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Absence of Quantum-Electrodynamic Corrections to the Charge of the Electron as Measured in Josephson Junction Experiments

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The ac Josephson effect enables one to measure the ratio $\Delta\mu/h$, where $\Delta\mu$ is the electrochemical potential difference between two weakly coupled superconductors. Combined with an independent measurement of the electrostatic potential difference V, this measurement determines the ratio e/h. This determination rests on the fact that a voltage difference is defined so that the electrochemical potential difference is $\Delta\mu = eV$, where e is the charge on the free electron. Further, while we argue that it is not directly relevant to the determination of e/h, there are no quantum-electrodynamic corrections to the electrostatic term in the chemical potential difference $\Delta\mu$.

The recent improvement in experimental accuracy[1–3] of the measurements of e/h through the ac Josephson effect lends new importance to efforts to determine the theoretical accuracy of this technique. Recently, for example, Nordtvedt[4] has argued that quantum-electrodynamic corrections will result in an e/h ratio from the Josephson measurements which depends upon the metal used in the junction and differs typically by parts in 10^{-9} from the same ratio for a free electron. We would like to argue here that in fact there are no such quantum-electrodynamic corrections to be expected. In particular, Nordtvedt's results (i) are not directly relevant to the Josephson measurement, (ii) do not in fact change the electrostatic part of the electrochemical potential, and (iii) represent a well-known difference between the $\vec{q} \to 0$ and the $\vec{q} = 0$ limit of the dielectric properties of a many-body system.

The present theories of superconductivity[5] predict that for a superconductor in equilibrium, the time rate of change of the gauge-invariant phase of the order parameter and the electrochemical potential[6] are related by

$$\frac{\partial\varphi}{\partial t} = -\frac{2\mu}{\hbar} . \tag{1}$$

This implies that if the relative phase between two regions of a superconductor is stationary, the two regions have the same electrochemical potential. Furthermore, if the relative phase changes, then the difference in electrochemical potential is set by (1). Ultimately it is upon the validity of Eq. (1) that the recent solid-state determinations of e/h rest. Briefly,[2] two superconductors are joined by a weak link such as a tunnel junction or thin bridge. Then the rate of change of the relative phase difference across this weak link is synchronized at a multiple $n\omega$ of the frequency ω of an applied microwave field. According to (1) this sets a difference in the electrochemical potential $\Delta\mu$ between the two superconductors which is faithfully propagated by the superconductors to the normal leads. Then a null potentiometer measurement compares this frequency-induced $\Delta\mu$ with the electrochemical potential of a standard cell which is calibrated in absolute volts times the free-electron charge.[7] Thus, the electrochemical potential and not some effective interaction charge of an electron in a metal is what enters the Josephson relation (1) and the measurement. From this it is clear that if quantum electrodynamics is to give corrections to the measurement they must modify Eq. (1).

It might appear that Nordtvedt's prediction of a shift in the charge of an electron in a metal would modify the electrostatic part of the chemical potential difference. Even if this were true, we would argue that it would not affect the determination of the e/h ratio because, as we have discussed, and as has been emphasized in Ref. 2, the Josephson condition establishes a relationship between frequency and chemical potential difference. This relationship does not involve e, which only enters when the chemical potential difference across the Josephson junction is compared to that of a standard cell. e enters here because the standard cell is calibrated so that $\Delta\mu = eV$ at zero current.

A simple model of the Josephson effect[8] illustrates that there are no quantum-electrodynamic corrections to the charge which enters into the electrostatic part of the chemical potential. The important assumption of this model for our considerations is that in the absence of external electromagnetic fields the stationary state Ψ of two weakly coupled superconductors may be written as a superposition of degenerate states ψ_n in which n pairs have moved from superconductor 1 to superconductor 2:

$$\Psi = \sum_n c(n)\psi_n \; . \tag{2}$$

In terms of the ground state $\psi_0(N)$ of a superconductor with a definite number, of electrons, N, the ψ_n are

$$\psi_n = \psi_0(N_1 - 2n)\,\psi_0(N_2 + 2n) \; . \tag{3}$$

If the superconductors are now maintained at a potential difference V, the time evolution of Ψ will be

$$\Psi(t) = e^{-i(E_1+E_2)t/\hbar} \sum_n c(n)\, e^{-i(Q_1-Q_2)Vt/\hbar}\, \psi_n \; , \tag{4}$$

where Q_1 is the total charge on the first superconductor, E_1 is its energy in the absence of any electromagnetic potentials, and Q_2 and E_2 are similarly defined. If we assume that the charges N_1 and N_2 are neutralized, and write the charge on each pair transferred, we have

$$\Psi(t) = e^{-i(E_1+E_2)t/\hbar} \sum_n e^{-i(2eVt/\hbar)n}\, c(n)\,\psi_n \; . \tag{5}$$

We may then conclude that in this model[9] the expectation value of any time-independent operator such as the tunneling current is a periodic function of t with period $\hbar/2eV$. The important point is that it is the total charge on one or the other superconductor which is an integral multiple of e. In quantum electrodynamics the total charge on any system is an integral multiple of the renormalized charge of the free electron. There are no small renormalization corrections. We would now like to show how one can arrive at this result through the familiar methods of Feynman graphs.[10]

It is enough to show that acting on any complete set of states of protons and electrons the total charge operator Q operating on each one gives back an integer times e times the state. We choose for our complete set of states the in states of scattering theory. Expanding them in a complete set of out states we then need to show

$$\langle j\,\mathrm{out}\,|\,Q\,|\,i\,\mathrm{in}\rangle = q\langle j\,\mathrm{out}\,|\,i\,\mathrm{in}\rangle \; , \tag{6}$$

where q is the sum of the renormalized charges of each particle in the in (or out) state. The quantum numbers of the in and out states are denoted by i and j. Posed in this way the necessary theorem is directly related to the Feynman diagrams of quantum electrodynamics.

To make the relation explicit just write

$$\langle j\,\mathrm{out}\,|\,Q\,|\,i\,\mathrm{in}\rangle = \int d^3x \langle j\,\mathrm{out}\,|\,J^t(\vec{x}, t)\,|\,i\,\mathrm{in}\rangle \; , \tag{7}$$

where $J^\mu(\vec{x}, t)$ is the electromagnetic current. Then note that the amplitude for a photon of momentum k^μ and polarization ϵ^μ to be emitted during the scattering from i to j is $\epsilon_\mu M^\mu$ where[11]

$$M^\mu(k) = -i \int d^4y\, e^{iky} \langle i\,\mathrm{out}\,|\,J^\mu(y)\,|\,j\,\mathrm{in}\rangle\, N_i N_j \; . \tag{8}$$

Here, N_i and N_j are normalization factors connecting S-matrix elements to Feynman amplitudes. One can then find the following expression for the matrix element of Q:

$$\langle j\,\mathrm{out}\,|\,Q\,|\,i\,\mathrm{in}\rangle = \frac{i}{2\pi} \int_{-\infty}^{+\infty} d\omega\, e^{-i\omega t} M^t(\omega, \vec{\mathrm{k}} = 0)(N_i N_j)^{-1} \; , \tag{9}$$

with $k^\mu = (\omega, \vec{\mathrm{k}})$.

Gauge invariance tells us that $k_\mu M^\mu = 0$. In our case of zero three-momentum this becomes $\omega M^t = 0$. Thus $M^t(\omega, \vec{\mathrm{k}} = 0)$ has the general form[12]

$$M^t(\omega, \vec{\mathrm{k}} = 0) = a\delta(\omega) \; , \tag{10}$$

so that

$$\langle j\,\mathrm{out}\,|\,Q\,|\,i\,\mathrm{in}\rangle = ia/2\pi N_i N_j \; . \tag{11}$$

We have only to evaluate a by looking at $M^t(\omega, \vec{\mathrm{k}} = 0)$ for small ω. The Feynman graphs which contribute

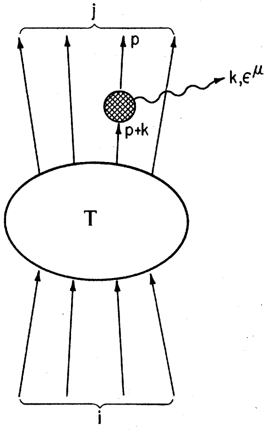

FIG. 1. Typical Feynman graph contributing to $M^\mu(k)$ in which a photon of momentum k^μ is being emitted from an external leg of the amplitude T for the process $i \rightarrow j$ but with the spinor left off.

to M^μ may be divided into those for which the photon is attached to an external line of the process $i \to j$ and those where it is attached to an internal line. The latter class of graphs are always nonsingular as $k^\mu \to 0$. A typical graph of the first type is shown in Fig. 1. If we write T for the sum of Feynman graphs contributing to the process $i \to j$ but with no spinor $\bar{u}(p)$ on the external leg in question, M^μ is

$$M^\mu(k) = \bar{u}(p)\, \Gamma^\mu(p, p+k)\, [i\gamma(p+k) - m + i\epsilon]^{-1}\, T \; . \tag{12}$$

The function $\Gamma^\mu(p, p+k)$ is the full vertex function and satisfies

$$\Gamma^\mu(p, p) = e\gamma^\mu \; , \tag{13}$$

where e is the renormalized charge of the free electron. Writing out the most general form for Γ^μ it is not difficult to see that the only singular contribution for small ω of $M^t(\omega, \vec{k}=0)$ is

$$M^t(\omega, \vec{k}=0) = \frac{ie}{\omega + i\epsilon}\, \bar{u}(p) T = \frac{ie}{\omega + i\epsilon} \times (\text{amplitude for } i \to j) \; . \tag{14}$$

If the photon were attached to an incoming line the result would have been similar except that the denominator would be $-\omega + i\epsilon$.

Now, every charged line can be traced through a Feynman graph. If the line goes from the initial to the final state the net contribution from the photons coupled to its external parts is

$$2\pi e \delta(\omega) \times (\text{amplitude for } i \to j) \; . \tag{15}$$

If the line enters the initial state and emerges there the net contribution is zero. Similarly for the final state. Using Eq. (15) to evaluate a and inserting in Eq. (11) we arrive at our result, Eq. (6). Thus there are no quantum-electrodynamic corrections to the charge of the free electron as it enters into the electrostatic part of the electrochemical potential in the ac Josephson effect.

What Nordtvedt calculates is the effect on the vacuum polarization of the presence of a degenerate sea of electrons. This correction to the photon propagator is relevant to the discussion of the interaction of two electrons inside the metal. Indeed, if such an interaction were partially described in terms of an effective charge, a calculation of this charge would have to take into account the type of correction he describes. Put differently, what is relevant for the Josephson effect is the charge on the superconductor which describes the interaction of the whole metal with an exterior electron. This, we have argued is an integral multiple of the renormalized charge of the free electron. It is clear, however, that corrections of the type Nordtvedt describes cannot sum up to give a correction to the metals total charge. Such a correction would be proportional to the Fermi momentum of the electron sea and alterable by compressing the metal. It is not possible to alter an object's total charge by compression.

This difference between the effective charge characteristic of the static long-wavelength field induced by an electron in a many-body medium and the total charge of the system with an additional electron is familiar in solid-state physics.[13] The formal distinction between these two quantities is whether one sets $\vec{k}$ equal to zero at the beginning or end of the calculation and physically corresponds to being inside or outside the solid. In the calculation discussed here $\vec{k}$ has been set equal to zero initially while the calculation of Ref. 4 corresponds to taking this limit at the end. Langenberg and Schrieffer have discussed this same effect in terms of an induced surface charge.[14]

In conclusion, we would remark that departures from equilibrium can modify Eq. (1) locally.[15] Furthermore, even in equilibrium, fluctuations about the mean values described by Eq. (1) will occur. However, it would seem very unlikely that quantum-electrodynamic effects play an important role in such phenomena. To the extent that Eq. (1) holds the Josephson effect indeed measures e/h.

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[6] The electrochemical potential can be defined in a number of equivalent ways, for example,

$$\mu = \left.\frac{\partial F}{\partial N}\right|_{T,V} = \left.\frac{\partial U}{\partial N}\right|_{S,V} ,$$

where F is the free energy, U is the energy, and N is the number of electrons.

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Comments on Effects of Electron-Electron Interactions on Nuclear Spin-Lattice Relaxation Times in Aluminum†

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The results of measurements of ^{27}Al spin-lattice relaxation and local field in the rotating frame in aluminum are inconsistent with the model presented by Tunstall and Brown.

Recently Tunstall and Brown[1] have reported the results of nuclear dipolar relaxation in aluminum for $1.3 < T < 295$ °K. They interpreted their results using a three-bath model and an analysis extending the coupled rate equations of Schumacher.[2] For their sample, filed from a "high-purity zone-refined aluminum single crystal," they found the ratio δ of the dipolar spin-lattice relaxation rate to the Zeeman spin-lattice relaxation rate to be 2.15 ± 0.07 independent of temperature. The quantity $\delta - 2.0$ is a measure of the effect of electron correlations. Their analysis relied on an estimate of the relative heat capacities of the dipolar, small quadrupolar, and large quadrupolar systems to be in the ratio of $2:1:5$. They pointed out that at low temperatures, where the spin-lattice relaxation times are long, all systems contribute to the measured relaxation rate. At high temperatures, where spin-lattice relaxation times are short, cross relaxation is ineffective at coupling the quadrupolar systems to the dipolar system and thus the quadrupolar systems do not contribute to the measured relaxation rate.

We have measured the spin-lattice relaxation time in the rotating frame $T_{1\rho}$ in nominally 99.9999% pure aluminum at 300 °K. Some of our results have been reported in earlier publications,[3,4] where details of the experimental procedure can be found. The spin system is prepared in the rotating frame using the spin-locking pulse sequence.[5] At exact resonance the initial quasiequilibrium ($t \ll T_{1\rho}$) magnetization in the rotating reference frame is given by[4]

$$M = M_0[H_1^2/(H_1^2 + H_D^2 + H_Q^2)], \qquad (1)$$

where M_0 is the equilibrium magnetization in the laboratory frame, H_1 is the amplitude of the rf field, H_D^2 is the square of the dipolar field in the rotating frame (equal to one-third[6] of the Van Vleck second moment), and H_Q^2 is the square of the effective quad-

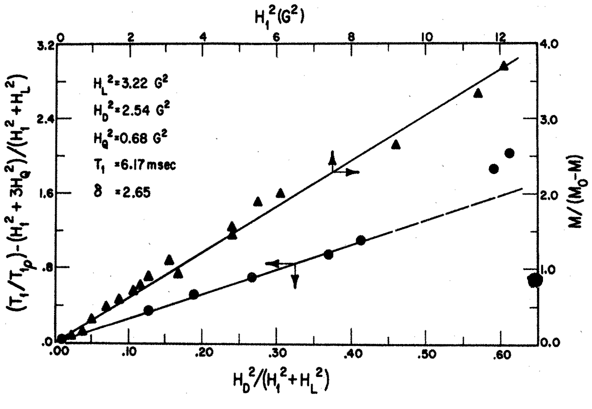

FIG. 1. Experimental determination of H_L^2 and δ for nominally 99.9999% purity aluminum at 300 °K. The triangles are for the data $M/(M_0 - M)$ vs H_1^2 and the circles are for the data $(T_1/T_{1\rho}) - (H_1^2 + 3H_Q^2)/(H_1^2 + H_L^2)$ vs $H_D^2/(H_1^2 + H_L^2)$.

the apparent values of δ found in a number of experiments on aluminum are considerably greater than 2.[10]