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Current-Induced Flow of Domains in the Intermediate State of a Superconductor

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Current-induced flow of superconducting domains similar to the flow of normal domains (flux flow) has been observed in the intermediate state. A theory (neglecting the Hall effect) is presented for the motion of domains in an intermediate state of arbitrary topology. Without pinning, the current in the normal regions is uniform and equal to J_0, the average current density. Domains move with velocity $v_D = cJ_0/\sigma H_c$. Both results agree with those previously derived for flux flow. Introducing pinning gives agreement between the predicted and measured velocities.

A dynamic current-carrying intermediate state was first discussed by Gorter,[1] who suggested that a transport current could induce motion of laminae perpendicular to the current. Although attempts to observe this particular phenomenon have produced conflicting results,[2-5] current-induced motion of simply connected normal domains (flux flow) has been demonstrated by several experiments.[5,6] Recently, we have observed that a transport current can also induce a flow of simply connected superconducting domains,[7] and it appears from our observations that any intermediate-state topology is unstable with respect to transverse motion in the presence of a transport current. In this paper we describe the characteristics of superconducting domain flow and present a theory of the current-induced motion of domains of arbitrary topology. In the theory, the magnetic field is assumed to be perpendicular to the transport current and the Hall effect has been neglected. The electric field within the normal regions is calculated using Maxwell's equations and the condition that the electric and magnetic fields vanish within the superconducting regions. Using this electric field, the domain velocity $\vec{v}_D$ is obtained by solving a power-balance equation which equates the power supplied by a battery with the power dissipated by Joule heating and by pinning or motion-induced thermal gradients.[8] This formulation avoids the difficult problem of defining the force on a domain.[9] For normal domains, where there is no dissipation in the absence of motion, the force has been obtained by thermodynamic arguments,[10] but this approach is not directly applicable to superconducting domains where there is dissipation in the absence of motion.

The intermediate state was observed using the magneto-optic rotation in a thin film of $EuSe_{(0.9)}EuF_{2(0.1)}$ evaporated onto the sample surface.[11]

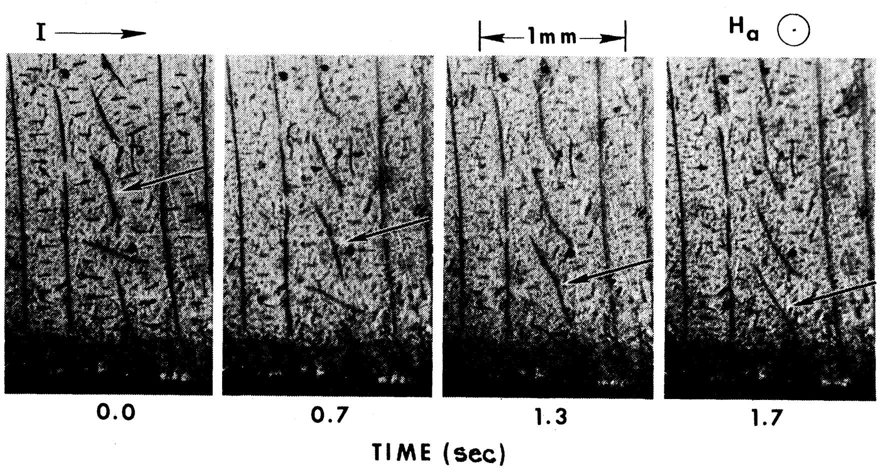

FIG. 1. Four sequential photographs illustrating the motion of superconducting domains (dark regions). The arrows point to the same domain in each picture. The sample is a rectangular Pb slab 4×12×40 mm. The small superconducting inclusions in the left-hand picture have been retouched for illustrative purposes.

When more than about half of the sample is normal, current-induced flow of simply connected superconducting domains occurs.[7] An example is shown in Fig. 1, which consists of four sequential frames of a motion picture film. Several superconducting laminae are perpendicular to the current[12] and several shorter superconducting laminae are at other angles to the current. The entire pattern, including the gaps in the vertical laminae, moves with only slight deformation from top to bottom. In addition, there is motion of small superconducting domains, aligned roughly with the direction of the current, which appear in the regions between the large laminae. These small inclusions are thought to be part of a Landau branching structure.

The pattern can be made to evolve continuously from normal domain flow to superconducting domain flow by slowly increasing the applied field at constant current. The direction of motion of superconducting domains is the same as that of normal domains. Quantitative measurements of the dependence of the superconducting domain velocity on current and applied field show a behavior similar to that for normal domain motion.

To calculate the domain velocity $\vec{v}_D$ induced by a steady transport current I, we construct a power-balance equation

$$P_B(I, \vec{v}_D) = P_R(I, \vec{v}_D) + P_L , \quad (1)$$

where $P_B(I, \vec{v}_D)$ is the power supplied to the sample by a battery, $P_R(I, \vec{v}_D)$ is the power dissipated through resistive losses in the sample, and P_L is the power dissipated through other losses (e.g., pinning or motion-induced thermal losses[8]). The terms $P_B(I, \vec{v}_D)$ and $P_R(I, \vec{v}_D)$ may be calculated by determining the electric fields in the presence of moving domains. Then, $\vec{v}_D$ is determined from Eq. (1) when P_L can be defined.

For calculating $P_B(I, \vec{v}_D)$ and $P_R(I, \vec{v}_D)$ the hypothetical geometry shown in Fig. 2(a) is useful because its cylindrical symmetry ensures that domains move continuously in circles without encountering a sample edge. The sample in the intermediate state (hatch-marked) is a tube with cross-sectional area $\mathcal{A}$, length l, normal-state conductivity σ, and critical field H_c. It is connected at top and bottom (d and c) by tubular superconducting leads (solid) to a variable voltage source at points a and b. The source maintains a constant current I. The average current density is $J_0 = I/\mathcal{A}$. The applied field $\vec{H}_a$ is directed radially through holes in the inner superconducting lead.

The electric fields $\vec{E}_N(\vec{r})$ and $\vec{E}_S(\vec{r})$ in the normal and superconducting regions, respectively, must satisfy the following conditions: (a) $\vec{E}_S(\vec{r}) = 0$ away from a domain boundary; (b) $\hat{n}\times\vec{E}_N(\vec{r}) = -\hat{n}\times(\vec{v}_D\times\vec{H}_c)/c$ near a moving domain boundary whose normal is $\hat{n}$,[13] where $\vec{H}_c$ is the magnetic field in

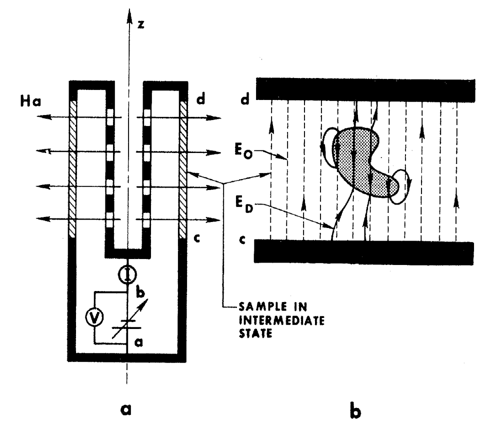

FIG. 2. (a) Hypothetical geometry used in the analysis. (b) Side view of the cylindrical sample in the intermediate state showing schematically the electric fields for an irregular superconducting domain (shaded).

the normal region; (c) $I = \int_{\mathcal{S}} \vec{J}(\vec{r}) \cdot d\vec{a}$, where $\mathcal{S}$ is any surface within the sample which breaks all current lines. If some $\mathcal{S}$ (called $\mathcal{S}_N$) is entirely normal, then one may write

$$I = \sigma \int_{\mathcal{S}_N} \vec{E}_N(\vec{r}) \cdot d\vec{a} .$$

These conditions appear to be sufficient to ensure uniqueness of the solution for the electric field.

The power-balance equation for moving domains may be written in terms of the electric field and dissipation associated with static domains, so we examine static domains first. Consider the simply connected superconducting domain (shaded) in a normal strip shown in Fig. 2(b). The total field $\vec{E}(\vec{r})$ may be written as $\vec{E}_0 + \vec{E}_D(\vec{r})$, where $\vec{E}_0 = \hat{z} J_0/\sigma$ is the field which would occur in the absence of any superconducting domains. Thus $\vec{E}_D(\vec{r})$ resembles a polarization field. Boundary conditions require $\vec{E}_D(\vec{r})$ to have the following properties: It must be perpendicular to the surfaces where the superconducting leads connect to the sample; within the superconducting regions $\vec{E}_D(\vec{r}) = -\vec{E}_0$; and if there is an entirely normal surface $\mathcal{S}_N$, then

$$\int_{\mathcal{S}_N} \vec{E}_D(\vec{r}) \cdot d\vec{a} = 0$$

[i.e., $\vec{E}_D(\vec{r})$ does not affect the total current]. If there is no $\mathcal{S}_N$, then $\vec{E}_D(\vec{r}) = -\vec{E}_0$ over the entire sample.

Expressing $P_B(I, 0)$ in terms of these field distributions, one obtains

$$P_B(I, 0) = IV = -I\int_a^b \vec{\nabla}\varphi(\vec{r}) \cdot d\vec{l} = I\int_c^d [\vec{E}_0 + \vec{E}_D(\vec{r})] \cdot d\vec{l} ,$$

the last expression being valid since $\vec{E}(\vec{r}) = 0$ in the superconducting leads. Because the line integral is the same for any c and d at the sample ends, it may also be converted to a volume integral:

$$P_B(I,\,0)=\frac{I}{\vec{E}_0\cdot\hat{z}}\int_c^d[\vec{E}_0+\vec{E}_D(\vec{r})]\cdot\vec{E}_0\,dz$$

$$=\sigma\int_{\text{sample}}[\vec{E}_0+\vec{E}_D(\vec{r})]\cdot\vec{E}_0\,d^3r$$

$$=\sigma\int_N[\vec{E}_0+\vec{E}_D(\vec{r})]\cdot\vec{E}_0\,d^3r\,,$$

where $\int_N$ stands for the integral over the normal regions.

The power dissipated resistively is

$$P_R(I,\,0)=\int_N\vec{E}(\vec{r})\cdot\vec{J}(\vec{r})\,d^3r=\sigma\int_N[\vec{E}_0+\vec{E}_D(\vec{r})]^2\,d^3r\,.$$

The power dissipated by the normal currents in the domain boundary region is neglected since this volume is small compared to the domain volume for bulk type-I superconductors. Although the supercurrents in the boundaries undergo accelerations as the domain moves, there is no net energy change in the supercurrents and the process is nondissipative.

For static domains there are no other dissipative processes, so

$$P_B(I,\,0)=P_R(I,\,0)\equiv P(I,\,0)\,. \qquad (2)$$

This equality yields the following result which will be useful later:

$$\int_N[\vec{E}_0+\vec{E}_D(\vec{r})]\cdot\vec{E}_D(\vec{r})\,d^3r=0\,. \qquad (3)$$

Now assume that domains move at the velocity $\vec{v}_D$ in the direction perpendicular to the z axis. The total field for the moving domain pattern at any instant of time is constructed by adding proper multiples of the fields $\vec{E}_0$ and $\vec{E}_D(\vec{r})$, for the static domain pattern which is congruent to the moving pattern, and the electric field contributed by the domain motion. Noting that the Meissner currents produce a magnetic field $-\vec{H}_c$ within the superconducting domains we see that the electric field produced by the motion is the same as that from a moving solenoid with a constant field $-\vec{H}_c$ over the same cross section as the superconducting domains.[14] The contribution from such a solenoid is $\vec{E}_{\text{sol}}=-\vec{v}_D\times(-\vec{H}_c)/c$ within the superconducting domain and $\vec{E}_{\text{sol}}=0$ outside. The total field is the sum of $[\vec{E}_0+\vec{E}_D(\vec{r})]$, $\vec{E}_{\text{sol}}$, and some multiple of the field $\vec{E}_D(\vec{r})$ such that the total field is zero within the superconducting regions. The use of $\vec{E}_D(\vec{r})$ as the additive field ensures that condition (c) is satisfied. Recalling that $\vec{E}_D(\vec{r})=-\vec{E}_0$ within the superconducting regions, it can be seen that the conditions (a)–(c) are satisfied by

$$\vec{E}_S(\vec{r})=[\vec{E}_0+\vec{E}_D(\vec{r})]+\vec{E}_D(\vec{r})\frac{\vec{v}_D\times\vec{H}_c\cdot\hat{z}}{c\vec{E}_0\cdot\hat{z}}+\frac{1}{c}\vec{v}_D\times\vec{H}_c=0\,,$$

$$\vec{E}_N(\vec{r})=[\vec{E}_0+\vec{E}_D(\vec{r})]+\vec{E}_D(\vec{r})\frac{\vec{v}_D\times\vec{H}_c\cdot\hat{z}}{c\vec{E}_0\cdot\hat{z}}\,. \qquad (4)$$

For moving domains, the power delivered by the battery may be written $P_B(I,\vec{v}_D)=I\int_a^b\vec{E}(\vec{r})\cdot d\vec{l}$, where the line integral is taken through the battery and $\vec{E}(\vec{r})=-\vec{\nabla}\varphi(\vec{r})-\dot{\vec{A}}(\vec{r})/c$ is the total electric field. A more useful expression for $P_B(I,\vec{v}_D)$ may be obtained by noting that $\langle\dot{\vec{A}}(\vec{r})\rangle=0$, where angular brackets denote the time average taken over a complete revolution of the domain pattern. Then $\langle\int_a^b\vec{E}(\vec{r})\cdot d\vec{l}\rangle$ must be independent of path since it equals $\langle-\int_a^b\vec{\nabla}\varphi(\vec{r})\cdot d\vec{l}\rangle$ and the latter is independent of path. The axially symmetric placement of the battery and voltmeter ensures that $P_B(I,\vec{v}_D)$ is constant in time for constant $\vec{v}_D$. Therefore, we have

$$P_B(I,\vec{v}_D)=\langle P_B(I,\vec{v}_D)\rangle=I\langle\int_a^b\vec{E}(\vec{r})\cdot d\vec{l}\rangle=I\langle\int_c^d\vec{E}(\vec{r})\cdot d\vec{l}\rangle\,.$$

Since the domain pattern simply rotates as a function of time, it is possible to convert the average of the line integral from c to d into a volume integral:

$$P_B(I,\vec{v}_D)=\sigma\int_{\text{sample}}\vec{E}(\vec{r})\cdot\vec{E}_0\,d^3r. \qquad (5)$$

Substituting Eq. (4) into Eq. (5) and using Eq. (3) yields

$$P_B(I,\vec{v}_D)=P(I,0)-\frac{\vec{v}_D\times\vec{H}_c\cdot\hat{z}}{c\vec{E}_0\cdot\hat{z}}\,\sigma\int_N\vec{E}_D^2(\vec{r})d^3r\,. \qquad (6)$$

Using $P_R(I,\vec{v}_D)=\sigma\int_N\vec{E}^2(\vec{r})\,d^3r$ and Eqs. (3) and (4) yields

$$P_R(I,\vec{v}_D)=P(I,0)+\left(\frac{\vec{v}_D\times\vec{H}_c\cdot\hat{z}}{c\vec{E}_0\cdot\hat{z}}\right)^2\sigma\int_N\vec{E}_D^2(\vec{r})\,d^3r. \qquad (7)$$

In the absence of pinning or other dissipative terms, Eq. (1) reduces to $P_B(I,\vec{v}_D)=P_R(I,\vec{v}_D)$, so the equality of Eqs. (6) and (7) results in a quadratic equation for $\vec{v}_D$, whose solutions are

$$\vec{v}_D=0 \quad\text{and}\quad \vec{v}_D\times\vec{H}_c\cdot\hat{z}/c=-\vec{E}_0\cdot\hat{z}=-J_0/\sigma.$$

The solutions are illustrated in Fig. 3, where $P_B(I,\vec{v}_D)$ and $P_R(I,\vec{v}_D)$ are plotted. For the moving domain solution $v_D=cJ_0/\sigma|\vec{H}_c|$, we see that according to Eq. (4) the field in the normal region is just $\vec{E}_0$, so the current density $\vec{J}=\sigma\vec{E}_0$ is uniform and equal to the average current density J_0.

The above results depend on J_0, H_c, and σ, not on sample geometry. It is reasonable to expect them to hold for any geometry for which $\vec{J}$ is perpendicular to $\vec{H}_a$ as long as the sample is large compared with the domain size.

To determine which solution will prevail we examine the stability at each solution to a fluctuation

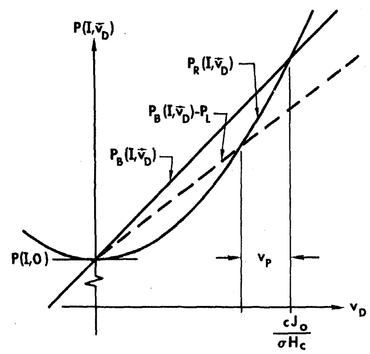

FIG. 3. The power supplied to the sample in the intermediate state by the battery $P_B(I, \vec{v}_D)$ and the power dissipated resistively in the normal regions $P_R(I, \vec{v}_D)$ as functions of the domain velocity $\vec{v}_D$ [Eqs. (6) and (7)]. In the absence of pinning, energy conservation requires that $P_B(I, \vec{v}_D) = P_R(I, \vec{v}_D)$. Thus, solutions for v_D occur at $v_D = 0$ and $v_D = cJ_0/\sigma H_c$. With pinning, additional energy is dissipated at a rate P_L so energy conservation requires that $P_B(I, \vec{v}_D) - P_L = P_R(I, \vec{v}_D)$. The dashed line illustrates the left-hand side of this equation when $P_L \propto \vec{v}_D$. The solutions occur for $v_D = 0$ and $v_D = cJ_0/\sigma H_c - v_P$, where v_P is the decrease in the domain velocity due to pinning [Eq. (8)].

δv_D in velocity. For nonstationary situations, we have

$$P_B(I,\vec{v}_D) - P_R(I,\vec{v}_D) = \frac{\partial \mathcal{E}}{\partial t},$$

where $\mathcal{E}$ is the energy added or subtracted to change the velocity of the domain. Expanding $P_B(I,\vec{v}_D) - P_R(I,\vec{v}_D)$ in a Taylor series and setting $\partial\mathcal{E}/\partial t = (\partial\mathcal{E}/\partial v_D)(\partial v_D/\partial t)$ yields

$$\left(\frac{\partial}{\partial v_D}\left[P_B(I,\vec{v}_D) - P_R(I,\vec{v}_D)\right]\right)\delta v_D = \frac{\partial\mathcal{E}}{\partial v_D}\,\frac{\partial v_D}{\partial t}.$$

The term in large parentheses is positive for positive velocity near $v_D = 0$ and negative near $v_D = cJ_0/\sigma|\vec{H}_c|$. The stability depends on the sign of $\partial\mathcal{E}/\partial v_D$. If $\mathcal{E}$ has the form of a kinetic energy, then $\partial\mathcal{E}/\partial v_D$ is positive for positive v_D and the solution at $v_D = 0$ is unstable to positive fluctuations in v_D (δv_D and $\partial v_D/\partial t$ are of the same sign) but stable to fluctuations about $v_D = cJ_0/\sigma|\vec{H}_c|$ (∂v_D and $\partial v_D/\partial t$ are of the opposite sign). This seems to agree with the experimental observations, but the dependence of $\mathcal{E}$ on v_D should be examined in more detail.

Since the theory is valid for domains of arbitrary topology, the equations for normal domain flow may be obtained as a special case. Here, $P(I,0) = 0$ and $\vec{E}_D(\vec{r}) = -\vec{E}_0$. We can compute the force $\vec{f}_L$ per flux quantum on a unit length of flux line by equating the total energy dissipated $\vec{f}_L\cdot\vec{v}_D(|\vec{B}|/\varphi_0)l\mathcal{A}$ (where $\vec{B}$ is the average magnetic induction which is $\vec{H}_c$ times the fraction of normal material) with the energy supplied by the battery $P_B(I,\vec{v}_D)$. Using Eq. (6) we obtain

$$\vec{f}_L = J_0\hat{z}\times\vec{\varphi}_0/c,$$

the usual result for flux flow.[9,10] We can also compute the average electric field $\langle\vec{E}\rangle$. When normal domains flow, Eq. (4) reduces to

$$\vec{E}_S(\vec{r}) = 0, \quad \vec{E}_N(\vec{r}) = -\vec{v}_D\times\vec{H}_c/c.$$

Since $|\vec{B}|/|\vec{H}_c|$ is the fraction of normal material, we obtain

$$\langle\vec{E}\rangle = -\vec{v}_D\times\vec{B}/c,$$

which is a well-known result.

As an example of nonzero pinning, assume that P_L is dissipation caused by a density p of pinning centers, each of which absorbs an average amount of energy ϵ each time a domain boundary passes it. (It is reasonable to assume that ϵ is independent of applied field and is proportional to the condensation energy $H_c^2/8\pi$ times a factor which involves the penetration depth and pinning-site geometry.) As the domains move there are $pS_2 l\mathcal{A}v_D$ pinning centers passed per unit time, where S_2 is a structure factor defined as the total area of domain boundary parallel to the current (and perpendicular to the domain flow) per unit volume of sample. Thus $P_L = (p\epsilon S_2 l\mathcal{A})v_D$, and inserting Eqs. (6) and (7) into Eq. (1) we obtain

$$v_D = v_0 - v_P,$$

where

$$v_0 = cJ_0/\sigma H_c, \quad v_P = (c^2 p)(1/\sigma)(\epsilon/H_c^2)(S_2/S_1),$$

$$S_1 = (1/l\mathcal{A})\int_N (E_D/E_0)^2\, d^3r.$$

The pinning term v_P is independent of J_0 and always positive. Thus v_0 is an upper bound on the domain velocity. Figure 3 graphically illustrates the solutions in the presence of pinning which occur when $P_B(I,\vec{v}_D) - P_L = P_R(I,\vec{v}_D)$.

To test this relation we have measured superconducting domain velocities in a Pb slab by timing their transit between fiducial marks scribed on the slab. In Fig. 4 we have plotted $\sigma(v_D - v_0)$ determined experimentally[15] as a function of reduced applied field for six values of J_0. In all cases $\sigma(v_D - v_0)$ is negative within the scatter in the data and the theoretical prediction that v_0 is an upper bound on v_D is therefore confirmed. Furthermore, it can be seen that $\sigma(v_D - v_0) = -\sigma v_P$ is nearly the same function of H_a/H_c for the four lowest values of J_0, in agreement with the above expression for v_P, which predicts that σv_P is independent of temperature for T sufficiently below T_c. Nearer T_c the temperature dependence of the penetration depth and coherence length will make ϵ/H_c^2, S_1, and S_2 temperature dependent. This may explain the dif-

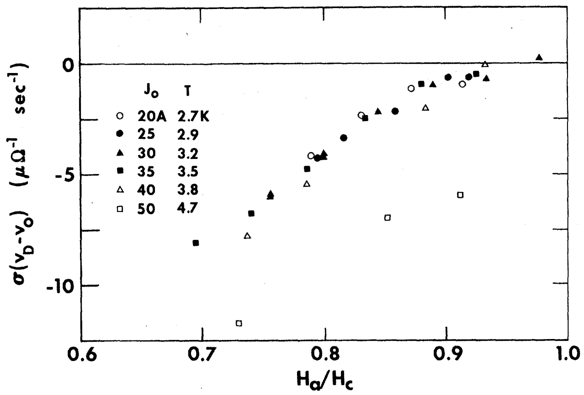

FIG. 4. Experimentally determined values of $\sigma(v_D - v_0)$ as a function of H_a/H_c for a wide range of J_0 and T.

ference in $\sigma(v_D - v_0)$ for the higher values of J_0.

It is probable that the previously unrecognized motion of superconducting domains has affected earlier data. Superconducting domain flow appears to be an explanation for the conflicting results of experiments performed in the geometry first used by Sharvin.[2–5] Following Sharvin we have used a magnetic field close to H_c and nearly parallel to the surface of the sample and have observed current-induced superconducting domain flow between static superconducting lamina. The lamina may be the static features of the intermediate state observed by Brandt and Parks.[4]

Improved agreement between theory and measurements of noise voltages[6] and thermomagnetic effects[5,16] at high fields should also result from inclusion of superconducting domain flow in those theories.

We are grateful for many helpful discussions with G. B. Yntema, D. G. Hamblen, and M. P. Shaw and for experimental assistance from L. F. Conopask.

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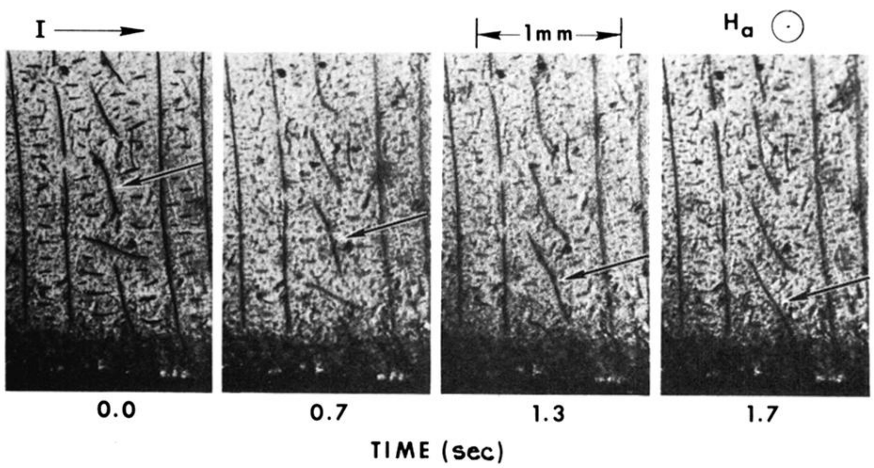

FIG. 1. Four sequential photographs illustrating the motion of superconducting domains (dark regions). The arrows point to the same domain in each picture. The sample is a rectangular Pb slab $4 \times 12 \times 40$ mm. The small superconducting inclusions in the left-hand picture have been retouched for illustrative purposes.