

## Semiclassical Estimation of Neutron Stopping Power

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The semiclassical impulse approximation is used to calculate the effect on the stopping power of the charge and magnetic moment of a heavy particle. A stopping-power formula applicable when  $1 < \gamma \lesssim 10$  is derived, where  $\gamma = (1 - \beta^2)^{-1/2}$ ,  $\beta$  being the particle's speed in terms of the speed of light. Applied to neutrons, for which, apparently, no quantum-mechanical result is available, the semiclassical theory predicts for the stopping power of a medium containing  $NZ$  electrons per unit volume  $-dE/ds = 1.54 \times 10^{-32} NZ (\gamma^2 - 1)(\gamma^2 + 3)$  MeV/cm. For most materials this is equivalent to  $-dE/\rho ds \sim 5 \times 10^{-9} (\gamma^2 - 1)(\gamma^2 + 3)$  MeV cm<sup>2</sup>/g. A method is suggested for observing neutron energy losses of this magnitude to atomic electrons in gases.

### I. INTRODUCTION

The slowing down of a fast, heavy, charged particle in matter has been calculated quantum mechanically in the first Born approximation by Bethe.<sup>1,2</sup> The relativistic formula obtained for the stopping power  $-dE/ds$  is

$$-\frac{dE}{ds} = \kappa \left( \ln \frac{2\gamma^2 m v^2}{I} - \beta^2 \right). \quad (1)$$

Here  $I$  is the mean excitation energy of the medium,  $m$  the electron mass,  $v$  the speed of the particle,  $\beta = v/c$ ,  $c$  being the speed of light, and  $\gamma = (1 - \beta^2)^{-1/2}$ . The factor  $\kappa$  is given by

$$\kappa = 4\pi z^2 e^4 NZ / m v^2, \quad (2)$$

where  $-e$  is the electronic charge,  $ze$  the charge of the particle, and  $NZ$  the number of electrons per unit volume in the medium. Characteristic features of charged particle slowing down are evident in the semiclassical impulse treatment made by Bohr.<sup>3</sup> A formula much like Eq. (1) can thus be derived from classical mechanics when one makes certain "reasonable" assumptions, which also give some physical insight into collision phenomena. In this paper, we use the semiclassical method to calculate the effect of magnetic moment on the slowing down of a fast, heavy particle. The result is used to estimate the stopping power of matter for neutrons, a quantity apparently not yet formulated quantum mechanically, although mentioned by several authors.<sup>4,5</sup>

### II. SEMICLASSICAL CALCULATION

Figure 1 shows schematically the interaction between a heavy particle and an electron at rest at

the position  $x_1 = b$ ,  $x_2 = 0$ ,  $x_3 = 0$  in the coordinate system  $K$  (laboratory system). The heavy particle (charge  $q$  and speed  $v$ ) is incident along the  $X_3$  axis. As shown in the figure, we form a coordinate system  $K'$  (rest system of the particle) with axes parallel to those of  $K$  and with the incident particle at rest at its origin. The magnetic moments of the electron and particle measured, respectively, in  $K$  and  $K'$  are  $\vec{\mu}$  and  $\vec{\mu}'$ . According to the assumptions made in the semiclassical approximation, the collision between the particle and electron is swift and the electron, which is treated as free, does not move appreciably during the encounter. Furthermore, the path of the incident particle is assumed not to deviate from a straight line.

#### A. Electromagnetic Fields

We find the electromagnetic field measured in  $K$  due to charge  $q$  and magnetic moment  $\vec{\mu}$  of the particle at the position of the electron. The components of the electric field measured in  $K'$  at a point  $(x'_1, x'_2, x'_3)$  due to  $q$  are

$$E'_1 = qx'_1/s'^3, \quad E'_2 = 0, \quad E'_3 = qx'_3/s'^3, \quad (3)$$

where  $\vec{s}'$  is the displacement of the point from the origin of  $K'$ . The magnetic field in  $K'$  at this point, due to  $\vec{\mu}'$ , is given by<sup>6</sup>

$$\vec{B}'(\vec{s}') = \frac{3\vec{s}'(\vec{s}' \cdot \vec{\mu}') - \vec{\mu}' s'^2}{s'^5}. \quad (4)$$

Applying the Lorentz transformation to the electromagnetic field and the coordinates,<sup>7</sup> we obtain the field components in  $K$ :

$$E_1 = \frac{\gamma q x_1}{s'^3} + \gamma \beta \left( \frac{3x_2 w'}{s'^5} - \frac{\mu_2}{s'^3} \right),$$

$$\begin{aligned}
E_2 &= -\gamma\beta\left(\frac{3x_1w'}{s'^5} - \frac{\mu_1}{s'^3}\right), \\
E_3 &= \frac{q\gamma(x_3 - vt)}{s'^3}, \\
B_1 &= \gamma\left(\frac{3x_1w'}{s'^5} - \frac{\mu_1}{s'^3}\right), \\
B_2 &= \gamma\left(\frac{3x_2w'}{s'^5} - \frac{\mu_2}{s'^3} + \frac{\beta qx_1}{s'^3}\right), \\
B_3 &= \frac{3\gamma(x_3 - vt)w'}{s'^5} - \frac{\mu_3}{s'^3},
\end{aligned} \tag{5}$$

where

$$s' = [x_1^2 + x_2^2 + \gamma^2(x_3 - vt)^2]^{1/2} \tag{6}$$

and

$$w' = x_1\mu_1 + x_2\mu_2 + \gamma(x_3 - vt)\mu_3. \tag{7}$$

The quantities  $\mu_1, \mu_2$ , and  $\mu_3$  represent the components of the heavy particle's magnetic moment measured in  $K'$ ;  $u_1, u_2$ , and  $u_3$  represent the components of the electron's magnetic moment in  $K$ .

#### B. Energy Loss

With  $\mu = u = w' = 0$ , the transfer of energy from the heavy particle to the electron at the position  $x_1 = b, x_2 = x_3 = 0$  results entirely from the first term in the electric field component  $E_1: \gamma qx_1/s'^3 = \gamma qb/r'^3$ . The force due to the component  $E_3$  is odd in time and averages to zero over the duration of the collision.<sup>3</sup> With  $\mu \neq 0$ , however, the magnetic moment of the particle contributes to the electric field components  $E_1$  and  $E_2$  perpendicular to the direction of motion in  $K$ . The magnetic field components in  $K$  depend on  $\vec{\mu}$  and, in addition,  $B_2$  depends on  $q$ . Because of its magnetic moment, the electron experiences a force<sup>8</sup>

$$\vec{F}_M = \nabla(\vec{u} \cdot \vec{B}) = \sum_{i,j=1}^3 \hat{i}_i u_j \frac{\partial B_j}{\partial x_i} = \sum_{i=1}^3 \hat{i}_i F_{Mi}, \tag{8}$$

where the  $\hat{i}_i$  are unit vectors in the directions of the coordinate axes in  $K$ .

The total force on the electron as a function of time is the sum of  $\vec{F}_E = -e\vec{E}$  and  $\vec{F}_M$ , evaluated at the position  $x_1 = b, x_2 = 0, x_3 = 0$ , where  $s' = r' = (b^2 + \gamma^2 v^2 t^2)^{1/2}$  and  $w' = b\mu_1 - \gamma v\mu_3 t$ . From (5) it follows that

$$\begin{aligned}
F_{E1} &= -(e\gamma/r'^3)(qb - \beta\mu_2), \\
F_{E2} &= (e\gamma\beta/r'^3)[(1/r'^2)(3b^2\mu_1 - \gamma v\mu_3 t) - \mu_1], \\
F_{E3} &= e\gamma qvt/r'^3.
\end{aligned} \tag{9}$$

Differentiating the components (5) of  $\vec{B}$  and combining them as indicated in (8), we find that

$$\begin{aligned}
F_{M1} &= \frac{3\gamma u_1}{r'^5} \left( 3b\mu_1 - \gamma\mu_3 vt - \frac{5b^2}{r'^2} (b\mu_1 - \gamma\mu_3 vt) \right) \\
&\quad + \frac{\gamma u_2}{r'^3} \left( qb + \frac{3b}{r'^2} (\mu_2 - qb) \right)
\end{aligned}$$

$$\begin{aligned}
&\quad + \frac{3u_3}{r'^5} \left( b\mu_3 - \gamma\mu_1 vt + \frac{5\gamma bvt}{r'^2} (b\mu_1 - \gamma\mu_3 vt) \right), \\
F_{M2} &= \frac{3\gamma}{r'^5} [bu_1\mu_2 + u_2(b\mu_1 - \gamma\mu_3 vt) - u_3\mu_2 vt], \\
F_{M3} &= \frac{3\gamma^2 u_1}{r'^5} \left( b\mu_3 - \gamma\mu_1 vt + \frac{5\gamma bvt}{r'^2} (b\mu_1 - \gamma\mu_3 vt) \right) \\
&\quad + \frac{3\gamma^3 u_2 vt}{r'^5} (-\mu_2 + \beta qb) \\
&\quad + \frac{3\gamma u_3}{r'^5} \left( b\mu_1 - 3\gamma\mu_3 vt - \frac{5\gamma^2 v^2 t^2}{r'^2} (b\mu_1 - \gamma\mu_3 vt) \right).
\end{aligned} \tag{10}$$

The impulses of these components are their time integrals:

$$\begin{aligned}
I_{E1} &= \int_{-\infty}^{\infty} F_{E1} dt = -\frac{2eq}{vb} + \frac{2e\mu_2}{cb^2}, \\
I_{E2} &= \frac{2e\mu_1}{cb^2}, \\
I_{M1} &= -\frac{4u_1\mu_1}{vb^3} - \frac{2qu_2}{cb^2} + \frac{4u_2\mu_2}{vb^3}, \\
I_{M2} &= \frac{4u_1\mu_2}{vb^3} + \frac{4u_2\mu_1}{vb^3}, \\
I_{E3} &= I_{M3} = 0.
\end{aligned} \tag{11}$$

The components of the total impulse received by the electron are

$$\begin{aligned}
I_1 &= -\frac{2eq}{vb} + \frac{2(e\mu_2 - qu_2)}{cb^2} + \frac{4(u_2\mu_2 - u_1\mu_1)}{vb^3}, \\
I_2 &= \frac{2e\mu_1}{cb^2} + \frac{4(u_1\mu_2 + u_2\mu_1)}{vb^3}, \\
I_3 &= 0.
\end{aligned} \tag{12}$$

The energy lost by the heavy particle to the electron in  $K$  is

$$T = (1/2m)(I_1^2 + I_2^2). \tag{13}$$

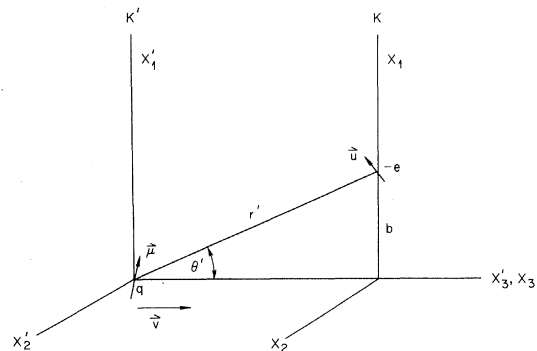


FIG. 1. Rest systems  $K'$  of incident particle (charge  $q$ ) and of electron (charge  $-e$ ). System  $K'$  moves with velocity  $\vec{v}$  in  $K$ , which is also the laboratory system for calculation of stopping power. Magnetic moments, shown schematically, are  $\vec{\mu}$  and  $\vec{u}$ .

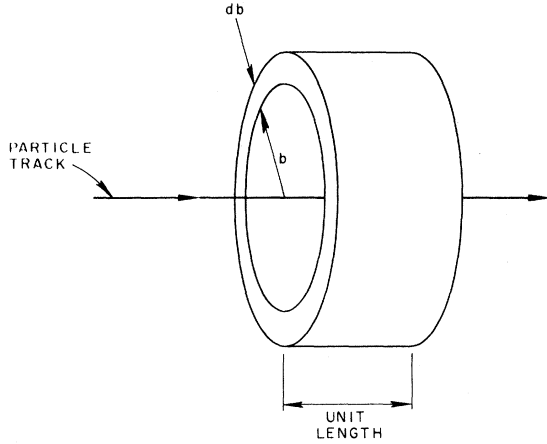


FIG. 2. Number of collisions with impact parameter between  $b$  and  $b+db$  per unit distance traveled is equal to the number of electrons in the annular cylinder shown.

As shown in the Appendix, use of this nonrelativistic form for the electron recoil implies that we make the restriction  $\gamma \lesssim 10$ .

### C. Stopping Power

The stopping power of the medium, assumed to be uniform and isotropic, through which the heavy particle passes is obtained by averaging (13) over directions of  $\vec{u}$ , integrating over impact parameters from  $b_{\min}$  to  $b_{\max}$  (given below), and averaging over directions of  $\vec{\mu}$ . As seen from (12) and (13), because  $T$  is a sum of powers of the magnetic-moment components, the order of performing the integration and averaging operations is arbitrary. The number of electrons in an annular ring, centered along the particle's track and having unit length and thickness  $db$  (Fig. 2), is  $2\pi NZb db$ , and so we obtain the stopping power from the expression

$$-\frac{dE}{ds} = 2\pi NZ \int_{b_{\min}}^{b_{\max}} \langle T \rangle b db, \quad (14)$$

where  $\langle T \rangle$  is the value of (13) averaged over directions of  $\vec{u}$  and  $\vec{\mu}$ . We make use of the following relations, which apply to both  $\vec{\mu}$  and  $\vec{u}$ :

$$\begin{aligned} \langle \mu_i \rangle &\equiv \frac{1}{4\pi} \int_0^\pi \int_0^{2\pi} \mu_i \sin\theta d\theta d\varphi = 0, & \langle \mu_i^2 \rangle &= \frac{1}{3} \mu^2, \\ \langle \mu_i \mu_j \rangle &= 0, & i &\neq j \\ \langle u_i \mu_j \rangle &= \langle u_i \rangle \langle \mu_j \rangle = 0. \end{aligned} \quad (15)$$

Combining (12) and (13) and carrying out the averaging in accordance with (15) gives

$$\langle T \rangle = \frac{2}{m} \left( \frac{e^2 q^2}{v^2 b^2} + \frac{2e^2 \mu^2 + u^2 q^2}{3c^2 b^4} + \frac{16u^2 \mu^2}{9v^2 b^6} \right). \quad (16)$$

From (14),

$$\begin{aligned} -\frac{dE}{ds} &= \frac{4\pi e^2 q^2 NZ}{mv^2} \left[ \ln \left( \frac{b_{\max}}{b_{\min}} \right) \right. \\ &\quad \left. - \frac{v^2}{3e^2 q^2} \left( \frac{2e^2 \mu^2 + u^2 q^2}{2c^2 b^2} + \frac{4u^2 \mu^2}{3v^2 b^4} \right) \right]_{b_{\min}}^{b_{\max}}. \end{aligned} \quad (17)$$

The maximum and minimum values of the impact parameters are<sup>9</sup>

$$b_{\max} = \gamma v / \bar{\nu} \quad \text{and} \quad b_{\min} = \hbar / \gamma m v, \quad (18)$$

in which  $\bar{\nu}$  represents an average frequency associated with the motion of the bound atomic electrons. The logarithm term in (17) is the same as that in the quantum-mechanical formula (1) if we write

$$\ln \frac{b_{\max}}{b_{\min}} = \ln \frac{\gamma^2 m v^2}{\hbar \bar{\nu}} = \ln \frac{2\gamma^2 m v^2}{I}, \quad (19)$$

the energy  $2\hbar \bar{\nu}$  replacing the precisely defined parameter  $I$  in the quantum-mechanical formula. Restricting ourselves to values of  $\gamma$  somewhat greater than unity, we can neglect  $(1/b_{\max})^2$  compared with  $(1/b_{\min})^2$  in (17) (see the Appendix) to obtain

$$\begin{aligned} -\frac{dE}{ds} &= \frac{4\pi e^2 q^2 NZ}{mv^2} \left[ \ln \frac{2\gamma^2 m v^2}{I} \right. \\ &\quad \left. + \frac{\gamma^2 m^2 v^4}{3e^2 q^2 \hbar^2} \left( \frac{2e^2 \mu^2 + u^2 q^2}{2c^2} + \frac{4u^2 \mu^2 \gamma^2 m^2}{3\hbar^2} \right) \right]. \end{aligned} \quad (20)$$

Finally, we write  $q = ze$  and let  $\bar{\mu}$  and  $\bar{u}$  represent the magnetic moments in units of the nuclear magneton and the Bohr magneton by writing

$$\mu = \bar{\mu} e \hbar / 2M_p c \quad \text{and} \quad u = \bar{u} e \hbar / 2mc, \quad (21)$$

where  $M_p$  is the proton mass.<sup>10</sup> Equation (20) becomes

$$\begin{aligned} -\frac{dE}{ds} &= \frac{4\pi z^2 e^4 NZ}{mv^2} \left[ \ln \frac{2\gamma^2 m v^2}{I} + \frac{1}{24} \gamma^2 \beta^2 \bar{u}^2 \right. \\ &\quad \left. + \frac{\gamma^4 \beta^4}{36z^2} \left( \frac{m}{M_p} \right)^2 \bar{\mu}^2 \bar{u}^2 + \frac{\gamma^2 \beta^4}{12z^2} \left( \frac{m}{M_p} \right)^2 \bar{\mu}^2 \right], \end{aligned} \quad (22)$$

with the restriction  $1 < \gamma \lesssim 10$  imposed, as described in the Appendix. The second term in the bracket of (22) arises from the interaction of the heavy particle's charge and the electron's magnetic moment. Since  $\gamma \lesssim 10$  and  $(m/M_p)^2 \ll 1$ , this term is the largest of the nonlogarithmic ones. The third term in the bracket results from the interaction between the magnetic moments of the particle and the electron. The last term comes from the interaction of the magnetic moment of the particle and the electron's charge.

### III. STOPPING POWER FOR NEUTRONS

Setting  $z = 0$  in Eq. (22), we see that only the last

two terms in the square bracket remain. We obtain for a neutral particle

$$-\frac{dE}{ds} = \frac{\pi e^4 NZ \gamma^2 \beta^2 \bar{\mu}^2}{9mc^2} \left( \frac{m}{M_p} \right)^2 (\gamma^2 \bar{\mu}^2 + 3). \quad (23)$$

The relative contribution from coupling of the particle's magnetic moment to the magnetic moment and to the charge of the electron is  $\frac{1}{3}\gamma^2$ . Introducing numerical values into (23) and writing  $\gamma^2 \beta^2 = \gamma^2 - 1$ , we obtain for the neutron ( $\bar{\mu} = -1.9135$ )

$$-\frac{dE}{ds} = 1.53 \times 10^{-32} NZ (\gamma^2 - 1) (\gamma^2 + 3) \text{ MeV/cm} \quad (24)$$

or

$$-\frac{dE}{\rho ds} = 9.24 \times 10^{-9} \frac{Z}{A} (\gamma^2 - 1) (\gamma^2 + 3) \text{ MeV cm}^2/\text{g}, \quad (25)$$

where  $\rho$  and  $A$  are the density and atomic weight of an elemental medium being traversed. Since for most materials  $Z/A \sim 0.5$ , the semiclassical estimate of stopping power for neutrons becomes

$$-\frac{dE}{\rho ds} \sim 5 \times 10^{-9} (\gamma^2 - 1) (\gamma^2 + 3) \text{ MeV cm}^2/\text{g}. \quad (26)$$

In aluminum, for example, Eq. (25) gives

$$-\frac{dE}{\rho ds} = 4.45 \times 10^{-9} (\gamma^2 - 1) (\gamma^2 + 3) \text{ MeV cm}^2/\text{g}, \quad (27)$$

with  $\gamma = 10$ ,  $-dE/\rho ds = 4.54 \times 10^{-5}$  MeV cm<sup>2</sup>/g, which is about five orders of magnitude smaller than the value 1.86 MeV cm<sup>2</sup>/g for a proton of the same energy ( $\sim 10$  GeV). The neutron mean free path in Al (based on a nuclear cross section of 400 mb) is about 40 cm. Thus a 10-GeV neutron would lose only about 5 keV in traveling a mean free path in aluminum, according to this semiclassical estimate.

Attempts have been made to detect ionization produced directly by the interaction of neutrons with electrons.<sup>4</sup> The order of magnitude of the direct ionization current to be expected with available neutron beams can be estimated with the help of Eq. (26). At the CERN synchrocyclotron, for example, the 400-MeV neutron beam has a flux density of  $\sim 2 \times 10^5$  neutrons cm<sup>-2</sup> sec<sup>-1</sup>. Assuming that a uniform beam of neutrons at this energy traverses a gas in which an average of 33 eV is needed to create an ion pair, we find from (26) that a current of  $\sim 3 \times 10^{-17}$  A is produced per gram of gas per cm<sup>2</sup> in the path of the beam. In one experiment at CERN a current of  $10^{-11}$  A was produced in an ionization chamber, containing 1.19 g cm<sup>-2</sup> of gas, placed in the neutron beam.<sup>11</sup> The estimated neutron direct ionization current thus appears to be smaller than that produced as a result of nuclear interactions by about a factor of  $10^6$ . This factor could be reduced some by an optimum choice of chamber gas and by

going to higher neutron energies.

A possible way of observing ionization produced directly by neutrons has been suggested by Hurst.<sup>12</sup> A gas-discharge tube located just beyond the collecting anode of a parallel-plate ionization chamber and open to the gas in the chamber through a hole in the anode is triggered by pulses of charge produced in the chamber. A similar arrangement has been used for other purposes.<sup>13</sup> A high-energy neutron beam, passing through the chamber parallel to its plates, would produce charge mostly in large pulses following the reactions of neutrons with nuclei. A small amount of charge—perhaps  $10^{-6}$  as little, based on the above estimate—would be produced in small pulses by the direct ionization of atoms by neutrons. The predicted response of the gas-discharge tube in time is a series of large pulses (nuclear reactions) with an approximately equal number of smaller ones (neutron ionizations) intermingled.

#### APPENDIX

The condition  $1 < \gamma \leq 10$  that applies to Eq. (20) and subsequent stopping-power formulas will be discussed. The ratio  $b_{\max}/b_{\min} \approx 10^6 \gamma^2 \beta^2 / I_{eV}$ , where  $I_{eV}$  ( $\lesssim 10^3$  for all elements) is the mean excitation energy expressed in electron volts. With  $I_{eV} = 900$ ,  $(b_{\max}/b_{\min})^2 \sim 700$  for a 10-MeV nucleon, for example, and decreases to  $\sim 7$  at 1 MeV. For convenience, we neglected  $(1/b_{\max})^2$  compared with  $(1/b_{\min})^2$  in the nonlogarithmic terms in going from Eq. (17) to Eq. (20), thereby introducing the restriction that  $\gamma$  not be too close to unity. This limitation is also consistent with the breakdown of the semiclassical concepts used here when applied to collisions in which a small amount of momentum is transferred.

From the relativistic expression  $T = (p^2 c^2 + m^2 c^4)^{1/2} - mc^2$  involving the electron's momentum  $p$  after collision it follows that the nonrelativistic recoil formula (13) is valid when

$$p^2/m^2 c^2 \ll 1. \quad (A1)$$

We examine each of the terms in Eq. (16) when they have their largest value ( $b = b_{\min}$ ). The first term gives

$$\frac{p_1^2}{m^2 c^2} \equiv \frac{4e^2 q^2}{m^2 c^2 v^2 b_{\min}^2} = 4\alpha^2 \gamma^2, \quad (A2)$$

where  $\alpha = e^2/\hbar c = 1/137$  is the fine-structure constant and we have set  $q = e$ . In the second term, applied to the nucleon,  $\mu \ll u$ , and we find that

$$\frac{p_2^2}{m^2 c^2} \equiv \frac{4u^2 q^2}{3m^2 c^4 b_{\min}^4} = \frac{1}{3} \alpha^2 (\gamma^2 - 1)^2. \quad (A3)$$

The third term gives

$$\frac{p_3^2}{m^2 c^2} = \frac{64u^2 \mu^2}{9u^2 b_{\min}^6} = \frac{4}{9} \alpha^2 \left( \frac{m}{M_p} \right)^2 \gamma^6 \beta^4. \quad (\text{A4})$$

For  $\gamma \lesssim 10$  these terms are small compared with unity and hence relativistic effects on the recoiling electron will not be large.

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<sup>1</sup>H. A. Bethe, *Ann. Physik* **5**, 325 (1930).

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<sup>5</sup>U. Fano, *Natl. Acad. Sci. Natl. Res. Council Publ.* No. 1133, 1964 (unpublished).

<sup>6</sup>J. D. Jackson, *Electrodynamics* (Wiley, New York, 1962), p. 143.

<sup>7</sup>J. D. Jackson, in Ref. 6, pp. 380–381. The transformation appropriate to Fig. 1 is obtained from Jackson's

Eq. (11.114) by interchanging the primed and unprimed symbols and reversing the sign in front of  $\beta$ . It is assumed that the origins of  $K$  and  $K'$  coincide at times  $t=t'=0$ .

<sup>8</sup>J. D. Jackson, in Ref. 6, p. 147.

<sup>9</sup>See, for example, E. Fermi, *Nuclear Physics* (Chicago U. P., Chicago, 1953).

<sup>10</sup>For the electron, of course,  $\bar{v}=1$ , but we write it in Eq. (21) explicitly to identify the physical origin of terms in Eq. (22).

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## Electron-Paramagnetic-Resonance Investigation of the Dynamic Jahn-Teller Effect in $\text{SrCl}_2:\text{La}^{2+}$

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The electron-paramagnetic-resonance spectrum for  $\text{SrCl}_2:\text{La}^{2+}$  has been observed between 1.2 and 40°K. At 1.2°K, the dominant structure is anisotropic and is described within experimental error by second-order solutions of the effective Hamiltonian for an isolated  ${}^2E_g$  state split by large random internal strains. Coexisting with the anisotropic structure at temperatures between 1.2°K and approximately 5°K is structure whose position is isotropic but whose intensity and linewidth are anisotropic and vary with temperature and sample treatment. This structure is shown to result from rapid direct relaxation between the strain-split vibronic states. At temperatures above approximately 6°K, only the isotropic structure is observed.

### I. INTRODUCTION

The instability of a symmetric nonlinear polyatomic complex in an orbitally degenerate state was demonstrated theoretically by Jahn and Teller.<sup>1</sup> Early experimental evidence<sup>2–4</sup> indicated that the effects of this instability could be conveniently divided into two categories. Either the complex spontaneously became distorted and was stabilized in a configuration of lower symmetry (static Jahn-Teller effect), or the complex became distorted but, instead of being stabilized, was rapidly reoriented between several distorted configurations (dynamic Jahn-Teller effect). Anisotropic electron-paramagnetic-resonance (EPR) spectra characteristic of a static Jahn-Teller effect were first reported<sup>2</sup> for  $\text{Cu}^{2+}$  in trigonal sites of  $\text{ZnSiF}_6 \cdot 6\text{H}_2\text{O}$

at temperatures in the liquid-hydrogen range. Somewhat earlier, an isotropic EPR spectrum had been observed<sup>3</sup> at elevated temperatures for the same system and interpreted<sup>4</sup> as a thermally induced reorientation of the complex between equivalent static distortions, a type of dynamic Jahn-Teller effect. Subsequently, both types of spectra have been reported for a number of systems. These investigations and other studies of the Jahn-Teller effect have been reviewed by Sturge.<sup>5</sup>

Recently, anisotropic EPR spectra have been observed<sup>6–11</sup> which indicate the occurrence of another type of dynamic Jahn-Teller effect for orbital doublets at liquid-helium temperatures. The first such spectrum was reported by Coffman<sup>6–8</sup> for  $\text{Cu}^{2+}$  in MgO. Similar spectra were reported