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Addendum to the Theory of Ultrasonic Attenuation in Pure Two-Band Superconductors in High Fields

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By modifying the sound-wave dispersion relation for a two-band superconductor, the ultrasonic attenuation coefficient for transverse waves in pure two-band superconductors near H_{c2} is obtained. An explicit expression for the attenuation is obtained by the use of an "unverified" conjecture used by the author previously to obtain the ultrasonic attenuation of longitudinal waves in pure superconducting niobium near H_{c2}. It is then shown that the Brandt-Pesch-Tewordt method can be applied to pure two-band superconductors near H_{c2} to produce results which are in agreement with those obtained by the use of the unverified conjecture that the effect of the magnetic field on the two energy gaps in a pure two-band superconductor near H_{c2} is similar to that of an uniform current in both bands.

I. INTRODUCTION

In a recent paper,[1] the present author was able to obtain the two-band expressions for the ultrasonic attenuation of longitudinal waves, which were applicable to pure type-II transition-metal superconductors in the mixed state. One of the purposes of this addendum is to show that with only a slight modification of Eq. (2.9) of Ref. 1, the attenuation coefficient for transverse waves can be obtained in the limit $ql \ll 1$ (q being the wave vector of the sound wave and l being the electronic mean free path) in pure two-band superconductors in the mixed state. The various correlation functions appearing in the two-band transverse-wave attenuation coefficient are evaluated using the same conjecture used in Ref. 1. The conjecture makes it possible to apply the technique developed by Maki[2] to circumvent the difficulties associated with the perturbation expansion[3] of the various correlation functions in powers of the order parameter in pure type-II superconductors near H_{c2}.

This brings us to the second purpose of this addendum. While the conjecture used by Maki for evaluating the correlation functions in a one-band superconductor in high fields has been verified, the conjecture used to evaluate the correlation functions for a two-band superconductor near H_{c2} has not been verified. In the second part of this addendum, we will show that results obtained by the use of the conjecture to evaluate the correlation functions can be obtained using a modification of the Brandt-Pesch-Tewordt[4] (BPT) method to evaluate the correlation functions. The BPT method allows us to obtain explicit expressions for the Green's functions for the two-band superconductors near H_{c2} without having to iterate the Gor'kov equations[5] for the superconductors near H_{c2} (this being the cause of the difficulties mentioned previously). We will see that the results for the attenuation of longitudinal waves in the limit $ql \gg 1$ obtained by the BPT method are in agreement with the results obtained by the use of the conjecture to evaluate the various correlation functions.

II. TRANSVERSE ATTENUATION COEFFICIENT

The propagation of transverse sound waves in a pure two-band superconductor is described by the same sound-wave dispersion relation (2.9) found in Ref. 1:

$$p_{\text{ion}}\,\omega^2\phi_i(\vec{q},\omega)-\tfrac{1}{5}(v_{sF}^2 m_s N_s+v_{dF}^2 m_d N_d)$$
$$\times\{2q_i[\vec{q}\cdot\vec{\phi}(\vec{q},\omega)]+q^2\phi_i(\vec{q},\omega)\}+(\langle[h_{si},h_{sj}]\rangle_{\vec{q},\omega}$$
$$+\langle[h_{di},h_{dj}]\rangle_{\vec{q},\omega})\,\omega^2\phi_j(\vec{q},\omega)=0\ , \quad (1)$$

where

$$h_{s(d)i}(\vec{r},t)=q_j\tau_{s(d)ij}(\vec{r},t)/\omega-m_{s(d)}j_{s(d)i}(\vec{r},t)\ . \quad (2)$$

The definitions of all the terms can be found in Ref. 1. To obtain the transverse attenuation coefficient, we note that for transverse waves, the displacement vector $\vec{\phi}(\vec{q},z)$ is perpendicular to the wave vector $\vec{q}$. Taking $\vec{q}$ to be pointing in the z direction and the displacement vector $\vec{\phi}(\vec{q},z)$ to be in the x direction, we obtain the transverse attenuation coefficient

$$\alpha_T=\mathrm{Re}\left[\frac{\omega^2}{i\omega\rho_{\text{ion}}v_s}(\langle[h_{sI}^T,h_{sI}^T]\rangle_{\vec{q}\omega}+\langle[h_{dI}^T,h_{dI}^T]\rangle_{\vec{q}\omega})\right], \quad (3)$$

where

$$h_{s(d)I}^T(\vec{r},t)=(q/\omega)\,\tau_{s(d)xz}(\vec{r},t)-m_{s(d)}j_x(\vec{r},t). \quad (4)$$

Remembering that the sound-wave dispersion relation (1) was obtained in a system which still contained the long-range electron-electron interactions we now transform to a fictitious system which does not contain any long-range Coulomb interactions.[6] While in most situations, the long-range Coulomb interactions will give rise to relativistic corrections, in the present situation, this interaction will produce some important corrections. Following Kadanoff and Fal'ko,[6] we arrive at

$$\alpha_T=\mathrm{Re}\left\{\frac{\omega^2}{i\omega\rho_{\text{ion}}v_s}\left[\langle[h_{sI}^T,h_{sI}^T]\rangle_{\vec{q}\omega}+\langle[h_{dI}^T,h_{dI}^T]\rangle_{\vec{q}\omega}\right.\right.$$
$$+\frac{4\pi e^2q^{-2}|\langle[h_{sI}^T,j_{sx}]\rangle_{\vec{q}\omega}|^2}{1-4\pi e^2q^{-2}\langle[j_{sx},j_{sx}]\rangle_{q\omega}}$$
$$\left.\left.+\frac{4\pi e^2q^{-2}|\langle[h_{dI}^T,j_{dx}]\rangle_{\vec{q}\omega}|^2}{1-4\pi e^2q^{-2}\langle[j_{dx},j_{dx}]\rangle_{q\omega}}\right]\right\}\ , \quad (5)$$

where the averages are now over the fictitious system.

In the limit of low frequencies ($\omega\ll\Delta_d$), Eq. (5) reduces to

$$\alpha_T=\mathrm{Re}\left\{\frac{q^2}{i\omega\rho_{\text{ion}}v_s}\left[\langle[\tau_{sxz},\tau_{sxz}]\rangle_{q\omega}+\langle[\tau_{dxz},\tau_{dxz}]\rangle_{q\omega}\right.\right.$$
$$\left.\left.-\frac{|\langle[\tau_{sxz},j_{sx}]\rangle_{\vec{q}\omega}|^2}{\langle[j_{sx},j_{sx}]\rangle_{q\omega}}-\frac{|\langle[\tau_{dxz},j_{dx}]\rangle_{\vec{q}\omega}|^2}{\langle[j_{dx},j_{dx}]\rangle_{q\omega}}\right]\right\}. \quad (6)$$

The first two terms in (6) are the "collision-drag" terms treated by Kadanoff and Fal'ko. The last two terms are the electromagnetic terms which may be neglected in the limit $ql\ll1$.

Since the Gor'kov equations for the two-band superconductors in high fields[5] are identical in form to the Gor'kov equations for a one-band superconductor near H_{c2}, we expect that the usual perturbation expansion of the various correlation functions appearing in (6) for pure two-band superconductors near H_{2c} will diverge in the same manner as the expansion of the correlation functions for a pure one-band superconductor near H_{c2}.[3] To circumvent this difficulty, we make the conjecture that the effect of the magnetic field on the two energy gaps is similar to that of an uniform current in both bands. This would then allow us to formally sum the divergent terms in the perturbation expansion *a la* Maki.[2] By making the conjecture, we can write the density of states in each band as

$$N_{s(d)}(\omega)=N_{s(d)0}\int\frac{d\Omega}{4\pi}\int_{-\infty}^{\infty}d\alpha\,\rho_{s(d)}(\alpha,\Omega)$$
$$\times\mathrm{Re}\left[\frac{\omega-\alpha}{[(\omega-\alpha^2)-\Delta_{s(d)}{}^2]^{1/2}}\right]\ , \quad (7)$$

where

$$\rho_{s(d)}(\alpha,\Omega)=[\pi^{1/2}\epsilon_{s(d)}\sin\theta]^{-1}\exp\left[-\left(\frac{\alpha}{\epsilon_{s(d)}\sin\theta}\right)^2\right]\ . \quad (8)$$

The definitions of the various terms in (7) and (8) can be found in Ref. 1. We can now evaluate the various correlation functions appearing in (6) using the techniques developed by Maki.[7] As was the case of transverse attenuation in a one-band superconductor in an external magnetic field, two geometries have to be considered.

(a) $\vec{q}\parallel\vec{H}$ (the propagation vector is parallel to $\vec{H}$): Since we are considering the limit $ql\ll1$, only the first two terms in (6) contribute to the transverse attenuation. Following Maki, we obtain

$$\frac{\alpha_T^s}{\alpha_T^n}=\frac{\alpha_T^n(s)}{\alpha_T^n}g(y_s)$$
$$\times\left[1-\frac{\Delta_s}{2T}\int_{-\infty}^{\infty}d\alpha\,\Phi''_{Ts}(\alpha,y_s)\cosh^{-2}\left(\frac{\alpha}{2T}\right)\right]$$
$$+\frac{\alpha_T^n(d)}{\alpha_T^n}g(y_d)\left[1-\frac{\Delta_d}{2T}\right.$$
$$\left.\times\int_{-\infty}^{\infty}d\alpha\,\Phi''_{Td}(\alpha,y_d)\cosh^{-2}\left(\frac{\alpha}{2T}\right)\right]\ , \quad (9)$$

where

$$\Phi''_{Ts(d)}=\frac{3y_{s(d)}^2}{1-g(y_{s(d)})}\int\frac{d\Omega}{4\pi}\,\rho_{s(d)}(\alpha,\Omega)\,\frac{z^2(1-z^2)}{2(1+y_{s(d)}^2z^2)}\ , \quad (10)$$

$y_{s(d)} = ql_{s(d)}$, $l_{s(d)}$ is the $s(d)$ electronic mean free path, $g(y)$ is the Pippard function, and $g(y) = \frac{3}{2} y^{-3} \times [-y + (y^2+1)\arctan y]$. The asymptotic forms of the attenuation are given by

$$\frac{\alpha_L^s}{\alpha_L^n} = \frac{\alpha_L^n(s)}{\alpha_L^n} g(y_s) \left\{1 - \frac{\Delta_s}{2T}\left[1 - \frac{1}{2}\left(\frac{\epsilon_s}{2T}\right)^2 \left(1 + y_s^{-2} - \frac{1}{5}\,\frac{1}{1-g(y_s)}\right)\right]\right\}$$

$$+ \frac{\alpha_L^n(d)}{\alpha_L^n} g(y_d)\left\{1 - \frac{\Delta_d}{2T}\left[\left(1 - \frac{1}{2}\;\frac{\epsilon_d}{2T}\right)^2 \left(1 + y_d^{-2} - \frac{1}{5}\,\frac{1}{1-g(y_d)}\right)\right]\right\}, \quad T \lesssim T_{cd} \qquad (11)$$

$$\frac{\alpha_T^s}{\alpha_T^n} = \frac{\alpha_T^n(s)}{\alpha_T^n} g(y_s) \left\{1 - \frac{3(\sqrt{\pi})\,\Delta_s}{2(1-g(y_s))\epsilon_s}\left[\frac{y_s^2}{2(1+(1+y_s^2)^{1/2})^2} - \frac{\pi^2}{3}\left(\frac{T}{\epsilon_s}\right)^2 \frac{y_s^2}{(1+y_s^2)^{1/2}(1+(1+y_s^2)^{1/2})}\right] + O(T^3)\right\}$$

$$+ \frac{\alpha_T^n(d)}{\alpha_T^n} g(y_d)\left\{1 - \frac{3(\sqrt{\pi})\,\Delta_d}{2(1-g(y_d))\epsilon_d}\left[\frac{y_d^2}{2(1+(1+y_d^2)^{1/2})^2} - \frac{\pi^2}{3}\left(\frac{T}{\epsilon_d}\right)^2 - \frac{y_d^2}{(1+y_d^2)^{1/2}(1+(1+y_d^2)^{1/2})}\right] + O(T^3)\right\},$$

$$T \ll T_{cd}\,. \qquad (12)$$

In the above equations, $\alpha_T^n(s)$ $[\alpha_T^n(d)]$ is the normal-state attenuation in a metal containing only $s[d]$ electrons.

(b) $\vec{q} \perp \vec{H}$ (the propagation vector is perpendicular to $\vec{H}$): In this case, the attenuation is given by

$$\frac{\alpha_T^s}{\alpha_T^n} = \frac{\alpha_T^n(s)}{\alpha_T^n} g(y_s)\left[1 - \frac{\Delta_s}{2T}\int_{-\infty}^{\infty} d\alpha\, \Phi_{Ts}^{\perp}(\alpha, y_s, k) \times \cosh^{-2}\left(\frac{\alpha}{2T}\right)\right] + \frac{\alpha_T^n(d)}{\alpha_T^n} g(y_d)\left[1 - \frac{\Delta_d}{2T} \times \int_{-\infty}^{\infty} d\alpha\, \Phi_{Td}^{\perp}(\alpha, y_d, k)\cosh^{-2}\left(\frac{\alpha}{2T}\right)\right], \qquad (13)$$

where

$$\Phi_{Ts(d)}^{\perp}(\alpha, y_{s(d)}, k) = \frac{3y_{s(d)}^2}{1-g(y_{s(d)})} \times \int \frac{d\Omega}{4\pi}\, \rho_{s(d)}(\alpha, \Omega)\, \frac{z'^2 x'^2}{1 + y_{s(d)}^2 z'^2}\,, \qquad (14)$$

with $z' = (1-z^2)^{1/2}\cos\phi$, $x' = zk - (1-z^2)^{1/2} \times (1-k^2)^{1/2}\sin\phi$, and $k = \cos\theta$ (θ being the angle between the polarization vector and $\vec{H}$).

The asymptotic forms of the above attenuation are

$$\frac{\alpha_L^s}{\alpha_L^n} = \frac{\alpha_L^s(s)}{\alpha_L^n} g(y_s)\left(1 - \frac{\Delta_s}{2T}\left\{1 - \frac{1}{2}\left(\frac{\epsilon_s}{2T}\right)^2 \left[\frac{3-2k^2}{4} + \frac{2k^2+1}{4}\left(\frac{1}{5[1-g(y_s)]} - y_s^{-2}\right)\right]\right\}\right)$$

$$+ \frac{\alpha_L^n(d)}{\alpha_L^n} g(y_d)\left(1 - \frac{\Delta_d}{2T}\left\{1 - \frac{1}{2}\left(\frac{\epsilon_d}{2T}\right)^2 \left[\frac{3-2k^2}{4} + \frac{2k^2+1}{4}\left(\frac{1}{5[1-g(y_d)]} - y_d^{-2}\right)\right]\right\}\right), \qquad T \lesssim T_{cd} \qquad (15)$$

$$\frac{\alpha_L^s}{\alpha_L^n} = \frac{\alpha_L^n(s)}{\alpha_L^n} g(y_s)\left\{1 - \frac{3(\sqrt{\pi})\,\Delta_s}{[1-g(y_s)]\epsilon_s}\left[\frac{1+k^2}{4} + y_s^{-2}(1-k^2) - \frac{1}{(1+y_s^2)^{1/2}}\right.\right.$$

$$\times \frac{2}{\pi}\{[1-k^2+y_s^{-2}(2-3k^2)]K((1+y_s^{-2})^{-1/2}) - (1+y_s^{-2})(1-2k^2)E((1+y_s^{-2})^{-1/2})\} - \frac{1}{3}\left(\frac{\pi T}{\epsilon_s}\right)$$

$$\left.\left.\times\left(\frac{1-3k^2}{2} - \frac{2}{\pi}\{y_s^{-2}(1-k^2)(1+y_s^2)^{1/2}K((1+y_s^{-2})^{-1/2}) - [k^2+y_s^{-2}(1-k^2)]E((1+y_s^{-2})^{-1/2})\}\right)\right]\right\}$$

$$+ \frac{\alpha_L^n(d)}{\alpha_L^n} g(y_d)\left\{1 - \frac{3(\sqrt{\pi})\,\Delta_d}{[1-g(y_d)]\epsilon_d}\left[\frac{1+k^2}{4} + \cdots\right]\right\}, \qquad T \ll T_{cd} \qquad (16)$$

and $K(z)$ and $E(z)$ are the complete elliptic integrals.

The recent discoveries of a second enery gap[8] and a second transition temperature[9] in pure niobium superconductors clearly point to the need to use the two-band model[10] as the basis for superconductivity in niobium and other transition-metal superconductors. That the two-band nature of the niobium superconductor will affect the ultrasonic attenuation is seen in the longitudinal attenuation measurements in niobium superconductors by Tsuda and Suzuki[11]. The longitudinal attenuation in zero field was seen to depart from the Bardeen, Cooper, and Schrieffer[12] behavior. A similar departure of the longitudinal attenuations in mercury[13] and lead[14] superconductors was explained by the use of a phenomenological multiband attenuation coefficient.[15]

A clear indication that the one-band mixed-state attenuation coefficients obtained by Maki[7] are wrong is the discrepancy in the density of states needed to achieve a fit of the longitudinal and transverse attenuation data on a niobium superconductor having a residual resistivity ratio (RRR) 150. To fit their data on the transverse attenuation in a RRR-150 niobium superconductor near H_{c2} and on the longitudinal attenuation in a RRR-300 niobium superconductor near H_{c2}, Kagiwada *et al.*[16] used the same value of 1.5×10^{34} state/cm^3 erg for the densities of states in both samples. The value of 1.5×10^{34} state/cm^3 erg for the density of states in a RRR-300 sample was consistent with the values obtained by Tsuda *et al.* from their mixed-state longitudinal attenuation data. The longitudinal data of Tsuda and Suzuki indicate that the density of states for the RRR-150 sample is 3.4×10^{34} and not 1.5×10^{34}. In addition, neither of these values are in agreement with the value 5.6×10^{34} state/cm^3 erg obtained from the specific-heat measurements.[17] The specific-heat measurements also indicate that the density of states does not change appreciably when a small amount of impurities are added to the transition-metal superconductor.

The advantage of the two-band attenuation coefficients is that the density of states in the d band, which governs the behavior of thermodynamic properties, can remain constant while the attenuation changes drastically when impurities are added. Unpublished results[18] of Vinen and Gough on the transverse attenuation in pure niobium superconductors show a purity dependence similar to that seen in the longitudinal attenuation in the same niobium superconductors.[18] At the present time, it is not possible to fit the transverse attenuation data on a RRR-150 niobium superconductor near H_{c2} to the two-band expression (12) since none of the two-band parameters for the RRR-150 sample are known.[19] The two-band parameters cannot be obtained by extrapolating from the known two-band parameters in a RRR-110 sample and a very pure sample (RRR $\geqslant$ 2100) since the two samples are at the opposite end of the purity spectrum that can be considered (the tunneling experiments of Hafstrom and MacVicar[8] indicate that a RRR-100 niobium sample is not clean enough to exhibit two-band effects).

III. BPT METHOD APPLIED TO TWO-BAND SUPERCONDUCTORS NEAR H_{2c}

As we have seen in Sec. II, the various correlation functions can be evaluated if we make the conjecture that the effects of the magnetic field on the two energy gaps in a pure two-band (type-II) superconductor near H_{c2} are similar to those of an uniform current in both bands. As was mentioned, the need for making his conjecture results from the fact that the usual perturbation expansion in powers of the order parameters (or energy gaps) diverges for the case of *pure* type-II superconductors in high magnetic fields. de Gennes[20] showed that the density of states for a pure one-band superconductor near H_{c2} obtained by means of a perturbation expansion in powers of the order parameter developed a logarithmic singularity at zero excitation energy. Since the Gor'kov equations for a pure two-band superconductor in high fields are identical in form to the one-band Gor'kov equations, we expect that the densities of states obtained by iterating the Gor'kov equations will develop the same logarithmic singularity. Therefore, the ultrasonic attenuation coefficients for two-band superconductors in high fields obtained by a perturbation expansion of the correlation functions should be as unphysical as the one-band attenuation coefficient obtained by the perturbation expansion technique.[3]

An alternative method for calculating the Green's function for a pure one-band superconductor near H_{c2} was devised by Brandt, Pesch, and Tewordt.[4] Their method was based on the existence of a periodicity in the order parameter near H_{c2} and did not involve an iteration of the Gor'kov equations. Using this method, Pesch[21] was able to calculate the nuclear-spin relaxation rate in a pure one-band superconductor near H_{c2}.

Since the solutions of the two-band analog of the Ginzburg-Landau equations[5] have the same periodicity[22] as the order parameter in the one-band superconductor, the BPT method can be employed to obtain the Green's functions for the pure two-band superconductor near H_{c2}. Utilizing the BPT method, we obtain the Green's functions for the two-band system, i.e.,

$$G_{s(d)\omega}(\vec{p},\vec{k}) = \delta_{k,0}\left\{ i\omega - \zeta_{s(d)} + \frac{i(\sqrt{\pi})\,\Delta_{s(d)}}{k_c v_{s(d)F}\sin\theta} \times W\left(\frac{i\omega+\zeta_{s(d)}}{k_c v_{s(d)F}\sin\theta}\right)\right\}^{-1}, \quad (17)$$

where the vector $\vec{k}_c = (2eH_{c2})^{1/2}$ is inversely proportional to the spacing betten the flux lines, $\Delta_{s(d)}$ is the energy gap in the $s(d)$ band, $\Delta^2_{s(d)} = |\Delta_{s(d)}|^2$, $\zeta_{s(d)} = k^2/2m_{s(d)}\epsilon_F$, θ is the angle between k and H, and

$$W(z) = \frac{i}{\pi}\int_{-\infty}^{\infty} dt\, \frac{e^{-t^2}}{z-t} \ . \tag{18}$$

Being in possession of explicit forms for the Green's function of a pure two-band superconductor near H_{c2}, we can now evaluate the various correlation functions without having to make the conjecture used in Sec. II. Since nothing would be gained by evaluating more than one type of correlation functions using the new BPT method, we shall calculate the longitudinal attenuation coefficient in the limit $ql \gg 1$. In this limit, the attenuation coefficient is given by

$$\alpha_L = \mathrm{Re}\left\{\frac{q^2}{i\omega_s\rho_{\mathrm{ion}}v_s}\left[\left(\frac{p_F^2}{3m_s}\right)^2 \langle [n_s, n_s]\rangle_{q\omega_s} + \left(\frac{p_F^2}{3m_d}\right)\langle [n_d, n_d]\rangle_{q\omega_s}\right]\right\}, \tag{19}$$

where the retarded products $\langle [n_s, n_s]\rangle_{q\omega_s}$ and $\langle [n_d, n_d]\rangle_{q\omega_s}$ are obtained by an analytic continuation of the thermal product $\langle [n_i, n_i]\rangle_{q\omega_0}$ from the set of discrete points $\omega_0 = 2m\pi T$ to $z = \omega_s - i\delta$ (ω_s being the angular frequency of the sound wave). Assuming that the magnetic field inside the superconductor can be replaced by its space average, the thermal products $\langle [n_i, n_i]\rangle_{q\omega_0}$ near H_{c2} can be written as[23]

$$\begin{aligned}\langle [n_i, n_i]\rangle_{q\omega_0} = 2T\sum_\omega \int d^3r \int d^3r'\, e^{-i\vec{q}\cdot(\vec{r}-\vec{r}')} \\ \times\{G_{i\omega+}(\vec{r},\vec{r}')\,G_{i\omega}(\vec{r}',\vec{r}) - \int d^3r_1 \int d^3r_2\, G^0_{i-\omega+}(\vec{r}-\vec{r}_1) \\ \times G_{i\omega}(\vec{r}',\vec{r}_1)\,V_i(\vec{r}_1,\vec{r}_2)G_{i\omega}(\vec{r}_2,\vec{r}')\,G^0_{i-\omega}(\vec{r}_2-\vec{r})\},\end{aligned} \tag{20}$$

where $G^0_{s(d)\omega}(\vec{r}-\vec{r}')$ is the normal-metal Green's function in the absence of the magnetic field, the function $V_{s(d)}(\vec{r}_1,\vec{r}_2)$ being defined as

$$V_i(r_1, r_2) = \Delta_i(r_1)\,\Delta_i^*(r_2) \times \exp(-ie\langle H\rangle(x_1+x_2)(y_1-y_2)), \tag{21}$$

$\omega = (2n+1)\pi T$, and $\omega_+ = \omega + \omega_0$.

After the integration over the magnitude of $\vec{k}$ and the polar angle, we have after the analytic continuation,

$$\begin{aligned}\alpha_L = \frac{\omega_s}{V_s^2\rho_{\mathrm{ion}}}\sum_i m_i^2\left(\frac{p_F^2}{3m_i}\right)^2 \times\int \frac{d\omega}{\partial\omega}\,\frac{\partial f(\omega)}{\partial\omega}\Bigg\{[2\,\mathrm{Re}K_i(z_{i0})-1]^2 \\ -2\left(\frac{\Delta_i}{k_cv_{iF}}\right)^2 \mathrm{Re}\left[K_i^2(z_{i0})\,i(\sqrt{\pi})\,W_i'(z_{i0}) + |K_i(z_{i0})|^2 \times\frac{2(\sqrt{\pi})(W_i(z_{i0})-W_i(z_{i0}^*))}{2\,\mathrm{Im}z_{i0}}\right]\Bigg\},\end{aligned} \tag{22}$$

where the summation is over the two bands, and where

$$z_{i0} = \frac{2(\omega + i/2\tau)}{k_cv_{iF}\sin\theta} + i(\sqrt{\pi})\left(\frac{\Delta_i}{k_cv_{iF}\sin\theta}\right)W_i(z_{i0}),$$

$$K_i(z_{i0}) = \left[1 - i(\sqrt{\pi})\left(\frac{\Delta_i}{k_cv_{iF}\sin\theta}\right)^2 W_i'(z_{i0})\right]^{-1}, \tag{23}$$

$$\sin\theta = (1-\sin^2\phi\,\sin^2\alpha)^{1/2}, \quad \alpha = \mathrm{angle}(\vec{q}, \vec{H}),$$

$$f(\omega) = (e^\omega + 1)^{-1}.$$

In obtaining the above expressions, we have assumed that $qv_{iF} > \Delta_i$, $\omega_S\tau_s \ll 1$, $\omega_S\tau_d \ll 1$, and that $\vec{q}\parallel H$. In addition, only terms of first order in ω_S have been kept. In the region $(H_{c2}-H)\ll H_{c2}$, where the parameters i/k_cv_{iF} are both less than one, the two functions $K_i(z_{i0})$ and $W_i'(z_{i0})$ can be expanded in powers of (Δ_i/k_cv_{iF}).[2] Noting that

$$\left(\frac{\Delta_i}{k_cv_{iF}}\right)\frac{k_cl_i\,\mathrm{Im}[i(\sqrt{\pi})\,W_i(z_{i0})]}{1+(\Delta_i/k_cv_{iF})^2k_cl_i\,\mathrm{Im}\,[i(\sqrt{\pi})\,W_i(z_{i0})]} = \left(\frac{\Delta_i}{k_cv_{iF}}\right)^2\frac{(\sqrt{\pi})[W_i(z_{i0})-W_i(z_{i0}^*)]}{2\,\mathrm{Im}z_{i0}}, \tag{24}$$

we have

$$\begin{aligned}\alpha_L = \frac{\omega_S}{v_S^2}\sum_i \frac{m_i^2}{2}\left(\frac{p_F^2}{3m_i}\right)^2\Bigg\{1-2\left(\frac{\Delta_i}{k_cv_{iF}}\right) \times\left[\frac{k_cl_i\,\mathrm{Im}[i(\sqrt{\pi})\,W_i(i/k_cl_i)]}{1+(\Delta_i/k_cv_{iF})^2k_cl_i\,\mathrm{Im}[i(\sqrt{\pi})\,W_i(i/k_cl_i)]} \\ - \mathrm{Re}[i(\sqrt{\pi})\,W_i'(i/k_cl_i)]\right]\Bigg\},\end{aligned} \tag{25}$$

where the terms of higher order in Δ_i/k_cv_{iF} have been dropped. A further reduction occurs when $(\Delta_i/k_cv_{iF})^2k_cl_i \ll 1$. So that the longitudinal attenuation coefficient finally becomes

$$\begin{aligned}\alpha_L = \frac{m_s^2}{V_S^2\rho_{\mathrm{ion}}}\left(\frac{p_F^2}{3m_s}\right)\frac{\omega_S}{2}\left[1-2(\sqrt{\pi})\,\frac{\Delta_s}{v_{sF}}\,l_s + O(\Delta_s^3)\right] \\ + \frac{m_d^2}{V_S^2\rho_{\mathrm{ion}}}\left(\frac{p_F^2}{3m_d}\right)^2\frac{\omega_S}{2}\left[1-2(\sqrt{\pi})\,\frac{\Delta_d}{v_{dF}}\,l_d + O(\Delta_d^3)\right].\end{aligned} \tag{26}$$

The above expression (26) for the longitudinal attenuation coefficient (in the limit $ql \gg 1$) of a pure two-band superconductor in the mixed state is in agreement with the results obtained by the use of Maki's conjecture. The attenuation resulting from the use of the conjecture regarding the effects of the magnetic field on the two energy gaps is

$$\alpha_L = \frac{\pi N_s m_s v_{sF}}{6\rho_{\text{ion}} v_S^2}\,\omega_S \left\{ 1 - \frac{4}{\pi^{3/2}}\,\frac{\Delta_s}{\epsilon_s} \times \left[K(k) - \frac{1}{3}\left(\frac{\pi T}{\epsilon_s}\right)^2 (K(k) + k K'(k)) + \cdots \right] \right\}$$

$$+ \frac{\pi N_d m_d v_{dF}}{6\rho_{\text{ion}} v_S^2}\,\omega_S \left\{ 1 - \frac{4}{\pi^{3/2}}\,\frac{\Delta_d}{\epsilon_d} \times \left[K(k) - \frac{1}{3}\left(\frac{\pi T}{\epsilon_d}\right)^2 (K(k) + k K'(k)) + \cdots \right] \right\}, \quad (27)$$

where the definitions of all the terms are found in Ref. 1.

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Temperature Dependence of the Weak Ferromagnetic Moment of Hematite

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A recent proposal by Searle and Dean, which ascribes the anomalous temperature dependence of the weak ferromagnetic moment of hematite to a *large temperature-dependent* inclination of the antiferromagnetic axis out of the basal plane above the Morin temperature, is demonstrated to be incompatible with Mössbauer data. Some possible explanations of this effect are noted.

Recently Searle and Dean[1] have measured the temperature dependence of the weak ferromagnetic moment m of hematite (α-Fe_2O_3) above its Morin transition[2] ($T_M \simeq 260$ °K), and, in agreement with a prior study by Flanders and Schule,[3] they found that m drops more slowly than the sublattice magnetization M. The observed increase of m/M is rather unexpected. The usual molecular-field treatment of the