Effects of Anisotropy on Magnetoplasma Waves in a Charged Fermi-Liquid System*

Y. C. Cheng
Department of Physics, University of Washington, Seattle, Washington 98105
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The propagation of the plasma modes in the presence of a static magnetic field and in an electron liquid is studied in terms of the Landau Fermi-liquid theory for a system with an anisotropic Fermi surface. Two types of anisotropy are studied: (a) Weak anisotropy; the resonant frequency of the normal modes is calculated in the long-wavelength limit, $q \to 0$, for a system with a nearly spherical Fermi surface. It is found that the deviation from the isotropic case is of the linear order of the anisotropy. Application to the alkali metals is briefly discussed. Effects of weak anisotropy of the Landau F function is also studied. (b) Strong anisotropy; the dispersion relations of the modes propagating parallel to the magnetic field near the cyclotron frequency ω_c and its harmonics $m\omega_c$ are calculated for a system with a simplified Fermi surface which satisfies the Gor'kov-Dzyaloshinskii condition. It is found that, except for the $m=1$ modes, all higher-m modes have large finite (or infinite) cutoff wave vectors even in the weak coupling limit; while in the isotropic case, all cutoff wave vectors approach zero in the same limit. It is suggested that this simplified model may be relevant to the noble metals.

I. INTRODUCTION

In recent years, the study[1-8] of Fermi-liquid effects on the conduction electrons in metals has been primarily concentrated on the investigation of the plasma modes propagating in a static magnetic field in an *isotropic* system. In this paper we would like to extend the same kind of analysis to an *anisotropic* system. There are two types of anisotropy which are of practical interest and we will confine ourselves to these two cases in the following.

(i) Weak anisotropy: All previous experiments were concerned with the measurements in the alkali metals and the experimental results were compared with the theoretical predictions of an isotropic case. However, the de Haas-van Alphan effects[9,10] show that the Fermi surface of the alkali metals is slightly different from a perfect sphere. We expect this small nonsphericity to affect the resonant frequency of the magnetoplasma modes, and by a careful study of the experimental results, one might be able to determine the anisotropy of the Fermi surface of the alkali metals through this kind of experiment.

In an isotropic case, the Landau F function $F(\vec{k},\vec{k}')$ depends only on the angle between $\vec{k}$ and $\vec{k}'$. But in real metals because of the presence of the crystal potential, $F(\vec{k},\vec{k}')$ will also have a dependence on the absolute directions of $\vec{k}$ and $\vec{k}'$. This dependence will be small in the alkali metals and can be considered as a small perturbation to the isotropic case. In this way we can also study the effect of anisotropic F function on the resonant frequency of the plasma modes.

(ii) Strong anisotropy: It is known that[4] in an isotropic system all the plasma modes propagating parallel to a static magnetic field near the cyclotron frequency have a cutoff wave vector. This cutoff wave vector decreases with decreasing Fermi-liquid interaction and goes to zero in the limit of vanishing Fermi-liquid interaction. Therefore these modes would be very difficult to be detected in the weak coupling limit. In an earlier paper[11] we pointed out that for certain suitable materials, high anisotropy would favor the existence of zero sound in the absence of a magnetic field. We believe that the same features responsible for allowing the propagation of zero sound in weakly interacting Fermi liquids in the absence of a magnetic field can materially alter the conditions for the propagation of modes in a magnetic field. In particular, we expect a finite (or even infinite) cutoff wave vector even in the weak coupling limit. Since the detection of the field-dependent modes is within the range of present experimental technique,[7] while field-free zero sound has yet to be observed, it may be easier to study the effects of high anisotropy through these field-dependent modes. The analysis is very complicated and difficult for a highly anisotropic case. In this paper we, therefore, use a simplified model of the Fermi surface to avoid the mathematical complexity. This model has its practical interest since it has several properties which may be relevant to the noble metals.

To study the propagation properties of the plasma modes, we make use of the collisionless kinetic equation for a charged Fermi-liquid system in the presence of a uniform static magnetic field[12] $\vec{H}(\omega\tau \gg 1)$:

$$\omega\nu - qv_{\parallel}\bar{\nu} - \frac{1}{i}\frac{e}{c}(\vec{v}\times\vec{H})\cdot\nabla_{\vec{k}}\,\bar{\nu} = \frac{k_0^2}{\omega}\,v_{\parallel}\langle v_{\parallel}\bar{\nu}\rangle + \frac{\omega k_0^2}{\omega^2 - q^2c^2}\,\vec{v}_{\perp}\cdot\langle\vec{v}_{\perp}\bar{\nu}\rangle\,. \quad (1)$$

Here $\vec{v}$ is the quasiparticle velocity and we have written the deviation from the equilibrium distribution as

$$\delta n = -\left(\frac{\partial n^0}{\partial \epsilon^0}\right) \nu(\vec{k})\, e^{i(\vec{q}\cdot\vec{r}-\omega t)}, \tag{2}$$

where $\vec{q}$ is the wave vector and ω the angular frequency of the disturbance, and $\vec{k}$ specifies a momentum state of a quasiparticle. The angular brackets represent an average over the Fermi surface[13]:

$$\langle \nu \rangle = \frac{1}{g(\epsilon_F)} \int \frac{dS}{4\pi^3 v}\, \nu, \quad g(\epsilon_F) = \int \frac{dS}{4\pi^3 v}, \tag{3}$$

and

$$\bar{\nu}(\vec{k}) = (1+F)\, \nu(\vec{k}) = \nu(\vec{k}) + \frac{1}{g(\epsilon_F)} \int \frac{dS(\vec{k}')}{4\pi^3 v(\vec{k}')} F(\vec{k},\vec{k}')\,\nu(\vec{k}'), \tag{4}$$

where $F(k, k')$ is the spin-independent part of the Landau F function. The quantity $k_0^2 = 4\pi e^2 g(\epsilon_F)$ is the square of the Fermi-Thomas screening wave vector, and $\parallel$ and $\perp$ refer to the direction of $\vec{q}$.

II. WEAK ANISOTROPY

It is known[2,5] that in an isotropic electron liquid, the resonant frequency of the lm mode of a magnetoplasma wave is shifted from its free-electron value $\omega_{lm} = m\omega_c$ to $m(1+A_l)\omega_c$ in the long-wavelength limit $(q \to 0)$, where ω_c is the cyclotron frequency and A_l is a Fermi-liquid parameter.[14] Therefore, the quantity

$$\omega_{lm_1}/m_1 - \omega_{lm_2}/m_2 \qquad (q \to 0) \tag{5}$$

is identical to zero in an isotropic system. This is a characteristic of an isotropic system. In a weakly anisotropic system, we expect that (5) will be different from zero and the magnitude of it will depend on the degree of anisotropy of the system. Therefore, by measuring the resonant frequency of the plasma modes with the same l but different m, one may be able to determine the degree of anisotropy of a nearly isotropic system from (5). Since we are only interested in the long-wavelength limit, we shall set $q = 0$ in the following analysis.

Since both the anisotropies of the Fermi surface and $F(\vec{k}, \vec{k}')$ are probably very small in the alkali metals, we shall discuss the effects of these two anisotropies separately, i.e., we assume only one kind of anisotropy at a time.

Fermi surface. Suppose we have a Fermi surface

$$\epsilon(\vec{k}) = \epsilon_F, \tag{6}$$

which is slightly different from a sphere. We define a "Fermi sphere" with radius k_F by requiring that the number of electronic states n inside the sphere is equal to those inside the Fermi surface, i.e.,

$$2\int \frac{d\vec{k}}{(2\pi)^3}\, \eta\,[\epsilon(\vec{k}) - \epsilon_F] = n = \frac{k_F^3}{3\pi^2}, \tag{7}$$

where

$$\eta(x) = \begin{cases} 0, & x > 0 \\ 1, & x < 0 . \end{cases} \tag{8}$$

We rewrite (6) as

$$\epsilon(\vec{k}) = k^2/2m^* + \lambda \epsilon_1(\vec{k}), \tag{9}$$

where the effective mass $m^* = k_F^2/2\epsilon_F$ and λ is a dimensionless quantity which measures the degree of anisotropy [i.e., $\epsilon_1(\vec{k})$ is of the order of ϵ_F when $|\vec{k}| = k_F$]. For weak anisotropy λ is very small. To the lowest order in λ, (7) requires

$$\int \frac{d\vec{k}}{(2\pi)^3}\, \epsilon_1(\vec{k}) = 0, \quad |\vec{k}| = k_F. \tag{10}$$

In the presence of a magnetic field $\vec{H}$ $(\parallel \hat{z})$, it is convenient to define a "phase variable" α, along with the energy ϵ and k_z, to describe the quasiparticle motion on its orbit, i.e.,

$$\frac{e}{c}(\vec{v}\times\vec{H})\cdot\nabla_{\vec{k}} = \omega_c(k_z)\frac{\partial}{\partial\alpha}, \quad \alpha = \omega_c(k_z)T, \tag{11}$$

where T is the time interval spent by the quasiparticle on its orbit. In an anisotropic system,[15] the cyclotron frequency ω_c is a function of k_z and α is not identical to the azimuthal angle ϕ of $\vec{k}$ on the Fermi surface unless the Fermi surface has a cylindrical symmetry around the direction of $\vec{H}$. The components of the velocity $\vec{v}$ will also have components outside the $l = 1$ subspace when we expand them in terms of the spherical harmonics.

We will show that to leading order in λ, the effects of anisotropy on the resonant frequencies will be of the linear order in λ. The contribution comes from the dependence of ω_c on k_z, the effects of the anisotropy of $\vec{v}$ and $\alpha \neq \phi$ being of the second or higher orders in λ.

To see this we write

$$\omega_c(k_z) = \omega_c^0 + \delta\omega_c(k_z) \tag{12}$$

and

$$\frac{\partial\phi}{\partial\alpha} = 1 + s(\theta, \phi), \tag{13}$$

where ω_c^0 is the cyclotron frequency when $\lambda = 0$ and (θ, ϕ) are the spherical angles on the Fermi surface. From the form of the Fermi surface (9), we see that the magnitude of $\vec{k}$ on the Fermi surface is a function of (θ, ϕ) and to linear order in λ, we have

$$k = k(\theta, \phi) = k_F[1 + \lambda a(\theta, \phi)], \tag{14}$$

where $a(\theta, \phi)$ depends on the form of $\epsilon_1(\vec{k})$. We expand $\omega_c(k_z)$ according to (14),

$$\omega_c(k_z)=\omega_c(k\cos\theta)=\omega_c(k_F\cos\theta)$$

$$+\left(\frac{\partial\omega_c}{\partial k_z}\right)\bigg|_{k_z=k_F\cos\theta}(\lambda k_F\cos\theta\, a(\theta,\phi)). \quad (15)$$

Since $\partial\omega_c/\partial k_z=0$ when $\lambda=0$, so that to linear order in λ we have $\omega_c(k_z)=\omega_c(k_F\cos\theta)$, i.e., ω_c is a constant for constant θ. Therefore, we can rewrite (12) as

$$\omega_c(\theta)=\omega_c^0+\delta\omega_c(\theta) \ . \quad (16)$$

With the aid of (11), (13), and (16), the kinetic equation (1) has the form

$$\frac{1}{i}\left(1+\frac{\delta\omega_c(\theta)}{\omega_c^0}+s(\theta,\phi)\right)\frac{\partial}{\partial\phi}\bar{\nu}+\frac{k_0^2}{\omega\omega_c^0}\vec{v}\cdot\langle\vec{v}\,\bar{\nu}\rangle=\frac{\omega}{\omega_c^0}\nu \ , \quad (17)$$

where we have neglected the term $s\ \delta\omega_c/\omega_c^0$, it being a higher-order term.

Equation (17) is to be solved in the space spanned by all Y_{lm}. We neglect the anisotropy of $\vec{v}$ and $F(\vec{k},\vec{k}')$, and then the last term on the left-hand side of (17) acts only in the $l=1$ subspace (the neglect of the anisotropies will introduce only higher-order corrections). In a weakly anisotropic case we consider $(\delta\omega_c/\omega_c^0+s)$ as a small perturbation. Then Y_{lm} $(l\neq 1)$ is an eigenfunction of the "unperturbed" system with eigenfrequency $m(1+A_l)\omega_c^0$. By using the perturbation method discussed in Ref. 2, we find

$$\omega_{lm}=m\gamma_l\,\omega_c^0+\gamma_l\left\langle lm\left|[\delta\omega_c(\theta)+\omega_c^0 s(\theta,\phi)]\frac{1}{i}\frac{\partial}{\partial\phi}\right|lm\right\rangle$$

$$=m\gamma_l\,\omega_c^0+m\gamma_l\langle lm\,|\,\delta\omega_c(\theta)\,|\,lm\rangle,$$

$$\gamma_l=1+A_l, \quad l>1. \quad (18)$$

There is no contribution from the term $s(\theta,\phi)$ in the linear order corrections. Since in the expansion $s(\theta,\phi)=\sum_m s_m(\theta)e^{im\phi}$, we should have $s_0(\theta)=0$ by noting that in one period of quasiparticle motion on the Fermi surface, α increases by 2π, and so does ϕ. Therefore, the effect of $\omega_c=\omega_c(k_z)$ will be proportional to $\delta\omega_c(\theta)$ which is linear in λ, while the effect of $\alpha\neq\phi$ will be at least in the second order.

We can also show that the corrections, due to the fact that the velocity $\vec{v}$ has components outside the $l=1$ subspace, are of the second order in λ. By neglecting the anisotropy in ω_c and α, the kinetic equation (1) can be written as

$$\omega\nu-\frac{1}{i}\,\omega_c\frac{\partial}{\partial\phi}\,\bar{\nu}=ie\,\vec{E}\cdot\vec{v}, \quad (19)$$

where we have written the right-hand side as its original form, $\vec{E}$ being the induced electrical field. Equation (19) has a solution

$$\nu=i\,e\sum_{l,m}[Y_{lm}(\theta,\phi)\int\vec{E}\cdot\vec{v}(\theta',\phi')Y_{lm}^*(\theta',\phi')\,d\Omega'/(\omega-m\gamma_l\,\omega_c)]. \quad (20)$$

From this we obtain the components of the conductivity tensor

$$\sigma_{\mu\nu}\propto\sum_{lm}\gamma_l\int v^\mu Y_{lm}\,d\Omega\int v^\nu Y_{lm}^*\,d\Omega/(\omega-m\gamma_l\,\omega_c). \quad (21)$$

The resonant frequencies can be obtained by setting the determinant of the conductivity tensor equal to zero,[1] i.e.,

$$|\overleftrightarrow{\sigma}|=0 \ . \quad (22)$$

Since in the isotropic case $\sigma_{xz}=\sigma_{zx}=\sigma_{yz}=\sigma_{zy}=0$, it is easy to see that to the lowest-order approximation (22) is separated into two equations:

$$\begin{vmatrix}\sigma_{xx} & \sigma_{xy}\\ \sigma_{yx} & \sigma_{yy}\end{vmatrix}=0 \quad (23)$$

and

$$|\sigma_{zz}|=0 \ . \quad (24)$$

For modes with frequency ω near $m\gamma_l\,\omega_c$, we write $\omega-m\gamma_l\,\omega_c=\Delta\omega_{lm}$. This is the effect of the anisotropy of $\vec{v}$ and is very small in comparison with $m\omega_c$. After some algebra, we find

$$\Delta\omega_{lm}=\frac{3}{4\pi}\,m\gamma_l\,\omega_c\times\frac{\gamma_l\,(|v_{lm}^x|^2+|v_{lm}^y|^2)-(2/m)\gamma_1\mathrm{Im}(v_{lm}^x v_{lm}^{y*})}{\gamma_1 v_F^2}, \quad l>1 \quad (25)$$

for modes contained in (23); and

$$\Delta\omega_{lm}=-\frac{3}{4\pi}\,m\gamma_l\,\omega_c\,\frac{\gamma_l|v_{lm}^z|^2}{\gamma_1 v_F^2}, \quad l>1 \quad (26)$$

for modes contained in (24). For $l\neq 1$, v_{lm}^μ is, at least, of the linear order in λ and therefore the effects of the anisotropy of $\vec{v}$ are of the second order in λ.

Therefore, to lowest order in λ, the effects of the anisotropy of the Fermi surface on the resonant frequencies will be linear in λ and its value is given by (18). The measure of the anisotropy is, by (5),

$$\omega_{lm_1}/m_1-\omega_{lm_2}/m_2=\gamma_l\,[\langle lm_1|\,\delta\omega_c(\theta)\,|\,lm_1\rangle-\langle lm_2|\,\delta\omega_c(\theta)\,|\,lm_2\rangle]. \quad (27)$$

One can calculate $\delta\omega_c(\theta)$ if $\epsilon_1(\vec{k})$ in (9) is known. From the equation of motion, we have

$$\frac{dk}{dT} = \frac{e}{c} v_\perp H, \tag{28}$$

where T is defined in (11) and $\perp$ here refers to the direction of $\vec{H}$. Note that on the quasiparticle orbit the component of $\vec{k}$ parallel to $\vec{H}$ is a constant of motion, therefore we have

$$dk = k_\perp d\phi = k_F \sin\theta\,[1+\lambda\, a(\theta,\phi)]\, d\phi, \tag{29}$$

where the last form is obtained from (14). From (9), we have to linear order in λ

$$v_\perp = \nabla_{\vec{k}_\perp} \epsilon(\vec{k}) = \frac{k_F}{m^*} \sin\theta\,[1+\lambda\, b(\theta,\phi)], \tag{30}$$

where $b(\theta,\phi)$ can be determined from $\nabla_{\vec{k}}\, \epsilon_1(\vec{k})$ evaluated at $|\vec{k}| = k_F$. From (28)–(30) we have the θ dependence of the period $T(\theta)$ of the quasiparticle motion

$$T(\theta) = \int_0^{T(\theta)} dT = \frac{c}{eH}\oint \frac{dk}{v_\perp} = \frac{m^* c}{eH}\int_0^{2\pi} [1+\lambda(a-b)]\, d\phi$$

$$= T_0 + \lambda T_1(\theta), \tag{31}$$

where $T_0 = 2\pi(m^*c/eH)$ and

$$T_1(\theta) = \frac{m^* c}{eH}\int_0^{2\pi} [a(\theta,\phi) - b(\theta,\phi)]\, d\phi. \tag{32}$$

Knowing $T_1(\theta)$, $\delta\omega_c(\theta)$ is readily calculated, i.e.,

$$\omega_c(\theta) = 2\pi/T(\theta) = \omega_c^0(1-\lambda\, T_1/T_0) = \omega_c^0 + \delta\omega_c(\theta). \tag{33}$$

For example, if we have a Fermi surface with cubic symmetry (as the alkali metals) of the following form:

$$\epsilon(\vec{k}) = \frac{k^2}{2m^*} + \lambda \sum_{i=x,y,z}\left(\frac{k_i^4}{m^* k_F^2} - \frac{7}{15}\frac{k_F^2}{m^*}\right), \tag{34}$$

which satisfies (10). For simplicity we take the magnetic field $\vec{H}$ along one of the crystal axes, and therefore we can choose the direction of $\vec{H}$ as the z axis without changing the form of (34). Following the analysis (29)–(33), we find

$$\delta\omega_c = 3\lambda\,\omega_c^0 \sin^2\theta. \tag{35}$$

Therefore, the measure of the anisotropy is, by (27),

$$\frac{\omega_{lm_1}/m_1 - \omega_{lm_2}/m_2}{\gamma_l\, \omega_c^0}$$

$$= 3\lambda\,(\langle lm_1|\sin^2\theta\,|lm_1\rangle - \langle lm_2|\sin^2\theta\,|lm_2\rangle)$$

$$= 6\frac{m_1^2 - m_2^2}{(2l-1)(2l+3)}\lambda, \quad l>1 \tag{36}$$

which is proportional to λ. For $l=2$ modes with $m_1 = 1$ and $m_2 = 2$, (36) is equal to $-\frac{6}{7}\lambda$. When the direction of $\vec{H}$ changes continuously over the crystal, the corresponding $\delta\omega_c(\theta)$ will also change accordingly. However, the order of magnitude of (27) will remain about the same as (36).

Experimental data[9,10] show that the deviation from a sphere of the Fermi surface is less than one part in 10^3 in potassium and sodium and a little less than one part in 10^2 in rubidium. This suggests that rubidium may be the best candidate for observing the weak anisotropy of the Fermi surface through the plasma-modes experiment.

Landau F function. Since the effects of the anisotropy of the Fermi surface of the alkali metals is rather small, in the following analysis we assume that the Fermi surface is a sphere, but $F(\vec{k},\vec{k}')$ is not invariant under an arbitrary rotation in $\vec{k}$ space. In general, $F(\vec{k},\vec{k}')$ will be invariant only under the operations of the crystal symmetry. For a particular choice of a coordinate system S_1 attached to the crystal (for example, xyz along cubic axes), we define the operator F as follows:

$$F\psi(\Omega) = \int d\Omega'\, F(\Omega,\Omega')\psi(\Omega'), \tag{37}$$

where Ω and Ω' are the spherical angles of the direction of $\vec{k}$ and $\vec{k}'$, respectively, and $F(\Omega,\Omega')$ can be expanded in terms of the spherical harmonics,

$$F(\Omega,\Omega') = \sum_{lm}\sum_{l'm'} F_{ll';mm'}\, Y_{lm}(\Omega)\, Y^*_{l'm'}(\Omega'). \tag{38}$$

In the practical situation it is most convenient to choose the direction of $\vec{H}$ as the z axis and therefore a coordinate system S_2 different from S_1. Since $F(\vec{k},\vec{k}')$ is not invariant under an arbitrary rotation, the operator F defined in (38) will have a different form in the coordinate system S_2; we call it F_R, i.e.,

$$F_R\psi(\Omega) = \int d\Omega'\, F(R\Omega, R\Omega')\psi(\Omega'), \tag{39}$$

where R is the rotation operator which transforms S_2 to S_1. From (38) we have

$$F(R\Omega, R\Omega') = \sum_{lm}\sum_{l'm'} F_{ll';mm'}\, Y_{lm}(R\Omega)\, Y^*_{l'm'}(R\Omega')$$

$$= \sum_{lm}\sum_{l'm'} F'_{ll';mm'}(R)\, Y_{lm}(\Omega)\, Y^*_{l'm'}(\Omega'). \tag{40}$$

In order to compare with the isotropic case, we define an operator $\overline{F}$ as follows:

$$\overline{F}\psi(\Omega) = \int d\Omega'\, \overline{F}(\Omega,\Omega')\psi(\Omega'),$$

$$\overline{F}(\Omega,\Omega') = \sum_{lm} \overline{F}_{lm}\, Y_{lm}(\Omega)\, Y^*_{lm}(\Omega'),$$

and

$$\overline{F}_{lm} = \int dR\,\langle lm|F_R|lm\rangle = \int dR\, F'_{ll;mm}(R), \tag{41}$$

where $\int dR$ means integration over all rotations. Note that $\overline{F}$ are the diagonal elements of the average of F over all orientations of the crystal. Moreover, we can show that $\overline{F}_{lm}$ is independent of m, and therefore $\overline{F}$ represents the isotropic part of F.

In general, we can always expand $F(\Omega, \Omega')$ in terms of the zero-order eigenfunction $\psi_{lj}^{(\alpha,p)}$ (belonging to the l subspace) of the symmetry group of the crystal,

$$F(\Omega, \Omega') = \sum_{\alpha, l} \sum_{p,p'} F_{lpp'}^{(\alpha)} \sum_{j=1}^{d_\alpha} \psi_{lj}^{(\alpha,p)}(\Omega)\, \psi_{lj}^{(\alpha,p')*}(\Omega')$$
$$+ \text{terms coupled with different } l'\text{s}, \quad (42)$$

where d_α is the dimension of the irreducible representation α, and $\psi_{lj}^{(\alpha,p)}$ is the jth eigenfunction of the pth appearance of the α representation in the subspace l. Note that when α appears more than once, the eigenfunctions with the same j but different p will couple to each other. We expand $\psi_{lj}^{(\alpha,p)}(\Omega)$ in terms of $Y_{lm}(\Omega)$,

$$\psi_{lj}^{(\alpha,p)} = \sum_{m=-l}^{l} a_{ljm}^{(\alpha,p)} Y_{lm}(\Omega) . \quad (43)$$

We have

$$F(\Omega, \Omega') = \sum_{\alpha, l} \sum_{p,p'} F_{lpp'}^{(\alpha)} \times \sum_{j=1}^{d_\alpha} \sum_{\substack{m=-l \\ m'=-l}}^{l} a_{ljm}^{(\alpha,p)} a_{ljm'}^{(\alpha,p')*} Y_{lm}(\Omega) Y_{lm'}^*(\Omega')$$
$$+ \text{other terms.} \quad (44)$$

Making use of the relation

$$Y_{lm}(R\Omega) = \sum_{m'=-l}^{l} Y_{lm'}(\Omega) D_{m'm}^{(l)}(R) , \quad (45)$$

where $D^{(l)}$ is the representation of the rotation group in the subspace l, we have from (40) and (44),

$$F(R\Omega, R\Omega') = \sum_{\alpha, l} \sum_{p,p'} F_{lpp'}^{(\alpha)} \sum_{j=1}^{d_\alpha} \sum_{\substack{m=-l \\ m'=-l}}^{l} a_{ljm}^{(\alpha,p)}\, a_{ljm'}^{(\alpha,p')*} \times \sum_{\substack{m_1=-l \\ m_2=-l}}^{l} Y_{lm_1}(\Omega)\, Y_{lm_2}^*(\Omega')\, D_{m_1 m}^{(l)}(R)\, D_{m_2 m'}^{(l)*}(R)$$
$$+ \text{other terms.} \quad (46)$$

Therefore, from (41) we have

$$\overline{F}_{lm} = \int dR\, \langle lm | F_R | lm \rangle$$
$$= \sum_\alpha \sum_{pp'} F_{lpp'}^{(\alpha)} \sum_{j=1}^{d_\alpha} \sum_{\substack{m_1=-l \\ m_2=-l}}^{l} a_{ljm_1}^{(\alpha,p)} a_{ljm_2}^{(\alpha,p')*} \times \int dR\, D_{mm_1}^{(l)}(R)\, D_{mm_2}^{(l)*}(R)$$

$$= \sum_\alpha \sum_p F_{lpp}^{(\alpha)} d_\alpha/(2l+1) = \overline{F}_l . \quad (47)$$

In obtaining the last form of (47), we have assumed that $\psi_{lj}^{(\alpha,p)}$ has been orthonormalized, i.e.,

$$\sum_{m=-l}^{l} a_{ljm}^{(\alpha,p)} a_{ljm}^{(\alpha,p')*} = \delta_{pp'} , \quad (48)$$

and have used the relation of the theory of group representation[16]

$$\int dR\, D_{mm_1}^{(l)*}(R) D_{mm_2}^{(l)}(R) = \delta_{m_1 m_2}/(2l+1). \quad (49)$$

Equation (47) tells us that $\overline{F}_{lm}$ is independent of m and therefore represents the isotropic part of F_R. In a weakly anisotropic system we expect $(F_R - \overline{F})$ to be very small and this anisotropy can be determined by (5).

If we choose the direction of $\vec{H}$ to be the z axis, the kinetic equation (1) has the form

$$\frac{1}{i} \frac{\partial}{\partial \phi} \overline{\nu} + \frac{k_0^2}{\omega \omega_c} \vec{v} \cdot \langle \vec{v} \overline{\nu} \rangle = \left(\frac{\omega}{\omega_c}\right) \nu, \quad (50)$$

where

$$\overline{\nu} = (1 + F_R)\nu. \quad (51)$$

In a weakly anisotropic system, we expand ν to the linear order of $(F_R - \overline{F})$

$$\nu = \frac{1}{1 + F_R} \overline{\nu} = \frac{1}{1 + \overline{F} + (F_R - \overline{F})} \overline{\nu}$$
$$= \frac{1}{1+\overline{F}} \overline{\nu} - \frac{1}{1+\overline{F}} (F_R - \overline{F}) \frac{1}{1+\overline{F}} \overline{\nu}. \quad (52)$$

Substituting (52) into (50), we have

$$(1+\overline{F}) \frac{1}{i} \frac{\partial}{\partial \phi} \overline{\nu} + \left(\frac{\omega}{\omega_c}\right) (F_R - \overline{F}) \frac{1}{1+\overline{F}} \overline{\nu}$$
$$= \left(\frac{\omega}{\omega_c}\right) \overline{\nu} - \frac{k_0^2}{\omega \omega_c} (1+\overline{F}) \vec{v} \cdot \langle \vec{v} \overline{\nu} \rangle . \quad (53)$$

We consider $(F_R - \overline{F})$ as a small perturbation. Since the components of $\vec{v}$ span the $l=1$ subspace, we find that $Y_{lm}(l \neq 1)$ is an eigenfunction of (53) (when $F_R - \overline{F} = 0$) with eigenfrequency $\omega_{lm}^0 = m(1 + \overline{F}_l)\omega_c$. To see how the anisotropy affects the resonant frequencies, we calculate the first-order perturbation,

$$\Delta\omega_{lm} = m(\langle lm | F_R | lm \rangle - \overline{F}_l)\omega_c , \quad (54)$$

where we have replaced ω by ω_{lm}^0 in the perturbation term. Therefore the measure of the anisotropy (5) has the value

$$\frac{\omega_{lm_1}/m_1 - \omega_{lm_2}/m_2}{\omega_c} = \langle lm_1 | F_R | lm_1 \rangle$$

$$-\langle lm_2 | F_R | lm_2 \rangle . \quad (55)$$

The relations (54) and (55) are derived for fixed direction of the crystal axes with respect to $\vec{H}$. If, however, in an actual experiment a polycrystal is used, the results of (54) and (55) are no longer true. We have to average over all crystal directions with respect to $\vec{H}$. Both (54) and (55) have zero average as can be seen from (47). Therefore, we have to go to the second-order perturbation to obtain the effects of the anisotropy.

We, therefore, conclude that single crystal favors the observation of the anisotropy of the F function.

III. STRONG ANISOTROPY

In the case without a magnetic field,[11] we pointed out that a system having multiple maxima of $\vec{q}\cdot\vec{v}$ on the Fermi surface favors the existence of zero sound in the weak coupling limit. In this section we would like to study the propagation properties of the magnetoplasma modes in such a system with the magnetic field $\vec{H}$ parallel to $\vec{q}$. Due to the presence of the magnetic field and the anisotropy of the Fermi surface, the mathematics involved becomes very difficult. It is therefore desirable to use a simplified model Fermi surface which satisfies the Gor'kov-Dzyaloshinskii condition and yet is free from mathematical complexity. In this suggested model it is also useful to study zero sound modes ($\vec{H}=0$) and their relations to the magnetoplasma modes ($\vec{H}\neq 0$).

We consider a Fermi surface which is essentially spherical (with radius k_F) except that there are two (circular) holes around the z axis (Fig. 1), i.e., there is no surface in the direction of z (or $-z$, the surface has an inversion symmetry). Therefore when $\vec{q}$ is parallel to the z axis, maximum of $\vec{q}\cdot\vec{v}$ will not occur on the z axis itself and hence the Gor'kov-Dzyaloshinskii condition is satisfied. (This simplified model can be considered as the Fermi surface of a free-electron system which is confined in two parallel zone planes with the Fermi surface touching the zone planes and neglecting all effects of the crystal potential.)

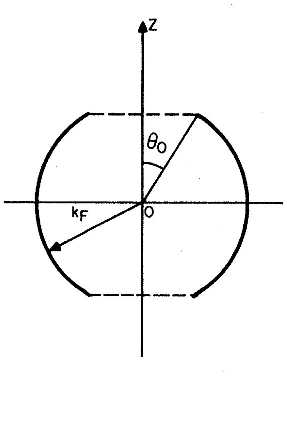

FIG. 1. The side view of the Fermi surface of the hole model, where $k_F \sin\theta_0$ is the radius of the hole.

We describe the Fermi surface by the usual spherical angles (θ, ϕ) with range $0 \leq \phi \leq 2\pi$ and $\theta_0 \leq \theta \leq \pi - \theta_0$, where $k_F \sin\theta_0$ is the radius of the hole. Therefore, when $\vec{q}$ is parallel to the z axis, $(\vec{q}\cdot\vec{v})_{\max}$ will have the value $qv_F\cos\theta_0$ and occur at those $\vec{k}$ on the Fermi surface with $\theta=\theta_0$. On this surface the spherical harmonics $Y_{lm}(\theta,\phi)$ are no longer orthonormal but are still linearly independent. Using Schmidt orthonormalization process,[17] we can find a new set of orthonormal functions $X_{nm}(\theta,\phi)$, such that

$$\int_0^{2\pi} d\phi \int_{\theta_0}^{\pi-\theta_0} \sin\theta\, d\theta\, X_{nm}(\theta,\phi) X^*_{n'm'}(\theta,\phi) = \int' d\Omega\, X_{nm}(\Omega) X^*_{n'm'}(\Omega) = \delta_{nn'}\delta_{mm'} , \quad (56)$$

where m is the index of the azimuthal dependence $e^{im\phi}$ and $\int'$ indicates the range of the integration of θ being from θ_0 to $\pi-\theta_0$. The first few X_{nm} are $(a=\cos\theta_0)$

$$X_{11} = -\left(\frac{3}{4\pi a(3-a^2)}\right)^{1/2} \sin\theta\, e^{i\phi} ,$$

$$X_{21} = -\left(\frac{15}{4\pi a^3(5-3a^2)}\right)^{1/2} \sin\theta\cos\theta\, e^{i\phi} , \quad (57)$$

$$X_{22} = -\left(\frac{15}{4\pi a(15-10a^2+3a^4)}\right)^{1/2} \sin\theta\, e^{2i\phi} , \quad \text{etc.}$$

All functions are defined only in the range $0 \leq \phi \leq 2\pi$ and $\theta_0 \leq \theta \leq \pi - \theta_0$.

When both $\vec{H}$ and $\vec{q}$ are parallel to the z axis, the cyclotron frequency ω_c is independent of k_z, and therefore the kinetic equation (1) can be written as

$$\omega\nu - qv_z\bar{\nu} - \frac{1}{i}\omega_c \frac{\partial}{\partial\phi}\bar{\nu} = \frac{k_0^2}{\omega} v_z \langle v_z \bar{\nu}\rangle + \frac{\omega k_0^2}{\omega^2 - q^2c^2}\left(v_x\langle v_x\bar{\nu}\rangle + v_y\langle v_y\bar{\nu}\rangle\right) . \quad (58)$$

Equation (58) is to be solved in a space spanned by all X_{nm}. If we assume that the F function can be expanded as[18]

$$F(\Omega,\Omega') = \sum_n 4\pi a A_n \sum_{m=-n}^{n} X_{nm}(\Omega) X^*_{nm}(\Omega') , \quad (59)$$

we see that Eq. (58) separates into independent equations for modes with azimuthal dependence $e^{im\phi}$.

Since we have discussed the $m=1$ modes for an isotropic system in Ref. 4, we would like to see the effects of anisotropy on these modes first. We are interested only in modes with frequency near ω_c, provided $k_0^2 v_F^2 \gg \omega_c^2 - q^2c^2$, we can solve the problem the same way as we did in Ref. 4. Since the components of the velocity $\vec{v}$ is in the $n=1$ subspace, Eq. (58) is equivalent to[19]

$$P(\omega P\nu - qv_z P\bar{\nu} - \omega_c P\bar{\nu}) = 0 \ , \tag{60}$$

where P projects on the subspace $m = 1$, $n \geq 2$.

If we keep only A_2 in (59), we find the solution of (60) as follows:

$$qav_F = A_2\omega_c/z[1 - (1 + A_2)h(z)], \quad z = (\omega - \omega_c)/qav_F \ , \tag{61}$$

where

$$h(z) = \frac{az[zK(z) - 1] - dK(z)}{a[zK(z) - 1]} \ , \quad d = \frac{(5 - 3a^2)a}{5(3 - a^2)} \tag{62}$$

and

$$K(z) = \int' \frac{X_{11}^* X_{11}}{z - \cos\theta/a} \, d\Omega = \frac{3z}{a(3 - a^2)} \int_0^a \frac{(1 - u^2)\,du}{z^2 - u^2/a^2} \ . \tag{63}$$

The function $h(z)$ decreases monotonically from $2(5 - a^2)/5(3 - a^2)$ at $z = 1$, to 0 at $z = \infty$. Therefore, we have a cutoff wave vector at [by putting $z = 1$ in (61)]

$$q_{co} = \omega_c \left| \frac{5(3 - a^2)A_2}{(5 - 3a^2) - 2(5 - a^2)A_2} \right| \Big/ av_F \ , \tag{64}$$

provided

$$A_2 < \frac{(5 - 3a^2)}{2(5 - a^2)} \ . \tag{65}$$

Note that the value of $K(z)$ at $z = 1$ has a discontinuity at $a = 1$, and therefore the cutoff wave vector does not reduce to the isotropic value obtained in Ref. 4 when we let $a = 1$ in (64). Therefore the anisotropy ($a < 1$) has changed the cutoff wave vector discontinuously; however, the over-all effect is not significant. It is easy to see from (64) and (65) that even though the cutoff wave vector is larger in comparison with the isotropic case for the same size of the interaction, the cutoff wave vector approaches zero when A_2 goes to zero, as in the isotropic case.

Therefore the anisotropy does not change the structure of the $m = 1$ mode too much. The reason for this is that, as we pointed out in Ref. 11, for a zero-sound mode to exist with an arbitrarily small F function we require the mode to belong to different irreducible representation from the density and current oscillations (in the frequency range $\omega^2 - q^2c^2 \ll k_0^2 v_F^2$, we require both the longitudinal and transverse currents to be negligible). Since the transverse currents belong to the $m = 1$ representation, zero sound cannot exist with an arbitrarily small F function for the $m = 1$ modes. Therefore, the effects of strong anisotropy cannot be fully understood from the $m = 1$ modes.

We, therefore, look for the $m = 2$ modes. Since these modes carry no density and current oscillations, the kinetic equation (58) becomes

$$\omega\nu - qa\,v_F \cos\theta\,\bar{\nu} - 2\omega_c \bar{\nu} = 0 \ . \tag{66}$$

If we expand

$$\nu = \textstyle\sum_n \nu_n X_{n2} \ , \tag{67}$$

and make use of the expansion (59), we have from (66),

$$\bar{\nu}_n = \left(z + \frac{2\omega_c}{qav_F}\right) \sum_{n'=2}^{\infty} K_{nn'}(z)\, B_{n'}\, \bar{\nu}_{n'}, \quad n \geq 2. \tag{68}$$

Here

$$z = (\omega - 2\omega_c)/qav_F \ , \quad B_n = A_n/(1 + A_n), \tag{69}$$

and

$$K_{nn'}(z) = \int' d\Omega \, \frac{X_{n2}^* X_{n'2}}{z - \cos\theta/a} \ . \tag{70}$$

It is easy to see that Eq. (66), and therefore (68), reduces to the equation for zero sound (where we define $z = \omega/qav_F$) if we let $\omega_c = 0$. Therefore, the solutions of zero sound ($\vec{H} = 0$) can be obtained from those of magnetoplasma modes ($\vec{H} \neq 0$) simply by letting $\omega_c = 0$.

If we keep only a finite number, say p, of B_n, then (68) will become a $p \times p$ homogeneous system. It is easy to see from the form of (68) that we can always solve $(2\omega_c/qav_F)$ as a function of z (with the B_n as fixed parameters). Therefore in general we have p solutions of the form

$$2\omega_c/qav_F = G(z) \ , \tag{71}$$

which determines z as a function of q and therefore the dispersion relation.

We discuss the case of zero sound first. By letting $\omega = 0$ we have from (71)

$$G(z) = 0 \ , \tag{72}$$

from which we solve z as a function of B_n. For zero-sound mode to exist, (72) must have a real solution z_0 such that

$$G(z_0) \equiv 0, \quad |z_0| > 1. \tag{73}$$

Condition (73) requires B_n to satisfy certain conditions; if they are not met, zero sound will not exist. If (73) is satisfied, zero sound will exist with velocity $z_0 a^0{}_F$.

It is easy to see from (71) that if (73) is satisfied, the corresponding magnetoplasma mode will have no cutoff wave vector, since as z approaches z_0, the wave vector q approaches infinity. Thus the mode exists for any value of q. Therefore we have established a one-to-one correspondence between zero-sound modes and magnetoplasma modes without cutoffs: If a zero-sound mode exists then there corresponds a magnetoplasma mode with no cutoff wave vector.

For instance, if we keep B_2 only we have from (68),

$$2\omega_c/qav_F = G(z) = z\,[(1+1/A_2)W(z)^{-1} - 1]\,, \qquad (74)$$

where

$$W(z) = zK_{22}(z) = z\int' d\Omega\,\frac{X_{22}^{*}X_{22}}{z-\cos\theta/a}$$

$$= 1 + \frac{15}{a^3(15-10a^2+3a^4)}\int_0^a \frac{u^2(1-u^2)}{z^2-u^2/a^2}\,du. \qquad (75)$$

The corresponding equation for zero sound is

$$1 + 1/A_2 = W(z). \qquad (76)$$

Note that the function $W(z)$ is even in z and decreases monotonically from ∞ at $z=1$ to 1 at $z=\infty$. The function approaches infinity logarithmically as z approaches 1 from above, due to the presence of the holes $0<a<1$. The function $W(z)$ is plotted against z in Fig. 2 for $0<a<1$. We distinguish two cases:

(i) $A_2 > 0$: It is easy to see from Fig. 2 that for any value of $A_2>0$, Eq. (76) will always have a real solution $z=z_0$, $z_0>1$. We therefore have an undamped zero sound with frequency qz_0av_F. Due to the symmetry of $W(z)$ there will also exist a mode with frequency $-qz_0av_F$. Note that the condition for the existence of zero sound depends only on the sign but not the strength of A_2. This is the condition we found in Ref. 11, and therefore we refer to the above two zero-sound modes as Gor'kov-Dzyaloshinskii modes (Fig. 3). (The mode with negative frequency can be interpreted as propagation along the negative z axis.)

When we turn on a magnetic field $\vec{H}$, we have from (74),

$$\frac{qav_F}{2\omega_c} = \frac{z^{-1}}{[1+(A_2)^{-1}]/W(z)-1} = \frac{1}{G(z)}\,. \qquad (77)$$

The function $1/G(z)$ is plotted against z in Fig. 4 for $A_2>0$. Note that $1/G$ approaches infinity as z approaches z_0, where $z_0>1$ is a solution of (76). It can be seen from Fig. 4 that for any given value of q, Eq. (77) will have at least one real solution $z=z_1(q)$, such that $z_1>1$. Therefore an undamped magnetoplasma mode exists with frequency $\omega = 2\omega_c + qz_1(q)av_F$. Since there is no restriction on the magnitude of q, there is no cutoff wave vector as we expected (since the corresponding zero-sound mode exists). Furthermore, two modes will propagate in the region $q > 2\omega_c/av_F$ (Fig. 5). We refer to these two modes as Gor'kov-Dzyaloshinskii zero-sound modes affected by the magnetic field $\vec{H}$.

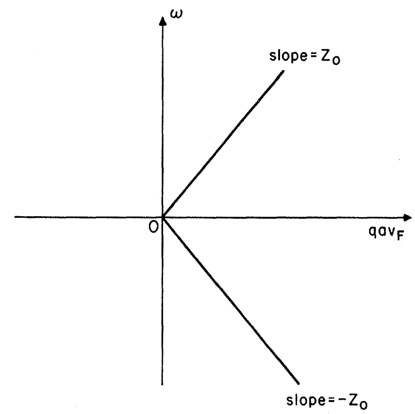

FIG. 3. The dispersion relation ω vs qav_F for zero sound ($\vec{H}=0$) in the hole model.

By comparing Figs. 3 and 5, we see that the magnetic field $\vec{H}$ destroys the equivalence of the propagation along the positive and negative directions of the z axis. In the presence of $\vec{H}$, the mode propagating along the field ($\omega>0$) exists for any value of $q>0$, while the mode propagating opposite to the field ($\omega<0$) exists only for $q>2\omega_c/av_F$.

We have seen that because of the presence of the holes ($a\neq 1$) the function $W(z)$ defined in (75) approaches infinity as z approaches 1 from above.

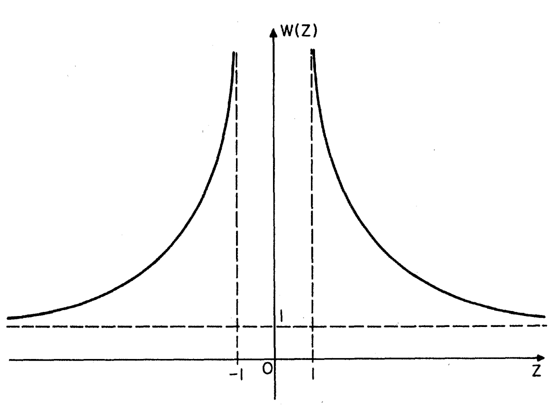

FIG. 2. $W(z)$ vs $z\,(0<a<1)$.

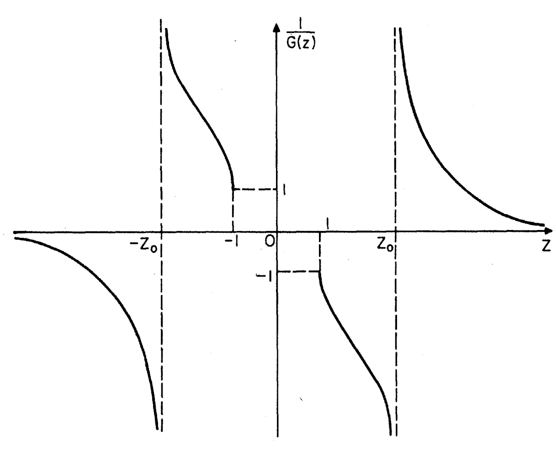

FIG. 4. $1/G(z)$ vs z, $A_2>0$.

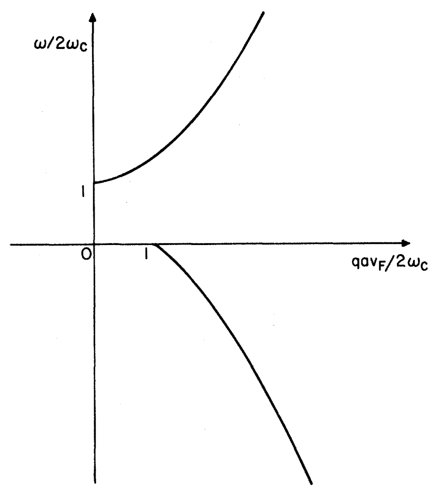

FIG. 5. The dispersion relation $\omega/2\omega_c$ vs $qav_F/2\omega_c$ in the hole model ($\vec{H} \neq 0$), $A_2 > 0$.

In the isotropic case ($a = 1$), $W(1) = \frac{5}{4}$, and therefore we require $A_2 > 4$ for the existence of zero sound and for the magnetoplasma model to have no cutoff wave vector. The effect of anisotropy is therefore very clear in this case.

(ii) $A_2 < 0$: In this case we can see from Fig. 2 that zero-sound modes cannot exist, and therfore the magnetoplasma mode will have a cutoff wave vector. However, the effect of the anisotropy plays an important role in the determination of the cutoff wave vector in this case.

The cutoff wave vector is obtained from (77) by putting $z = -1$, we have

$$q_{co} = 2\omega_c / av_F . \tag{78}$$

Note that due to the anisotropy, $W(1) = \infty$, q_{co} is independent of A_2. This means that the cutoff wave vector remains at a large constant value for an arbitrarily small negative A_2. In the isotropic case, however, $W(1) = \frac{5}{4}$ and q_{co} will be proportional to A_2.

There is no point to go to any higher-m modes. The general conclusions would be the same as in the $m = 2$ modes. We emphasize here that these higher-m modes carry no current and density oscillations. Therefore, as we discussed in Ref. 11, due to the fact that $(\vec{q} \cdot \vec{v})_{max}$ occurs at more than one point on the Fermi surface, these modes would exist in the absence of a magnetic field for an arbitrarily small positive F function. The application of a magnetic field to the system affects only the propagation properties of the modes but not their existence. In the case where zero-sound mode ($\vec{H} = 0$) does not exist, the anisotropy of the Fermi surface still plays an important role in the propagation of the modes. The reason is that the multiplicity of $(\vec{q} \cdot \vec{v})_{max}$ makes it possible for the modes to have nonzero amplitude around the regions where $(\vec{q} \cdot \vec{v})$ have maximum and yet to carry no current and density oscillations for a system with very small F function.[20] This makes the function $K_{nn'}(z)$ defined in Eq. (70) diverge at the edge of the continuum ($z = 1$), and therefore enhances the coupling and the existence of the modes [see Eq. (68)].

Therefore, in the case of weak Fermi-liquid interactions, a system with this simplified Fermi surface favors the existence of the $m \geq 2$ plasma modes. However, there arises a difficulty. Since all these modes do not couple to the current oscillations, it seems that they would be very difficult to detect experimentally. Fortunately, due to the anisotropy of the Fermi surface, all the $m \geq 2$ modes will be slightly coupled to the currents if the applied magnetic field $\vec{H}$ lies along a direction slightly different from the symmetry axis of the Fermi surface. The deviation of the direction of $\vec{H}$ should be very small so that all the modes described above are still well defined.

We conclude this section by suggesting that the Fermi surface of the noble metals has several properties similar to those of the "hole model" described above, and they may be considered as the best candidates for the observation of magnetoplasma modes in anisotropic metals. The Fermi surface of the noble metals are essentially spherical, except there are four pairs of necks in the [111] directions. When the magnetic field $\vec{H}$ and the wave vector $\vec{q}$ of a propagating mode are both parallel to the axis of a particular pair of necks, we may assume that all the other three pairs of necks will have negligible effects on the mode. Therefore we may consider the Fermi surface as a sphere with a pair of necks (having cylindrical symmetry) in the direction of $\vec{H}$ (and $\vec{q}$).

We take the symmetry axis of the necks as the z axis and calculate the simplified Fermi surface by using nearly free-electron theory. The system can

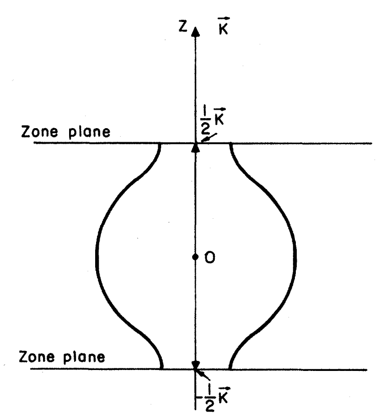

FIG. 6. The cross section for a simplified model of the Fermi surface of the noble metals containing the axis of a pair of necks.

be considered as a free-electron system bounded by two parallel zone planes perpendicular to the z axis. We assume that the upper half of the Fermi surface ($k_z > 0$) is affected only by the zone plane at $\frac{1}{2}\vec{K}$ and the lower half ($k_z < 0$) only by the zone plane at $-\frac{1}{2}\vec{K}$, where $\vec{K}$ is a reciprocal-lattice vector in the [111] direction. We find

$$\epsilon(\vec{k}) = \frac{k^2}{2m^*} + \left(\frac{K^2}{4m^*} - \frac{|k_z|K}{2m^*}\right) - \left[\left(\frac{K^2}{4m^*} - \frac{|k_z|K}{2m^*}\right)^2 + |U_{\vec{K}}|^2\right]^{1/2}, \quad |k_z| \le \tfrac{1}{2}K, \tag{79}$$

where $U_{\vec{K}}$ is the Fourier component of the crystal potential. The cross section of the Fermi surface containing the axis of the necks is shown in Fig. 6.

The Fermi surface, $\epsilon(\vec{k}) = \epsilon_F$, described by (79) has several properties which are similar to those of the hole model:

(a) The Fermi surface has cylindrical symmetry around the z axis. (b) The cyclotron frequency ω_c is independent of k_z when $\vec{H}$ is parallel to the z axis. (c) There is no surface in the direction of z and therefore $\vec{q}\cdot\vec{v}(\vec{k})$ will have multiple maxima when $\vec{q}$ is parallel to the z axis. Because of the above properties, the general conclusions we reached for the hole model will remain true for a system described by (79) and might be relevant to the noble metals.

There is an advantage of a system described in (79) over the hole model. In such a system, $(\vec{q}\cdot\vec{v})_{max}$ will occur on the Fermi surface where $(\partial v_z/\partial k_z) = 0$, $\vec{q} \parallel \hat{z}$ (while in the hole model it does not). Therefore the residue at the poles of the response function of such a system will be proportional to A_2^2 rather than $e^{-D/|A_2|}$ (D is some positive number) as we have in the hole model. Therefore in the weak coupling limit, system (79) favors the excitation of the mode.

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[1]P. M. Platzman and W. M. Walsh, Jr., Phys. Rev. Letters 19, 514 (1967); 20, 89 (E) (1968).

[2]N. D. Mermin and Y. C. Cheng, Phys. Rev. Letters 20, 839 (1968).

[3]S. C. Ying and J. J. Quinn, Phys. Rev. Letters 20, 1007 (1968).

[4]Y. C. Cheng, J. S. Clarke, and N. D. Mermin, Phys. Rev. Letters 20, 1486 (1968).

[5]P. M. Platzman, W. M. Walsh, Jr., and E-Ni Foo, Phys. Rev. 172, 689 (1968).

[6]S. C. Ying and J. J. Quinn, Phys. Rev. 173, 473 (1968).

[7]G. A. Baraff, C. C. Grimes, and P. M. Platzman, Phys. Rev. Letters 22, 590 (1969).

[8]S. C. Ying and J. J. Quinn, Phys. Rev. 180, 193 (1969).

[9]D. Shoenberg and P. J. Stiles, Proc. Roy. Soc. (London) A281, 62 (1964).

[10]M. J. G. Lee, Proc. Roy. Soc. (London) A295, 440 (1966).

[11]Y. C. Cheng and N. D. Mermin, Ann. Phys. (N.Y.) 60, 27 (1970).

[12]See, for example, D. Pines and P. Nozières, *The Theory of Quantum Liquids* (Benjamin, New York, 1966), Vol. I, p. 181; see Ref. 2 for the explanation of the right-hand side of Eq. (1). We have set $\hbar = 1$.

[13]The differential dS is a surface element on the Fermi surface.

[14]In the isotropic case, we expand $F(\vec{k},\vec{k}')$ in terms of Lengendre polynomials $F(\vec{k},\vec{k}') = \sum_l F_l P_l(\cos(\hat{k}\cdot\hat{k}'))$; and $A_l = F_l/(2l+1)$.

[15]For a system with an ellipsoidal Fermi surface, ω_c is still a constant for fixed direction of the magnetic field.

[16]M. Hamermesh, *Group Theory* (Addison-Wesley, Reading, Mass., 1964), p. 103.

[17]See, for example, L. Collatz, *Functional Analysis and Numerical Mathematics* (Academic, New York, 1966), p. 42.

[18]Here A_n corresponds to A_l, defined in Ref. 14, in the isotropic case.

[19]See Y. C. Cheng, J. S. Clarke, and N. D. Mermin, Ref. 4; Y. C. Cheng, Ph.D. thesis (Cornell University, 1970) (unpublished).

[20]See Ref. 11 for the reasoning. Since we are interested only in modes with $\omega \ll k_0 v_F$ and $\omega^2 - q^2c^2 \ll k_0^2 v_F^2$, we require the modes to have negligible current oscillations. Because zero-sound mode does not exist in this case, this condition can be satisfied only when $\vec{H} \neq 0$. This is another reason why we prefer to observe the field-dependent modes.