PHYSICAL REVIEW B VOLUME 3, NUMBER 7 1 APRIL 1971

Flux-Flow Resistivity and Hall Angle in Pure Transition-Metal Superconductors near H_{c2}

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(Received 12 August 1970)

The transport equations in the mixed state of a pure type II transition-metal superconductor are obtained by noting that the physically observed transport currents (i.e., the electric current and the heat current) in the two-band superconductors near H_{c2} are just the sum of the physical observables in the individual bands. Since the two moving energy gaps in the transition-metal superconductors have essentially the same form as the moving order parameter in the one-band superconductors, the calculational techniques devised by Caroli and Maki for evaluating the observable in pure one-band superconductors near H_{c2} can be applied directly to the evaluation of the physical observables in the individual bands of the two-band superconductors near H_{c2}. Using the transport equations with the definitions of the flux-flow resistivity and Hall angles, expressions for these properties are obtained which should be valid for pure transition-metal superconductors near H_{c2}. The two-band expressions are used to analyze the experimental data on the flux-flow resistivity and Hall angle in pure superconducting niobium.

I. INTRODUCTION

In a recent paper by the author[1] (hereafter referred to as I), it was seen that while the two energy gaps in a pure transition-metal superconductor in a high magnetic field and a transverse electric field moved with a common velocity $u = E/H_{c2}$, their motions were governed by separate diffusion equations. The two diffusion constants $\mathfrak{D}_s$ and $\mathfrak{D}_d$ were chosen such that

$$\Delta_{s(d)} = \sum_n C_{s(d)n} \exp[ikn(y+ut)] \times \exp\left[-eH\left(x + \frac{kn}{2eH} + \frac{iu}{4e\mathfrak{D}_{s(d)}}H\right)^2\right] \quad (1)$$

are the physically observed energy gaps, the physically observed energy gaps being the expectation values of the operators

$$\Delta^\dagger_{s(d)} = -g_{s(d)}\psi^\dagger_{s(d)\dagger}(r,t)\psi^\dagger_{s(d)\downarrow}(r,t) - g_{sd}\psi^\dagger_{d(s)\dagger}(r,t)\psi^\dagger_{d(s)\downarrow}(r,t) \quad (2)$$

in the system described by the interaction Hamiltonian[2]

$$\mathcal{H}_I = e\int n_s(r,t)\phi(r,t)d^3r + \int[\Delta_s(r,t)\Psi^\dagger_s(r,t) + \Delta^\dagger_s(r,t)\Psi_s(r,t)]d^3r + e\int n_d(r,t)\phi(r,t)d^3r + \int[\Delta_d(r,t)\Psi^\dagger_d(r,t) + \Delta^\dagger_d(r,t)\Psi_d(r,t)]d^3r, \quad (3)$$

where the operators $n_{s(d)}$, $\Psi_{s(d)}$, and $\Psi^\dagger_{s(d)}$ are defined in I.

The purpose of this paper is to obtain the physical observables of quantities such as the electric-current and heat-current operators in the mixed state of a pure transition-metal superconductor in which the two energy gaps are moving with a common velocity u. In this way, the transport equations for the two-band superconductor can be obtained. Then by using the resulting transport equations in the definitions[3] of various resistive-state properties, such as the flux-flow resistivity and Hall angle, we will be able to obtain expressions for these properties in pure transition-metal superconductors in high magnetic fields.

The approach used is an extension of the microscopic theory of Caroli and Maki[4] to a two-band superconductor. Since the physical observables in the two-band superconductors are just the sums of the same physical observables in the individual bands, they can be evaluated using the calculational techniques developed by Maki[5] for obtaining explicit forms of the one-band observables. The only modification that has to be taken in order that Maki's techniques may be used is that the role of the order parameter in the one-band system is taken by the energy gaps in the two-band calculations. Since the energy gaps in the two-band superconductor have essentially the same form as the order parameter in the one-band superconductor, no difficulty will be encountered in the evaluations because of the substitution.

The need for using the two-band model of Suhl, Matthias, and Walker[6] (SMW) to describe superconductivity in the transition metals is clearly indicated by the recent discoveries of a second energy gap[7] and a second transition temperature[8] in niobium. Prior to these discoveries, the existence of a second energy gap in pure transition-metal superconductors had been inferred from the specific-heat measurements on vanadium[9] and niobium.[10] Using the numerical values inferred by Sung and Shen[11] from the specific-heat measurements on niobium in the SMW two-band model, Radhakrishnan[12] was able to fit the experimental data on the temperature variation of the penetration depth in niobium[13] and to obtain the effective masses of the electrons in the two bands. It should be expected that any properties of pure transition-metal superconductors which depend on the motion of the energy gaps would reflect the presence of a second energy gap.

II. TWO-BAND TRANSPORT EQUATIONS

We begin by writing down the transport equations in their most general forms:

$$
\begin{aligned}
j_x^{(e)} &= \sigma_1 E_x - \sigma_2 E_y + (\alpha_2/T)(\nabla T)_x + (\alpha_1/T)(\nabla T)_y,\\
j_y^{(e)} &= \sigma_2 E_x + \sigma_1 E_y - (\alpha_1/T)(\nabla T)_x + (\alpha_2/T)(\nabla T)_y,\\
j_x^{(h)} &= -\alpha_2 E_x - \alpha_1 E_y + K_1(-\nabla T)_x - K_2(-\nabla T)_y,\\
j_y^{(h)} &= \alpha_1 E_x - \alpha_2 E_y + K_2(-\nabla T)_x + K_1(-\nabla T)_y,
\end{aligned}
\tag{4}
$$

where $j^{(e)}$ and $j^{(h)}$ are the electric and the heat current, respectively. The coefficients needed for the above equations to be the transport equations for the mixed state in a pure transition-metal superconductor in a high magnetic field and a transverse electric field can be determined by looking at the forms of certain physical observables in the resistive state of a pure two-band superconductor.

As was shown in I, the physical observable $\tilde{A}(r, t)$ of the operator $A(r, t)$ is given by

$$
\begin{aligned}
\tilde{A}(r,t) = &-ie\int_{-\infty}^{t} dt' \int d^3r' \sum_i \langle [A_i(r,t), n_i(r',t')]\rangle \phi(r',t') + \int_{-\infty}^{t} dt_1 \int_{-\infty}^{t} dt_2 \int d^3l\, d^3m \\
&\times \sum_i \{\langle [[A_i(r,t), \Psi_i(l,t_1)], \Psi_i^\dagger(m,t_2)]\rangle + \langle [[A_i(r,t), \Psi_i^\dagger(m,t_2)], \Psi_i(l,t_1)]\rangle\} \Delta_i^\dagger(l,t_1)\Delta_i(m,t_2),
\end{aligned}
\tag{5}
$$

where the summation is over the two bands. In obtaining (5), we have assumed that the operator $A(r, t)$ in the two-band system is just the sum of the operator in the individual band.[14] In (5), Δ_i is the energy gap in the ith band and the operators Ψ_j and n_i are defined as

$$
\begin{aligned}
\Psi_j(r,t) &= \psi_{j\uparrow}(r,t)\psi_{j\downarrow}(r,t),\\
n_i(r,t) &= \sum_\sigma \psi_{i\sigma}^\dagger(r,t)\psi_{i\sigma}(r,t).
\end{aligned}
\tag{6}
$$

The second term in (5) is the quantity which results from the uniform motion of the energy gaps.

By considering the physical observable in the two-band superconductor as being just the sum of the physical observables in the individual bands, we can easily evaluate (5) since the individual band observable is in exactly the same form as the observable in the one-band superconductor,[4,5] except for the band designation and the use of the energy gaps in place of the order parameter. This substitution will not cause any deviation in the calculations since the two moving energy gaps have essentially the same form as the moving order parameter in the one-band superconductor.

Using the results of the appendices, we obtain

$$
\sigma_1 = \sum_i \frac{\sigma_i}{1+\eta_i^2}(1+X_{0i}), \quad \sigma_2 = \sum_i \phi\frac{\sigma_i}{1+\eta_i^2}\left(1+\frac{\eta_i}{\phi}X_{0i}\right),
$$

$$
\alpha_1 = -\sum_i \frac{1}{1+\eta_i^2}(\phi TP + X_{1i}),
$$

$$
\alpha_2 = \sum_i \frac{1}{1+\eta_i^2}(TP - \eta_i X_{1i}),
$$

$$
K_1 = \sum_i \frac{1}{1+\eta_i^2}K_{Si}, \qquad K_2 = \sum_i \phi\frac{1}{1+\eta_i^2}K_{Si},
$$

with

$$
X_{0j} = \frac{m_j(H_{c2}-H)}{\pi e N_j\{1.16[2\kappa_2^2(t)-1]+1\}} + \frac{m_j(1-H/H_{c2})}{4\pi e^2\tau_j N_j \mathfrak{D}_j\{1.16[2\kappa_1^2(t)-1]+1\}},
\tag{7}
$$

$$
X_{1j} = \frac{H_{c2}-H}{4\pi\{1.16[2\kappa_2^2(t)-1]+1\}} L_{0j}(t),
$$

where the subscript i denotes a particular band. In Eqs. (7),

$$
\eta_i = \tau_i(eH/m_i),
\tag{8}
$$

ϕ is the Hall angle for the Hall effect caused by both bands, K_{Si} is the ith-band contribution to the thermal conductivity in the mixed state, σ_i is the ith-band contribution to the normal state dc conductivity, N_i is the density of state in the ith band, m_i is the effective mass of the ith-band electron, τ_i is the lifetime of the electrons in the ith band, P is the coefficient which appears in the thermoelectric power in the normal state, κ_2 is the second Ginzburg-Landau parameter, and

$$
L_{0i}(t) = 2\pi^{5/2}\tau_i T\int_0^1 \frac{dz}{(1-z^2)^{1/2}}\int_{-\infty}^{\infty}\frac{\alpha^2\,d\alpha}{\epsilon_i^3}\left[\exp\left(-\frac{\alpha^2}{\epsilon_i^2(1-z^2)}\right)\cosh^{-2}\left(\frac{\alpha}{2T}\right)\right]
$$

$$\times\left(\frac{1}{2}\int_0^1 \frac{dz}{(1-z^2)^{1/2}}\int_0^\infty d\zeta\,\zeta^2 \frac{\exp(-\rho^2\zeta^2)}{\sinh[\zeta(1-z^2)^{1/2}]}\right)^{-1}, \tag{9}$$

where

$$\rho=(2\pi T)^{-1}v_F(\tfrac{1}{2}eH_{c2})^{1/2}.$$

The two-band coefficients (7) were obtained by simply summing the contributions of the individual bands to the observables, i.e.,

$$\begin{aligned}\tilde{j}_x^{(e)} &= \tilde{j}_{xs}^{(e)}+\tilde{j}_{xd}^{(e)}\\ &=\sigma_{1s}E_x-\sigma_{2s}E_y+(\alpha_{1s}/T)(\nabla T)_y\\ &\quad+\sigma_{1d}E_x-\sigma_{2d}E_y+(\alpha_{1d}/T)(\nabla T)_y\\ &=(\sigma_{1s}+\sigma_{1d})E_x-(\sigma_{2s}+\sigma_{2d})E_y\\ &\quad+[(\alpha_{1s}+\alpha_{1d})/T](\nabla T)_y,\end{aligned} \tag{10}$$

where we have set $(\nabla T)_x=0$.

Since we will be interested in comparing our two-band expressions for the flux-flow resistivity and the Hall angle with various experimental results, we set

$$j_y^{(e)}=j_y^{(h)}=0, \quad (\nabla T)_x=0, \tag{11}$$

which reduces (4) to

$$\begin{aligned}E_x&=\frac{1}{\mathrm{Det}}\left(\sigma_1K_1-\frac{\alpha_2^2}{T}\right)j_x^{(e)},\\ E_y&=\frac{1}{\mathrm{Det}}\left(\sigma_2K_1+\frac{\alpha_1\alpha_2}{T}\right)j_x^{(e)},\\ (\nabla T)_y&=\frac{1}{\mathrm{Det}}(\sigma_2\alpha_2+\alpha_1\sigma_1)j_x^{(e)},\\ j_x^{(h)}&=-[\alpha_2E_x+\alpha_2E_y-K_2(\nabla T)_y],\end{aligned} \tag{12}$$

with

$$\mathrm{Det}=K_1(\sigma_1^2+\sigma_2^2)+(1/T)[2\sigma_2\alpha_1\alpha_2+\sigma_1(\alpha_1^2-\alpha_2^2)]. \tag{13}$$

III. FLUX-FLOW RESISTIVITY AND HALL ANGLE

Properties such as the flux-flow resistivity and Hall angle, which are related to the motion of the energy gaps, can be obtained for pure transition-metal superconductors near H_{c2} and in a transverse electric field simply by using (7) and (12) together with the definitions of the above properties. As was done by Caroli and Maki[4,5] in the case of one-band superconductors, terms of higher order in Δ^2 will be neglected in order to obtain fairly simple expressions.

A. Flux-Flow Resistivity

When a fluxoid in a pure type II superconductor near H_{c2} is caused to move by a transverse electric field, a dissipative effect is seen. This gives rise to a flux-flow resistance in the superconductor. A macroscopic model based on hydrodynamic considerations was introduced by Bardeen and Stephen[15] to describe the motion of the fluxoid. A different model which gave the same resistance as the Bardeen-Stephen model but a different Hall angle was proposed by Nozières and Vinen.[16] A phenomenological relation has been obtained by Kim *et al.*[17]:

$$\frac{R_f}{R_n}\cong\frac{B}{H_{c2}(0)}, \tag{14}$$

where B is the magnetic induction. A microscopic theory for the flux flow near H_{c2} has been proposed by Caroli and Maki.[4,5] Their approach was a generalization of an earlier theory of Schmid.[18] With the inclusion of the thermal effects on the flux flows, they were able to obtain for the flux-flow resistivity in pure type II superconductors near H_{c2}

$$\frac{R_f}{R_n}=1-\frac{m(H_{c2}-B)}{\pi eN\{1.16[2\kappa_2^2(t)-1]+1\}}, \tag{15}$$

where all the terms have been previously defined. For dirty superconductors near H_{c2}, they found the flux-flow resistivity was given by

$$\frac{R_f}{R_n}=1-\frac{4\kappa_1^2(0)}{1.16[2\kappa_2^2(t)-1]+1}\left(1-\frac{H}{H_{c2}}\right), \tag{16}$$

where κ_1 is the normal Ginzburg-Landau parameter. The reason for presenting the flux-flow resistivity in the dirty limit will be seen in the later discussion of the flux-flow data on niobium.

As we have already said, the flux-flow resistivity in pure transition-metal superconductors near H_{c2} can be obtained from the definition of the flux-flow resistivity,

$$R_f\equiv E_x/j_x. \tag{17}$$

Substituting the definition of E_x [See Eq. (12)] into the definition, we obtain

$$R_f\sim(\sigma_1+\alpha_1^2/TK_1)^{-1}, \tag{18}$$

where only terms of the same magnitude have been kept. Then by substituting the two-band coefficients (7) into the above equation, we obtain for the flux-flow resistivity of pure transition-metal superconductors in high magnetic fields

$$R_f=R_n\left[1-\frac{m_s}{m_s+m_d}\frac{N_s+N_d}{N_s}\frac{m_d(H_{c2}-H)}{\pi eN_d\{1.16[2\kappa_2^2(t)-1]+1\}}-\frac{m_dm_s}{(m_s+m_d)4\pi e^2\tau_sN_s\mathfrak{D}_s\{1.16[2\kappa_2^2(t)-1]+1\}}\left(1-\frac{H}{H_{c2}}\right)\right.$$

$$-\frac{m_s m_d}{(m_s+m_d)4\pi e^2\tau_d N_d \mathfrak{D}_d\{1.16[2\kappa_2^2(t)-1]+1\}}\left(1-\frac{H}{H_{c2}}\right)\Bigg], \tag{19}$$

where the last two terms are of the order ξ_0/l (ξ_0 being the coherence length and l being the mean free path). In extremely pure transition metals, these last two terms vanish.

B. Hall Angle

One of the possible end effects of an external magnetic field at an applied transverse electric field on the superconductor is an electric field inside the superconductor which is perpendicular to both the applied magnetic and electric field. This induced electric field is called the Hall field. The Hall angle is defined as the ratio of the induced electric field and the applied electric field, i.e.,

$$\tan\phi \sim \phi \equiv E_y/E_x\,. \tag{20}$$

In the Bardeen-Stephen model, the Hall angle is given by

$$\tan\phi = \omega_c \tau\,, \tag{21}$$

where $\omega_c = eH/mc$; while in the Nozières-Vinen model, it is given by

$$\tan\phi = \omega_{c2}\tau\,, \tag{22}$$

where $\omega_{c2} = eH_{c2}/mc$. By including the effects of the thermal current in the microscopic theory, Maki[5] obtained

$$\phi = \frac{\tau e B}{mc} + \frac{\pi e T P}{\sigma K} S_D(t), \tag{23}$$

where $S_D(t)$ is the entropy carried by a single vortex and the other terms have been defined in Sec. II. As is seen, the microscopic theory predicts a Hall angle consistent with the Bardeen-Stephen model.

As was done to obtain the flux-flow resistivity in transition-metal superconductors, the Hall angle in a pure two-band superconductor near H_{c2} is obtained by substituting the appropriate equations (12) into the definition (20). Keeping only terms of the same magnitude, we get for the Hall angle in a pure transition-metal superconductor

$$\tan\phi \sim \phi = -\frac{\sigma_2}{\sigma_1} - \frac{\alpha_1\alpha_2}{TK_1\sigma_1} \tag{24}$$

In extremely pure two-band superconductors, the last term in the above expression give rise to a negligible contribution, so that when one substitutes the two-band coefficients (7) into (24), the Hall angle is given by

$$\phi = \left(1-\frac{\sigma_d}{\sigma_s}\,\frac{1+\eta_s^2}{1+\eta_d^2}\,\frac{X_{0d}}{X_{0s}}\right)\eta_s + \frac{\sigma_d}{\sigma_s}\,\frac{1+\eta_s^2}{1+\eta_d^2}\,\frac{X_{0d}}{X_{0s}}\,\eta_d\,, \tag{25}$$

where $\eta_{s(d)}$ and $X_{0s(d)}$ were defined in Sec. II.

IV. COMPARISON WITH EXPERIMENTAL MEASUREMENTS

In attempting to analyze the experimental data on those properties which are related to the moving vortex structure in transition-metal superconductors using the two-band expressions obtained in Sec. III, it is important to realize that it is necessary to categorize the transition-metal superconductors according to their purities. This categorization is important since it has been shown by Garland[19] and Gusman[20] that only for those transition-metal superconductors satisfying the pure limit condition $l \gg \xi_0$ (l being the electronic mean free path and ξ_0 being the coherence length) or the intermediate-purity limit condition $l\sim\xi_0$, will the two-band model be appropriate. In these two limits, there exist two different sets of wave states, one for each band. There is only one energy gap in the intermediate limit, while there are two energy gaps in the pure limit. For transition-metal superconductors in the dirty limit $l \ll \xi_0$, the impurity concentration is sufficient to completely mix the electrons in the two bands together so that the BCS theory holds. It goes without saying that there is only one energy gap in the dirty transition-metal superconductors. Because the calculational technique used in this paper requires that $l\gg\xi_0$, our attention must center only on those transition-metal superconductors falling within the pure limit. A clear indication that it would be wrong to use the two-band expressions of Sec. III to analyze the data on the intermediate-purity superconductors is seen in the field dependence in the thermal conductivity in intermediate-purity niobium superconductors in high magnetic fields.[21] The two-band expressions for the thermal conductivity in pure transition-metal superconductors near H_{c2} predict a $(H_{c2}-H)^{1/2}$ dependence instead of the linear dependence seen by Wasim and Zebouni in their measurements on intermediate-purity niobium superconductors.

A. Flux-Flow Resistivity

Recent measurements of the flux-flow resistivity in pure niobium superconductors having residual resistivity ratios (RRR) of 550 and 620 by Huebener *et al.*[22] have cast some doubt on the microscopic theory of Caroli and Maki[4] (as modified later by Maki).[5] In analyzing their data by looking at the slope

$$\frac{\partial(R_f/R_n)}{\partial(H/H_{c2})}\,,$$

Huebener *et al.* came to the conclusion that the flux-flow resistivity in their samples obeyed the dirty-limit equation (16), i.e., the experimental slope looked more like

$$\frac{H_{c2}}{R_n}\frac{dR_f}{dH}=\frac{4\kappa_1^2(0)}{1.16\,[2\kappa_2^2(t)-1]+1}\,, \tag{26}$$

than the pure-limit slope

$$\frac{H_{c2}}{R_n}\frac{dH_c}{dH}=\frac{m}{Ne\pi}\,\frac{H_{c2}}{1.16[2\kappa_2^2(t)-1]+1}\,. \tag{27}$$

When they substituted numerical values into the pure-limit slope, they obtained for the values of the slope 0.058 at 1.98 K and 0.095 at 4.22 K. The observed slopes for the RRR 620 sample at the two temperatures were 0.8 at 1.98 K and 1.07 at 4.22 K, which are close to the values predicted by the dirty-limit slope.

Since the residual electrical resistivity of the specimens used clearly establishes that the specimens belong to the pure limit category, one must conclude that the microscopic theory of Caroli and Maki is not valid for pure niobium superconductors in high magnetic fields. It is the opinion of the author that the difficulty arises from the neglect of the two-band nature of niobium. Recent tunneling experiments on niobium of less purity (RRR 300) have shown the existence of a second energy gap.[7] Therefore, the flux-flow resistivity should be given by the two-band expression (19). Using the two-band values obtained for a RRR 110 sample [$m_d = 70\,m$,[12] $m_s = 1.9\,m$ (m being the free-electron mass), and $N_s/(N_s+N_d)=0.015$[11]] into the two-band expression

$$\frac{H_{c2}}{R_n}\frac{dR_c}{dH}=\frac{m_s}{m_s+m_d}\,\frac{N_s+N_d}{N_s}\,\frac{m_d}{\pi e N_d}\,\frac{H_{c2}}{1.16[2\kappa_2^2(t)-1]+1}\,, \tag{28}$$

we obtain for the value of the slope 0.73 at 1.98 K and 1.19 at 4.22 K. The behavior of the two-band slope along with the observed values are shown in Fig. 1.

A better fit may occur if the two-band parameters for the samples used by Huebener *et al.* were used instead of the numerical parameters of a RRR 110 niobium superconductor.

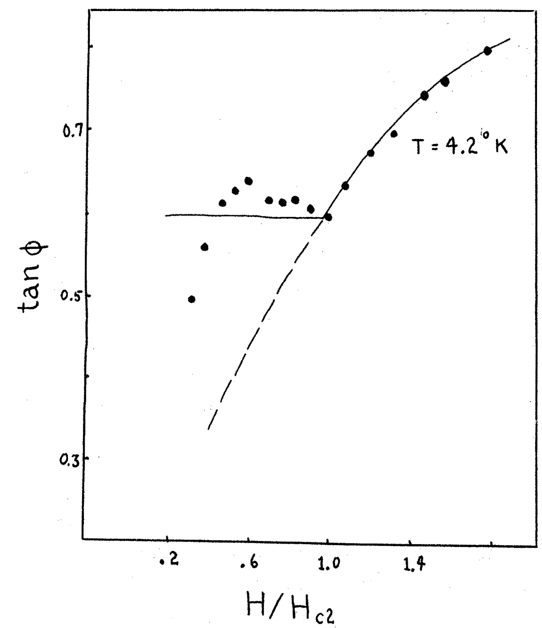

FIG. 2. Hall angle tanϕ vs the reduced field H/H_{c2}. (●) are the experimental points for a RRR 6500 niobium superconductor at 4.2 K as measured by Fiory and Serin. The horizontal line in the middle of the figure would represent the behavior of the Hall angle if the model of Noziéres and Vinen correctly describes the flux flow in pure type II superconductors. The dashed line represents the behavior if the Bardeen-Stephen model describes the flux flow.

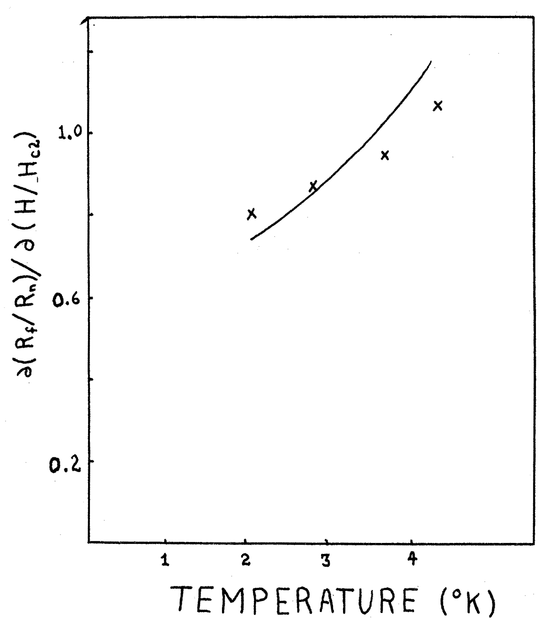

FIG. 1. $\partial(R_f/R_n)/\partial(H/H_{c2})$ vs temperature at fields just below H_{c2}. (×) are the experimental points for a niobium superconductor (RRR 620) as measured by Huebener, Kampwirth, and Seher. Solid curve represents the behavior of the two-band flux-flow resistivity slope when the numerical parameters for a RRR 110 niobium superconductor are used in the two-band flux-flow expression.

B. Hall Angle

For extremely pure type II superconductors, the second term in angle (23) obtained by Maki vanishes so that his results are the same as that in the Bardeen-Stephen (BS) model. In the BS model, the Hall angle is the same for both the normal and superconducting states of the metal. Therefore one would not expect the slope of the Hall angle versus the magnetic field to change at H_{c2}.

However, recent measurements of the Hall angle in niobium samples with RRR 6500[23] give support to the model of Nozières and Vinen, which predicts a constant Hall angle below the transition point in Fig. 2, i.e.,

$$\tan\phi = \tau e H_{c2}/mc = \text{const.} \tag{29}$$

Fiory and Serin[23] interpreted the experimental points as lying on a straight line (a constant Hall angle) and that the few points at the lower fields were spurious. If, however, we interpret the spurious points as the beginning of a new field dependence in the Hall angle, the two-band expression for the Hall angle can explain qualitatively the observed behavior.

Above H_{c2}, both X_{0s} and X_{0d} vanish so that the Hall angle is just equal to η_s. Thus the linear field dependence of the Hall angle above H_{c2} is a natural consequence of our two-band expression. Since the X_0's become finite below H_{c2}, the slope of the field dependence of the Hall angle changes at the critical point in agreement with the observed behavior. (The Hall angle obtained by Maki cannot explain the change in the slope at H_{c2}.)

APPENDIX A: PHYSICALLY OBSERVED ELECTRIC CURRENT

As was indicated in Sec. II, the physically observed electric current in a pure two-band superconductor containing moving energy gaps is obtained by substituting the two-band electric-current operator

$$j^{(e)}(r,t) = \sum_i j_i^{(e)}(r,t) = -\sum_{i,\sigma} [\nabla - \nabla' - 2ie\,A(r)]\, \frac{ie\,\psi^\dagger_{i\sigma}(r't)\psi_{i\sigma}(r,t)}{2m_i}\bigg|_{r=r'} \tag{A1}$$

into (5) to get

$$\tilde{j}^{(e)}(r,t) = \textstyle\sum_i \tilde{j}_i^{(e)}(r,t), \tag{A2}$$

where the individual band's contribution $j_i^{(e)}$, the physically observed current, is of the form of the one-band observable found in Refs. 4 and 5. Because the energy gaps in the two-band superconductor are of the same form as the order parameter in the one-band superconductor, the evaluations of the $j^{(e)}$'s will follow exactly the procedures in the two references cited.

By adding together the explicit expressions for $j_i^{(e)}$, the physically observed electric current in pure transition-metal superconductors near H_{c2} is obtained; i.e.,

$$j_x^{(e)} = \left(\frac{\sigma_s}{1+\eta_s^2} + \frac{\sigma_d}{1+\eta_d^2}\right) E + eN_s\left(\frac{V_{sF}}{2\pi T}\right)^2 \Delta_s^2 g(\rho_s)\left(\frac{u}{\mathfrak{D}_s} + \frac{4\tau_s eE}{1+\eta_s^2}\right) + eN_d\left(\frac{V_{dF}}{2\pi T}\right)^2 \Delta_d^2\, g(\rho_d)\left(\frac{u}{\mathfrak{D}_d} + \frac{4\tau_d eE}{1+\eta_d^2}\right), \tag{A3}$$

$$j_y^{(e)} = -\phi\left(\frac{\sigma_s}{1+\eta_s^2} + \frac{\sigma_d}{1+\eta_d^2}\right) E - eN_s\left(\frac{V_s F}{2\pi T}\right)^2 \Delta_s^2\, g(\rho_s)\, \frac{\eta_s}{1+\eta_s}\, 4\tau_s eE - e\,N_d\left(\frac{V_d F}{2\pi T}\right)^2 \Delta_d^2 g(\rho_d)\, \frac{\eta_d}{1+\eta_d}\, 4\tau_d\, eE, \tag{A4}$$

where

$$g(\rho_i) = \frac{1}{2}\int_0^1 \frac{dz}{(1-z^2)^{1/2}} \int d\zeta\, \zeta^2\, \frac{\exp(-\rho_i\zeta^2)}{\sinh[\zeta(1-z^2)^{-1/2}]}\,. \tag{A5}$$

The first two terms in both $j_x^{(e)}$ and $j_y^{(e)}$ are the contributions of the electric current in the normal state. The use of the Hall angle in these terms ensures that the electric current in the y direction is the result of the full Hall effect.

APPENDIX B: PHYSICALLY OBSERVED HEAT CURRENT

The physically observed heat current in a pure two-band superconductor containing moving energy gaps is obtained by substituting the two-band heat operator[24]

$$j^{(h)} = \sum_i j_i^{(h)} = -\tfrac{1}{2}\sum_i\sum_\sigma \left[[\nabla - 2ieA(r)]\left(\frac{\partial}{\partial t'} - 2ie\phi\right) + [\nabla' + 2ieA(r')]\left(\frac{\partial}{\partial t} - 2ie\phi\right)\right] \frac{\chi^\dagger_{i\sigma}(r',t)\chi_{i\sigma}(r,t)}{m_i}\bigg|_{r=r'} \tag{B1}$$

into (5). As was the case with the electric current, the physically observed two-band heat current is just the sum of the physically observed heat currents in the individual bands. Using the same calculational techniques found in Ref. 5, we obtain

$$\tilde{j}_x^{(h)} = \sum_i \left(\frac{1}{1+\eta_i^2}\, TPE + M_i L_{0i}\, \frac{\eta_i}{1+\eta_i}\, E\right), \tag{B2}$$

$$\tilde{j}_y^{(h)} = -\sum_i \left(\frac{\phi}{1+\eta_i^2}\, TPE + M_i L_{pi}(t)E + \frac{M_i L_{0i}}{1+\eta_i^2}\, E\right), \tag{B3}$$

where M_i, L_{0i}, and L_{pi} are defined in Ref. 5. Since L_{0i} is much larger than L_{pi}, the terms containing L_{pi} can be neglected so that we get

$$\alpha_1 = \sum_i \frac{1}{1+\eta_i^2}\,(TP - \eta_i X_{1i}),$$

$$\alpha_2 = -\sum_i \frac{1}{1+\eta_i^2}\,(\phi\, TP + X_{1i}). \tag{B4}$$

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[1]I. M. Tang, Phys. Rev. B 2, 3592 (1970).

[2]Interaction Hamiltonian is written in the mean field approximation of the Suhl-Matthias-Walker Hamiltonian:

$$\mathcal{H} = \sum_\sigma \int \left[(2m_s)^{-1} \nabla\psi^\dagger_{s\sigma} \nabla\psi^\dagger_{s\sigma} + \frac{1}{2m_d}\nabla\psi^\dagger_{d\sigma}\nabla\psi_{d\sigma}\right] d^3r + c\sum_\sigma\int(\psi^\dagger_{s\sigma}\psi_{s\sigma}+\psi^\dagger_{d\sigma}\psi_{d\sigma})\,\phi\, d^3r - g_s\int \psi^\dagger_{s\uparrow}\psi^\dagger_{s\downarrow}\psi_{s\downarrow}\psi_{s\uparrow}\, d^3r - g_d \int \psi^\dagger_{d\uparrow}\psi^\dagger_{d\downarrow}\psi_{d\downarrow}\psi_{d\uparrow}\, d^3r - g_{sd}\int\{\psi^\dagger_{s\uparrow}\psi^\dagger_{s\downarrow}\psi_{d\downarrow}\psi_{d\uparrow} + \psi^\dagger_{d\uparrow}\psi^\dagger_{d\downarrow}\psi_{s\downarrow}\psi_{s\uparrow}\}\, d^3r.$$

[3]See, for instance, Y. B. Kim and M. T. Stephen, in *Treatise on Superconductivity*, edited by R. D. Parks (Dekker, New York, 1969).

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[24]It can be easily shown that the two-band heat-current operator satisfies the heat equation $\partial/\partial t\, h(r) + \nabla\cdot\vec{j}^{(h)}(r) = 0$, where $h(r)$ is the Hamiltonian density in the SMW model.

PHYSICAL REVIEW B VOLUME 3, NUMBER 7 1 APRIL 1971

Tunneling into Weakly Coupled Films of Aluminum and Tin in Proximity*

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(Received 24 August 1970)

Tunneling experiments have been performed into the N side of N-S (aluminum-tin) proximity sandwiches evaporated at room temperature onto an oxidized aluminum electrode B. The coupling between the N and S films was made weak by allowing slight oxidation to occur at the interface. When B is normal, the normalized tunneling conductance of these junctions in the vicinity of the critical temperature of the proximity sandwich is markedly different from that of junctions formed between B and an ordinary (BCS) superconductor. When B is superconducting and the thickness of the N film is made about $d_N \simeq 100$ Å $\simeq \frac{1}{10} d_S$, where d_S is the thickness of the S film, a double-peaked structure is observed in the tunnel conductance as a function of applied voltage. The properties of the proximity sandwich depend on the amount of oxidation at the N-S interface. Self-consistent calculations have been performed using the McMillan model of proximity sandwiches and treating the barrier transmission as a parameter. Comparison of these calculations with the experimental results shows satisfactory quantitative agreement.

INTRODUCTION

Recently a simple theoretical model of the proximity effect between superposed normal (N) and superconducting (S) metal films has been proposed by McMillan[1] and calculations of the transition temperature, energy gap, and electronic density of states were made for comparison with the results of tunneling experiments. He treats, by second-order self-consistent perturbation theory, a model in which thin metal films are coupled by tunneling through a barrier at the interface. Experiments by Adkins and Kington[2] and by Freake and Adkins[3] showed reasonable agreement with general features of the theory, which is as much as could be expected, since their films were rather strongly coupled and the theory only applies strictly to weak coupling between the two films.

The coupling may be weakened by allowing a very thin oxide layer with an electron transmission prob-