$$+\cos\left[\tfrac{1}{2}a(k_x+k_z)\right]\}$$

$$+4J_2\{\cos(ak_x)+\cos(ak_y)+\cos(ak_z)\}\ ,$$

where a is the lattice constant. It is to be noted that if one computes the quantities $\mu+\lambda$ and $\mu-\lambda$ from these expressions, making proper use of trigonometric identities, the results will differ from those quoted in Lines's paper by negative signs, and factors of $\frac{1}{2}$ in the arguments of the trigonometric functions. We substituted the values of J_1 and J_2 from our specific-heat calculation into the expression for T_N. The average was done numerically over the paramagnetic Brillouin zone rather than the antiferromagnetic Brillouin zone, with proper account taken of the fact that the integral was over twice the volume in k space as was needed. We sought values of the exchange constants J_1 and J_2 in the range given above, obtained from the specific-heat calculation, which would yield a Néel temperature within 0.5 °K on either side of the experimental value 9.81 °K. It was found that T_N is rather more sensitive to changes in J_2 than in J_1. For $J_1=(0.07\pm0.02)$ °K and $J_2=-(0.215\pm0.005)$ °K, we obtained $T_N=(9.76\pm0.41)$ °K, in quite good agreement with the above experimental value.

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Influence of Normal-State Electrical Resistance on the Superconducting Transition Temperature of Ultrathin Films*

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Recent data showing decreases in the T_c of ultrathin films are reconsidered in the light of results from work on fluctuations in films, which indicate the possibility of a large "pair-breaker" in high-resistance films.

Recently, we have made a study of the decrease in transition temperature T_c in ultrathin films.[1] In this present paper we wish to amplify some of the conclusions in Ref. 1 by taking into account recent work on fluctuations in films, and in particular we wish to make a speculative estimate of the T_{c0} of films in the absence of the "pair-breaker" which appears to be associated with the large $R_\square$ of the films. We briefly review some of the results. With the theoretical work of McMillan,[2] it became evident how the increases in the T_c of soft-metal films could be related to "softening" of the phonon spectrum, either by disorder or by soft phonons caused by surfaces.[3,4] Unfortunately, the maximum T_c observed in either disordered[4,5] or ultrathin films[1,3] never approached the limits predicted by the theories. In fact, it was found that in the very thinnest films the T_c actually decreased again. In Fig. 1 we show data on Al films, previously reported,[1] showing that T_c rises as the thickness is decreased and finally decreases again in the thinnest films. In the case of Pb, where the enhancement is expected to be small, it was found that T_c decreased as the films were made thinner,[1,6,7] and the depression (of T_c) correlated with the sheet resistance $R_\square$. The question of why the T_c decreased was discussed, but was never convincingly explained on the basis of any existing theoretical ideas, and this is still the case at present. It is the purpose of this note to estimate how the effect of the high $R_\square$ in the films can affect the transition temperature of Al films. The clue to how to treat this problem is provided by Kajimura and Mikoshiba (KM),[8] although we emphasize that this phenomenological approach certainly does not imply that there is an understanding of why large $R_\square$ reduces T_c as rapidly as it does.

In recent work on fluctuation phenomena in Al films,[9] large deviations have been found from the mean-field theory of Aslamazov and Larkin (AL).[10] Thompson[11] has modified the AL theory by including a term originally proposed by Maki. This addition-

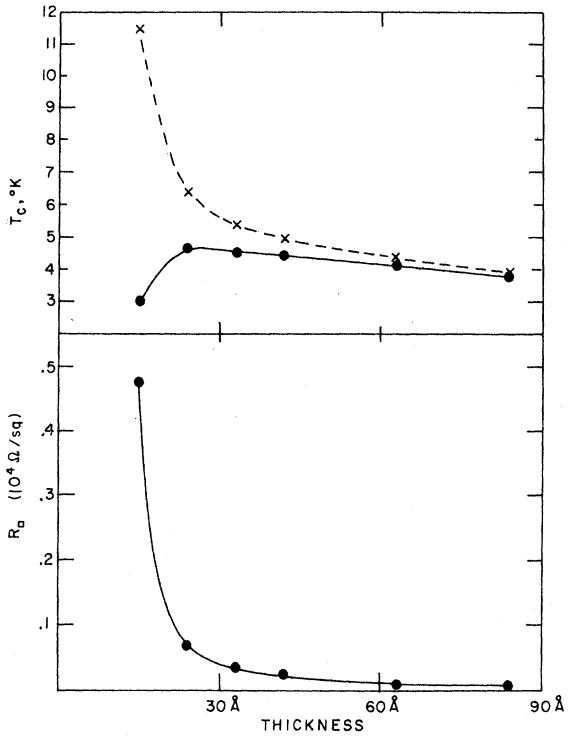

FIG. 1. Solid curves show T_c and $R_\square$ as a function of thickness. Dashed curve shows T_{c0} as a function of thickness and was calculated from the relationship of KM that $(T_{c0}-T_c)/T_c = 6\times10^{-4}R_\square$.

al contribution to the fluctuation spectrum is, however, divergent for long-wavelength fluctuations in one- and two-dimensional superconductors. Thompson showed that this long-wavelength divergence was cut off in the presence of a pair-breaking interaction, leading to a temperature dependence of the paraconductivity above T_c, which is given by

$$\frac{\sigma_p}{\sigma_n} = \frac{\tau_0}{\tau} + \frac{2\tau_0}{\tau-\delta}\ln\left(\frac{\tau}{\delta}\right) , \qquad (1)$$

with $\tau = (T-T_c)/T_c$ and $\delta = (T_{c0}-T_c)/T_c$. The first term is the AL contribution, and the second term is commonly referred to as the Maki-Thompson contribution. Crow *et al.*[12] have shown that by varying a pair-breaker in a predictable manner, i.e., with parallel magnetic field, the pair conductance varies with temperature and pair-breaker as indicated by Eq. (1). Other pair-breakers, such as current and magnetic impurities, have yielded similar agreement.[13] Recently, KM[8] have fitted the fluctuation data in Al films to the Thompson theory. For each film, with a given $R_\square$, they have obtained a value of δ which gives a fit to Eq. (1), and they find from this procedure that $\delta = 6\times10^{-4}R_\square$. It is interesting that this rate of depression of T_c is similar to that previously reported for Pb,[1] where $\delta \sim 7\times10^{-4}R_\square$. This, of course, could be a coincidence.

The thing we wish to do in this paper is to take our data for Al films, and assume that the above variation of the pair-breaker with $R_\square$ is present, a possibility suggested by KM. Given the variation of T_c with $R_\square$ we can solve for T_{c0}. The data showing T_c and $R_\square$ as a function of thickness are shown by the solid curves, and the dashed curve (which gives T_{c0} as a function of thickness) was calculated from the relationship of KM that $(T_{c0}-T_c)/T_c = 6\times10^{-4}R_\square$. It is seen that T_{c0} goes up to about 12 °K for the thinnest Al films. For Sn films we do not have complete pair-breaker data, but the highest T_{c0} obtained from our thin film and fluctuation data is about 8 °K. These values are in reasonable agreement with the estimates from the Garland *et al.*[4] modifications to the McMillan theory. Hence we can speculate that if films can be constructed that are about 3 to 5 monolayers thick with an $R_\square$ of about 100 Ω instead of about 5000 Ω, then the highest T_c's due to phonon softening may be obtained, since in this case $T_{c0} \sim T_c$. We emphasize that this argument is quite speculative, since the nature of ultrathin films prepared at cryogenic temperatures is not necessarily the same as the films used in fluctuation experiments. We also wish to mention that above about 500 Ω/sq, the normal-state resistance has a negative temperature coefficient. This may alter slightly the theoretical expressions for the fluctuation conductivity,[14] although we do not expect that it will greatly modify the δ-vs-$R_\square$ curve given by KM. The negative temperature coefficient can also be expected to reduce T_c by an amount given by $T_c = T_{c0} - \frac{1}{2}A$, for small changes in T_c, where A is the activation energy in the normal state.[1]

In our work on the depression of transition temperatures in Pb and Bi films,[1] we indicated that the best correlation was between T_c and $R_\square$, rather than thickness or resistivity. It is clear that the construction of long-mean-free-path films with low $R_\square$ will provide much information about what factors reduce T_c. If $R_\square$ is, indeed, the crucial factor determining the decrease in T_c, then we should expect higher transition temperatures in long-mean-free-path films. On the other hand, if this correlation is fortuitous and merely reflects the unknown effects at the film-substrate interface as the films are made thinner and more resistive, then we would expect a T_c reduced from the theoretical estimates.

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Effects of Phonons and Impurities on Single-Particle-Mode Neutron Scattering in Chromium

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Contrary to a comment made in a recent paper on inelastic neutron scattering in antiferromagnetic chromium, it is argued here that at low temperatures the electron-phonon interaction does not cause significant broadening of the single-particle-mode peak in the inelastic-neutron-scattering cross section at the gap energy. The addition of vanadium impurities is then suggested as a means of lowering the gap energy in order to make it easier to perform the experiment to observe this peak in the scattering and thus measure the gap energy by neutron diffraction. Estimates are given of the scattering cross section.

In a previous publication[1] (to be referred to as I), it was argued that there should be a peak [actually a singularity of the form $(\omega-\Delta)^{-1/2}$, where Δ is the gap energy] in the inelastic-neutron-scattering cross section for antiferromagnetic chromium metal at neutron energy transfers equal to the gap between the magnetically split hands. In a recent publication, Liu has argued that such a peak will be rendered unobservable when the effects of the electron-phonon interaction are included.[2] It will be shown in this note that although the arguments of Ref. 2 may be possibly valid at room temperature, they are certainly not correct at temperatures far below room temperature, at which not too many phonons exist to scatter electrons. At such low temperatures (e.g., liquid-nitrogen temperature), the results of I are not invalidated by the electron-phonon interaction. A method of lowering the gap energy to make it more easily observable by neutron diffraction will also be discussed.

Reference 2 treats the phonons as impurities, but phonons are different from impurities. For example, whereas there are impurities at all temperatures, there are no phonons at zero temperature. Our treatment of the electron-phonon interaction will follow Abrikosov, Gorkov, and Dzyaloshinski.[3] We find the self-energy of the one-electron Green's function from Eq. (21.26) in Ref. 3,

$$\Sigma(\vec{p},\epsilon)=\frac{g^2}{(2\pi)^4\pi}\int d^3p_1\int_{-\infty}^{\infty}d\omega\int_{-\infty}^{\infty}d\epsilon_1\frac{\mathrm{Im}G(\vec{p}_1,\epsilon_1)\,\mathrm{Im}D(\vec{p}-\vec{p}_1,\omega)}{\omega+\epsilon_1-\epsilon-i\delta}\left[\tanh\left(\frac{\epsilon_1}{2KT}\right)+\coth\left(\frac{\omega}{2KT}\right)\right], \quad (1)$$

where ϵ, ϵ_1, and ω are electron and phonon energies, $\vec{p}$ and $\vec{p}-\vec{p}_1$ are electron and phonon momenta, and G and D are electron and phonon Green's functions, respectively. Since G is a 2×2 matrix for the spin-density wave state, Σ is a 2×2 matrix.

In order to consider the rounding out of the peak at the gap energy in the density of states, following Zittartz,[4] we find the off-diagonal component of Eq. (1). Using Zittartz's one-electron Green's function,[4] we find