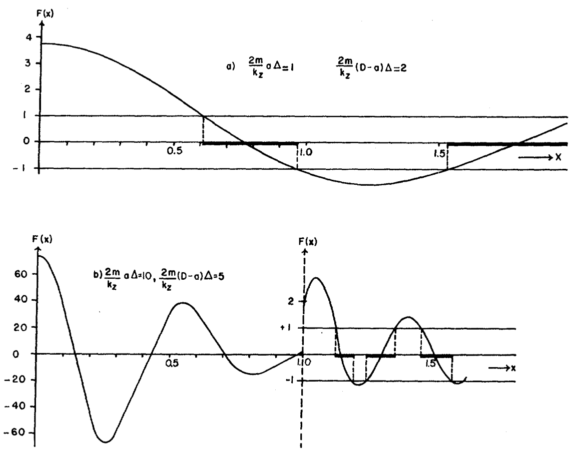

FIG. 3. Energy bands in the intermediate state with a Kronig model for the pair potential. For $x \equiv E/\Delta$ smaller [larger] than 1, $F(x)$ is given by the right-hand side of Eq. (3.22) [Eq. (3.25)]. The energy bands, which are indicated by heavy lines on the x axis, are given by the values of x for which $|F(x)| \lesssim 1$.

(b) the term in curly brackets vanishes, which implies that

$$\cos(k-k_z)2D = \cos[\eta_r(a) - 2qa]\cos 2[\xi_r(a) - \xi_r(D)] - \coth\eta_i(a)\sin[\eta_r(a) - 2qa] \times \sin 2[\xi_r(a) - \xi_r(D)]. \quad (3.24)$$

For a steplike pair potential with

$$\eta_r(a) = 0, \quad \eta_i(a) = \operatorname{arcosh}(E/\Delta)$$

and

$$\xi_r(a) - \xi_r(D) = (m/k_z)(E^2 - \Delta^2)^{1/2}(a - D),$$

Eq. (3.24) becomes

$$\cos(k-k_z)2D = \cos\frac{2m}{k_z}Ea\cos\left(\frac{2m}{k_z}(E^2-\Delta^2)^{1/2}(D-a)\right) - E(E^2-\Delta^2)^{-1/2}\sin\frac{2m}{k_z}Ea \times \sin\left(\frac{2m}{k_z}(E^2-\Delta^2)^{1/2}(D-a)\right), \quad (3.25)$$

which turns into Eq. (3.22) for $E < \Delta$.

IV. CALCULATION OF η AND ξ IN SUPERCONDUCTING REGIONS

In order to calculate the functions η and ξ from Eqs. (2.9), (3.5), (3.8), and (3.10) we use the same approximation method as in the calculation of the excitation spectrum of a paramagnetic-superconducting contact.[6] It gives approximate analytical solutions of the Bogoliubov equations by changing Eqs. (2.9) and (3.8) into integral equations which are solved in second order by the method of successive approximations. It does not require detailed assumptions about the spatial variation of the pair potential $\Delta(z)$ save one, as we believe, unimportant exception. Only over-all properties of the spatial variation expressed by an effective variation length d remain in the final expressions.

A. η and ξ for Bound States $E < \Delta$

We are going to consider positive-energy eigenvalues only. We integrate Eq. (2.9a) subject to the boundary conditions (3.5) and obtain

$$\eta_{1,2}(z) = \pm\eta_D - \frac{2m}{k_z}\int_z^D [E - \Delta(z)\cos\eta_{1,2}]\,dz. \quad (4.1)$$

In zero-order approximation the solutions of Eq. (4.1) are

$$\eta_{1,2}^{(0)} = \pm\eta_D = \pm\arccos(E/\Delta).$$

Substituting them for $\eta_{1,2}$ under the integral we find in first order

$$\eta_{1,2}^{(1)}(z) = \pm\eta_D - \frac{2m}{k_z}E\int_z^D\left(1 - \frac{\Delta(z)}{\Delta}\right)dz, \quad (4.2)$$

and resubstitution yields in second order

$$\eta_{1,2}^{(2)} = \pm\eta_D - \frac{2m}{k_z}\int_z^D\left\{E - \Delta(z)\cos\left[\pm\eta_D - \frac{2m}{k_z}E\int_z^D\left(1 - \frac{\Delta(z')}{\Delta}\right)dz'\right]\right\}dz. \quad (4.3)$$

As in Ref. 6, we may add to and subtract from this expression

$$\frac{2m}{k_z}\Delta\int_z^D\cos\left[\pm\eta_D - \frac{2m}{k_z}E\int_z^D\left(1 - \frac{\Delta(z')}{\Delta}\right)dz'\right]dz,$$

use

$$u_{1,2} \equiv \eta_D \mp \frac{2m}{k_z}E\int_z^D\left(1 - \frac{\Delta(z')}{\Delta}\right)dz'$$

as a convenient integration variable, define

$$F_{1,2}(z) \equiv \frac{2m}{k_z}\int_z^D\left\{\Delta\cos\left[\eta_D \mp \frac{2m}{k_z}E \times \int_z^D\left(1 - \frac{\Delta(z')}{\Delta}\right)dz'\right] - E\right\}dz, \quad (4.4)$$

and obtain

$$\eta_{1,2}^{(2)}(z) = \pm\eta_D \mp\left(\frac{\Delta^2}{E^2} - 1\right)^{1/2}\left\{1 - \cos\left[\frac{2m}{k_z}E \times \int_z^D\left(1 - \frac{\Delta(z')}{\Delta}\right)dz'\right]\right\}$$

$$-\sin\left[\frac{2m}{k_z}E\int_z^D\left(1-\frac{\Delta(z')}{\Delta}\right)dz'\right]+F_{1,2}(z). \qquad (4.5)$$

Since the pair potential only appears in the integrated form of

$$f(z)\equiv\int_z^D\left(1-\frac{\Delta(z)}{\Delta}\right)dz, \qquad (4.6)$$

the details of its spatial variation should not matter much. Therefore, in order to calculate $F_{1,2}(z)$ we may approximate $f(z)$ by the simplest possible function which satisfies the conditions imposed by the physically important features of the pair potential:

(a) Beyond a certain limit $z=\pm b$ in a periodicity interval $-D<z<+D$ there exists a region of constant pair potential $\Delta(z)=\Delta$ which does not contribute to $f(z)$, so that

$$f(z\geq b)=0; \qquad (4.7)$$

and

(b)

$$f(z)=\int_z^b\left(1-\frac{\Delta(z')}{\Delta}\right)dz'. \qquad (4.8)$$

(c) The effective length of the spatial variation of the pair potential defined by

$$d\equiv\int_a^D\left(1-\frac{\Delta(z)}{\Delta}\right)dz=f(a) \qquad (4.9)$$

is, because of Eq. (4.8), equal to

$$d\equiv\int_a^b\left(1-\frac{\Delta(z)}{\Delta}\right)dz.$$

This relates the limit b to the variational parameter d.

(d) Since $\Delta(z)=0$ for $-a<z<a$,

$$\left.\frac{df}{dz}\right|_a=-\left.\left(1-\frac{\Delta(z)}{\Delta}\right)\right|_a=-1;$$

(e) $\left.\frac{df}{dz}\right|_{z\geq b}=0.$

(g) $f(z)$ decreases monotonously from $f(a)=d$ to $f(b)=0$. The approximation

$$f(z)\approx\left[\frac{z^2}{4d}-\frac{2d+a}{2d}z+\frac{1}{4d}(2d+a)^2\right]\theta(2d+a-z),$$

$$b=2d+a \qquad (4.10)$$

is the simplest possible function which has the properties (a)–(g), and we will use it for the calculation of $F_{1,2}(z)$. Without further approximations we obtain

$$F_{1,2}(z)=\left[4\pi\Delta d\frac{m}{k_z}\right]^{1/2}\left\{\left(\frac{E}{\Delta}\right)^{1/2}C\left[\frac{2m}{k_z}Ef(z)\right]\right.$$

$$\left.\pm\left(\frac{\Delta}{E}-\frac{E}{\Delta}\right)^{1/2}S\left[\frac{2m}{k_z}Ef(z)\right]\right\}$$

$$-(2m/k_z)E(2d+a-z)\theta(2d+a-z), \qquad (4.11)$$

where

$$C[y]\equiv(2\pi)^{-1/2}\int_0^y\frac{\cos t}{\sqrt{t}}dt,$$

$$S[y]\equiv(2\pi)^{-1/2}\int_0^y\frac{\sin t}{\sqrt{t}}dt \qquad (4.12)$$

are the Fresnel integrals, [20] and

$$\theta(x)=\begin{cases}1 & \text{for } x>0\\ 0 & \text{for } x<0\ .\end{cases}$$

We suppose that $\eta_{1,2}^{(2)}(z)$ as given by Eqs. (4.5) and (4.11) is a sufficiently good approximation of $\eta(z)$.

For the calculation of ξ let us put the zero-order solutions $\eta_{1,2}^{(0)}$ in Eq. (2.9b) and integrate it subject to the boundary conditions (3.5). We obtain in first order

$$\xi_{1,2}^{(1)}=\pm i\frac{m}{k_z}(\Delta^2-E^2)^{1/2}\int_z^D\left(1-\frac{\Delta(z)}{\Delta}\right)dz$$

$$\pm i\frac{m}{k_z}(\Delta^2-E^2)^{1/2}z.$$

The second-order approximation $\xi_{1,2}^{(2)}$ is calculated similarly to $\eta_{1,2}^{(2)}$, inserting $\eta_{1,2}^{(1)}$ into Eq. (2.9b) and evaluating integrals of the same type as in Eqs. (4.3) and (4.4). This results in

$$\xi_{1,2}^{(2)}=\pm i\frac{m}{k_z}(\Delta^2-E^2)^{1/2}z$$

$$-i\frac{1}{2}\left\{1-\cos\frac{2m}{k_z}Ef(z)\mp\left(\frac{\Delta^2}{E^2}-1\right)^{1/2}\sin\frac{2m}{k_z}Ef(z)\right]$$

$$-i\left(\pi\Delta d\frac{m}{k_z}\right)^{1/2}\left[\pm\left(\frac{\Delta}{E}-\frac{E}{\Delta}\right)^{1/2}C\left[\frac{2m}{k_z}Ef(z)\right]\right.$$

$$\left.-\left(\frac{E}{\Delta}\right)^{1/2}S\left[\frac{2m}{k_z}Ef(z)\right]\right\}.$$

$$\pm i\frac{m}{k_z}(\Delta^2-E^2)^{1/2}(2d+a-z)\theta(2d+a-z), \qquad (4.13)$$

and we approximate $\xi_{1,2}$ by $\xi_{1,2}^{(2)}$.

Defining

$$A(z)\equiv\left(4\pi d\Delta\frac{m}{k_z}\right)^{1/2}\left(\frac{E}{\Delta}\right)^{1/2}C\left[\frac{2m}{k_z}Ef(z)\right]-\sin\left[\frac{2m}{k_z}Ef(z)\right]$$

$$-(2m/k_z)E(2d+a-z)\theta(2d+a-z), \qquad (4.14)$$

$$B(z)\equiv\arccos\frac{E}{\Delta}-\left(\frac{\Delta^2}{E^2}-1\right)^{1/2}\left(1-\cos\left[\frac{2m}{k_z}Ef(z)\right]\right)$$

$$+\left(4\pi d\Delta\frac{m}{k_z}\right)^{1/2}\left(\frac{\Delta}{E}-\frac{E}{\Delta}\right)^{1/2}S\left[\frac{2m}{k_z}Ef(z)\right],$$

we may write

$$\eta_1(z)=A(z)+B(z),\quad \eta_2(z)=A(z)-B(z). \tag{4.15}$$

With that and

$$Z\equiv i[\xi_2(a)-\xi_1(a)+2\xi_1(D)]$$
$$=(2m/k_z)(\Delta^2-E^2)^{1/2}(2d+a-D)$$
$$+\left(\frac{\Delta^2}{E^2}-1\right)^{1/2}\sin\left[\frac{2m}{k_z}Ed\right]$$
$$-\left(4\pi d\Delta\frac{m}{k_z}\right)^{1/2}\left(\frac{\Delta}{E}-\frac{E}{\Delta}\right)^{1/2}C\left[\frac{2m}{k_z}Ed\right],$$

the eigenvalue equation (3.20) for the bound states can be rewritten in the form

$$\cos(k-k_z)2D=\cosh Z\cos[A(a)-2qa]$$
$$-\sinh Z\sin[A(a)-2qa]\cot B(a). \tag{4.16}$$

B. η and ξ for the Scattering States $E>\Delta$

We write Eqs. (3.8) in integrated form using the boundary conditions (3.10)

$$\eta_r=\frac{-2m}{k_z}\int_z^D[E-\Delta(z)\cos\eta_r\cosh\eta_i]dz, \tag{4.17a}$$

$$\eta_i=\operatorname{arcosh}\frac{E}{\Delta}-\frac{2m}{k_z}\int_z^D\Delta(z)\sin\eta_r\sinh\eta_i\,dz, \tag{4.17b}$$

$$\xi_r=\frac{m}{k_z}(E^2-\Delta^2)^{1/2}D-\frac{m}{k_z}\int_z^D\Delta(z)\cos\eta_r\sinh\eta_i\,dz, \tag{4.17c}$$

$$\xi_i=\frac{m}{k_z}\int_z^D\Delta(z)\sin\eta_r\cosh\eta_i\,dz. \tag{4.17d}$$

With $\eta_1(D)$ and $\eta_i(D)$ as zero-order approximations, we obtain as first-order solutions

$$\eta_r^{(1)}=-\frac{2m}{k_z}E\int_z^D\left(1-\frac{\Delta(z)}{\Delta}\right)dz, \tag{4.18a}$$

$$\eta_i^{(1)}=\operatorname{arcosh}(E/\Delta), \tag{4.18b}$$

$$\xi_r^{(1)}=\frac{m}{k_z}(E^2-\Delta^2)^{1/2}z$$
$$+\frac{m}{k_z}(E^2-\Delta^2)^{1/2}\int_z^D\left(1-\frac{\Delta(z)}{\Delta}\right)dz, \tag{4.18c}$$

$$\xi_i^{(1)}=0. \tag{4.18d}$$

The second iteration of Eqs. (4.17) with the first-order solutions (4.18a) and (4.18b) is not very different from the calculation of $\eta_{1,2}^{(2)}$ and $\xi_{1,2}^{(2)}$ for the bound states. We encounter the same type of integrals, use the same approximation (4.10) for $f(z)=\int_z^D(1-\Delta(z)/\Delta)\,dz$, and find

$$\eta_r^{(2)}(z)=-\sin\left(\frac{2m}{k_z}Ef(z)\right)+\left(4\pi d\Delta\frac{m}{k_z}\right)^{1/2}$$
$$\times\left(\frac{E}{\Delta}\right)^{1/2}C\left[\frac{2m}{k_z}Ef(z)\right]$$
$$-(2m/k_z)E(2d+a-z)\theta(2d+a-z), \tag{4.19a}$$

$$\eta_i^{(2)}(z)=\operatorname{arcosh}\frac{E}{\Delta}-\left(1-\frac{\Delta^2}{E^2}\right)^{1/2}\left[1-\cos\left(\frac{2m}{k_z}Ef(z)\right)\right]$$
$$+\left(4\pi\,d\Delta\frac{m}{k_z}\right)^{1/2}\left(\frac{E}{\Delta}-\frac{\Delta}{E}\right)^{1/2}S\left[\frac{2m}{k_z}Ed\right], \tag{4.19b}$$

$$\xi_r^{(2)}(z)=\frac{m}{k_z}(E^2-\Delta^2)^{1/2}z+\frac{1}{2}\left(1-\frac{\Delta^2}{E^2}\right)^{1/2}\sin\left(\frac{2m}{k_z}Ef(z)\right)$$
$$-\left(\pi d\Delta\frac{m}{k_z}\right)^{1/2}\left(\frac{E}{\Delta}-\frac{\Delta}{E}\right)^{1/2}C\left[\frac{2m}{k_z}Ef(z)\right]$$
$$+\frac{m}{k_z}(E^2-\Delta^2)^{1/2}(2d+a-z)\theta(2d+a-z), \tag{4.19c}$$

$$\xi_i^{(2)}(z)=\frac{1}{2}\left\{1-\cos\left(\frac{2m}{k_z}Ef(z)\right)\right.$$
$$\left.-\left(\pi d\Delta\frac{m}{k_z}\right)^{1/2}\left(\frac{E}{\Delta}\right)^{1/2}S\left[\frac{2m}{k_z}Ef(z)\right]\right\}. \tag{4.19d}$$

Inserting these results into the energy-eigenvalue equation

$$\cos(k-k_z)2D=\cos[\eta_r(a)-2qa]\cos2[\xi_r(a)-\xi_r(D)]$$
$$-\coth\eta_i(a)\sin[\eta_r(a)-2qa]$$
$$\times\sin2[\xi_r(a)-\xi_r(D)], \tag{3.24}$$

we see that it becomes identical to Eq. (4.16), if one lowers E below Δ, because then

$$\eta_r(a)\to A(a),\quad \eta_i(a)\to iB(a),$$
$$2[\xi_r(a)-\xi_r(D)]\to iZ=[\xi_1(a)-\xi_2(a)-2\xi_1(D)]. \tag{4.20}$$

Remember that

$$2qa=\frac{2m}{k_z}Ea,\quad f(a)=\int_a^D\left(1-\frac{\Delta(z)}{\Delta}\right)dz\equiv d,\ \text{and}\ f(D)=0.$$

Therefore, taking into account the relations (4.19) and (4.20), it is sufficient to consider Eq. (3.24) as the eigenvalue equation for all quasiparticle energies $0<E<\infty$.

For the steplike pair potential we have $d=0$ and recover Eqs. (3.22) and (3.25).

V. SUMMARY

The quasiparticle wave functions in the intermediate state have been calculated. They are given by Eqs. (2.5), (2.8), (3.7), (3.11), (3.13)–(3.15), (4.15), and (4.19). In the direction of periodic change of the pair potential they are Bloch-type functions whose periodic part represents a linear combination of electrons and holes of nearly equal momenta but opposite group velocities. Parallel to the phase boundaries they have plane-wave character. The knowledge of the wave functions is required for the computation of the acoustic attenuation, nuclear spin relaxation, and other dynamical properties of the system involving coherence effects.

The derived energy-eigenvalue equations (3.24) and (4.16) allow the following general conclusions:

(i) In a weak magnetic field H where, according to Eq. (2.3), the normal layers are practically isolated from each other, the bound-state spectrum of the intermediate state is the same as that of a normal-superconducting contact. One may conclude this by comparing Eqs. (3.21) and (4.15) with their counterparts of Ref. 6.

(ii) Within the approximation of neglecting quadratic terms of the magnetic field and its penetration into the superconducting regions, the quasiparticle energies do not explicitly depend upon H. But clearly H influences the energy spectrum by determining the width of the normal and superconducting layers.

(iii) Owing to the periodicity of the pair potential, the energy spectrum for a given set of parameters exhibits band structure. One must expect, however, that the band structure will be obscured by the overlap of the different band sequences for the different momenta parallel to the phase boundaries. For widths of the normal layers considerably larger than the effective variation length of the pair potential, the excitation spectrum should exhibit a vanishing energy gap.[6]

It is hoped that soon numerical solutions of the energy-eigenvalue equations will be obtained so that the thickness of the normal and superconducting regions and the effective spatial variation length of the pair potential can be determined by minimizing the free energy (1.1) with respect to a and d. Once the wave functions and the energy spectrum are completely known, the way should be open to the computation of the thermal and electrical properties of the intermediate state at any given temperature below T_c.

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