*Work supported by the U. S. Atomic Energy Commission.

†Present address: Department of Physics, University of Toronto, Toronto, Ontario, Canada.

[1]S. C. Moss, J. Appl. Phys. 35, 3547 (1964).

[2]R. A. Tahir-Kheli, Phys. Rev. 169, 517 (1969).

[3]K. P. Wang and H. Amar, Phys. Rev. B 1, 582 (1970); see also J. B. Gibson [Phys. Chem. Solids 1, 27 (1956)] for a different approach.

[4]J. M. Ziman, *Electrons and Phonons* (Oxford U. P., New York, 1963).

[5]C. C. Bradley, T. E. Faber, E. G. Wilson, and J. M. Ziman, Phil. Mag. 1, 865 (1962).

[6]I. Giaever, Ph. D. thesis, Rensselaer Polytechnic Institute, 1964 (unpublished); see also W. A. Harrison, *Pseudopotentials in the Theory of Metals* (Benjamin, New York, 1966).

[7]A. O. E. Animalu (private communication).

[8]G. Airoldi, M. Asdente, and E. Rimini, Phil. Mag. 10, 43 (1964).

PHYSICAL REVIEW B VOLUME 3, NUMBER 4 15 FEBRUARY 1971

On High-Electric-Field Conductivity in n-Type Silicon†

Michele Costato and Lino Reggiani
Istituto di Fisica dell'Università di Modena, Modena, Italy
(Received 1 July 1970)

The theoretical results of Basu and Nag on the high-field mobility of electrons in Si for any arbitrary direction of field are discussed, taking into account the intervalley electron-phonon scattering mechanisms. An up-to-date picture of these scattering mechanisms is outlined and shown to fit satisfactorily the electron drift velocity between 77 and 300 °K, from the Ohmic up to the saturation region. Nag, Paria, and Basu's (NPB) ratio of cool-to-hot valley population for electric fields along the ⟨100⟩ direction is also successfully interpreted.

In a recent paper, Basu and Nag[1] (hereafter referred as BN) have derived theoretical expressions for the high-electric-field mobility of carriers in n-type Si, for any arbitrary direction of field. They have also criticized the lattice-scattering picture used by Costato and Scavo (CS).[2,3]

We wish to show here that excellent agreement between experimental and theoretical findings on high-field electron drift velocity in Si can be obtained making use of Dumke's[4] model in place of Long's[5] model for intervalley scattering mechanisms, that the theoretical expressions of BN are inconsistent with the intervalley scattering mechanisms they assumed, and that BN erroneously interpreted the approach of CS.

The theoretical approach of BN can be summarized as follows: They assumed a carrier concentration for which the symmetric part f_0 of the distribution function is Maxwellian. Making use of the intervalley phonon energies of Long,[5] and taking into account lattice scattering only, they obtained results on hot-carrier mobility not in good agreement with the experimental findings of Nag, Paria, and Basu[6] (NPB), and found the ratio of cool-to-hot valley population (n_c/n_h) in sharp disagreement with the values obtained from the analysis of experimental results of NPB. BN believe their discrepancies arose because of a large effect of intervalley scattering in the energy and momentum exchange processes.

We would like to comment on the theoretical procedure used by BN and to show how results of NPB on n_c/n_h may be satisfactorily interpreted by means of our theoretical approach.[7,8]

BN used for the intervalley electron-phonon scattering mechanisms the phonon energies assumed by Long,[5] i.e., the f-type and g-type scattering mechanisms are caused by interaction with phonons with an equivalent temperature of $\theta_f = 630$ °K, and of $\theta_g = 190$ °K, respectively. Long[5] pointed out that these energies were obtained through an average and were not necessarily to be considered as "true values for real phonons."[5] The ratio of f-type to g-type coupling constants was taken by BN to be $W_1/W_2 = 3.3$, thus, with an enhanced predominance of f-type over g-type scattering. To solve the problem for the asymmetric part f_1 of the distribution function for any arbitrary direction of the applied electric field, they used the transformation of Herring Vogt.[9] However, let us point out that the above transformation cannot be applied to many-valley semiconductors, if it is not possible to treat the valleys as independent of each other, as was inferred by Asche and Sarbei.[10] Consequently, since BN assumed f-type scattering to predominate over g-type scattering, their procedure in using Herring and Vogt's[9] transformation is inconsistent. However, the results of BN can be thought of as an approximation to the problem, since it may be assumed that "the collision terms are given in the same form as if there would be spherical surfaces of constant energy, and in the

field term the nonsphericity is taken into account, taking different effective masses for different orientations of the field by using $\vec{E}\alpha^{(i)}\vec{E}$, with $\alpha^{(i)}$ as a tensor of the reciprocal effective mass in the ith valley instead of E^2/m^*, in the case of spherical energy surfaces."[10]

On the other hand, we have developed a theoretical approach[7,11] quite different from that of BN. Our approach[7] explains satisfactorily the drift velocity of electrons in pure Si for a large range of electric fields (10^0–10^5 V/cm) running from the Ohmic region through the longitudinal anisotropy of the ⟨111⟩, ⟨110⟩, and ⟨100⟩ crystallographic directions in warm and hot regions, up to the saturation region in the temperature range between 77 and 300 °K.[7] See Fig. 1 and Fig. 2. Let us emphasize that as an ancillary result, the ratio of cool-to-hot valley population n_c/n_h we have obtained is in good agreement with the results of NPB as represented in Fig. 3.

Our theoretical approach can be summarized as follows. (i) g-type intervalley scattering predominates over f type[15,16] ($W_1/W_2 = 0.38$), where the former is caused by interaction with an LA phonon[4,17] with an equivalent temperature of $\theta_{g\mathrm{LA}} = 307$ °K.[16,18] The latter is caused by interaction with a mixture of LA + TO phonons[17] with $\theta_{f\mathrm{LA}} = 540$ °K,[16,18] and $\theta_{f\mathrm{TO}} = 680$ °K.[18,19] This enables us to decouple the many-valley structure of n-type Si and treat every valley independently. (ii) The electron-distribution function is taken to be of displaced Maxwell-Boltzmann type[20–23] and used in diffusion approximation. (iii) The problem is solved through the energy- and momentum-balance conditions for lattice scattering only. Making use of acoustic intravalley scattering mechanism in the "energy equipartition" and "zero-point scattering" approximations in their respective range of validity,[24] the equations for any valley (j) transformed into spherical shape are thus obtained in the starred space

$$(E^* v_d^*)_{\mathrm{ac\,eq}} = (m_\perp^2 m_{\parallel n})^{1/2}\{cE_1^2[1-(T/T_e)]+\Phi(T)\}(K_B T_e)^{3/2}\,, \tag{1}$$

$$(E^*/v_d^*)_{\mathrm{ac\,eq}} = (m_\perp^2 m_{\parallel n})^{1/2}[aE_1^2 T+\Theta(T)](K_B T_e)^{1/2}\,, \tag{2}$$

$$(E^* v_d^*)_{\mathrm{ac\,zp}} = (m_\perp^2 m_{\parallel n})^{1/2}[cE_1^2+\Phi(T)](k_B T_e)^{3/2}, \tag{1'}$$

$$(E^*/v_d^*)_{\mathrm{ac\,zp}} = (m_\perp^2 m_{\parallel n})^{1/2}[lE_1^2 T_e^{1/2}+\Theta(T)](K_B T_e)^{1/2}\,, \tag{2'}$$

where

$$m_\perp^{-1} = m_{\perp n}^{-1} - \frac{K_B T_e}{\Delta(T)}\left(\frac{2}{2m_{\parallel n}+m_{\perp n}}+\frac{4m_{\parallel n}}{m_{\perp n}(2m_{\parallel n}+m_{\perp n})}\right)\,, \tag{3}$$

$$\Phi(T) = d\frac{(DK)_g^2}{\theta_g}[N_g-(N_g+1)e^{-r_g}][R(r_g)-e^{-r_g}R(-r_g)]+d\sum_{(\mathrm{LA,TO})_f}\frac{(DK)_f^2}{\theta_f}[N_f-(N_f+1)e^{-r_f}][R(r_f)-e^{r_f}R(-r_f)]\,, \tag{4}$$

$$\Theta(T) = b\frac{(DK)_g^2}{\theta_g}[N_g R(r_g)+(N_g+1)R(-r_g)]+b\sum_{(\mathrm{LA,TO})_f}\frac{(DK)_f^2}{\theta_f}[N_f R(r_f)+(N_f+1)R(-r_f)], \tag{5}$$

$$N_{f,g} = (e^{\theta_{f,g}/T}-1)^{-1}, \tag{6}$$

$$R(\pm r_{f,g}) = \int_0^\infty e^{-\chi}\chi^{3/2}(\chi\pm r_{f,g})^{1/2}\,d\chi; \tag{7}$$

$$r_{f,g} = \frac{\theta_{f,g}}{T_e}\,;\quad (DK)_i\frac{\hbar s}{K_B\theta_i}\equiv W_i\,,$$

with

$$a = \frac{2^{9/2}m_0 K_B}{27\pi^{3/2}\hbar^4 e\rho s^2}\,;\quad b = \frac{2^{3/2}m_0}{3\pi^{3/2}\hbar^2 K_B e\rho}\,;\quad c = \frac{2^{9/2}m_0}{3\pi^{3/2}\hbar^4 e\rho}\,;$$

$$d = \frac{2^{1/2}}{\pi^{3/2}\hbar^2 e\rho K_B}\,;\quad l = \frac{2^3 m_0 m_\perp^{1/2}K_B^{1/2}}{9\pi\hbar^4 e\rho s}\,,$$

where the subscripts (ac eq) and (ac zp) refer to the "energy equipartition" and "zero-point scattering" approximations, respectively. Subscripts g and f refer to g-type and f-type scattering mechanisms, and $\theta_{g,f}$ are the characteristic equivalent temperatures of intervalley phonons. E_1 is the deformation potential constant (in spherical approximation) of the electron intravalley acoustic-phonon scattering mechanism. $(DK)_{g,f}$ are the coupling constants of

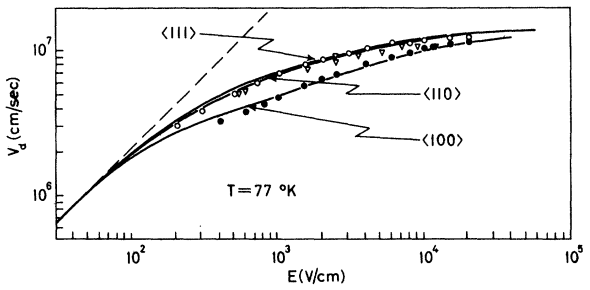

FIG. 1. Electron drift velocity v_d vs applied electric field E at 77 °K for different crystallographic directions in Si. Experimental values of Canali *et al.* (Ref. 12): ○ for $E \parallel \langle 111 \rangle$, ▽ for $E \parallel \langle 110 \rangle$, ● for $E \parallel \langle 100 \rangle$.

the electron intervalley scattering mechanisms of g type and f type, according to the subscript. 2Δ is the energy separation between the absolute minimum of the conduction band and the valence band in a vertical transition (its temperature dependence has been taken to be that of the energy gap). $m_{\parallel n}$ and $m_{\perp n}$ are the longitudinal and transverse components of the electron effective-mass tensor at the bottom of the conduction band, ρ the crystal density, s the sound velocity, and e the unit charge. According to the experimental evidence,[25] only $m_\perp$ has been taken to be energy dependent in Eq. (3). The summation $\sum_{(\mathrm{LA,TO})_f}$ is extended to f-type LA and TO phonons considered here as participating in independent scattering mechanisms characterized by the same coupling constant (see Table I).

Through a retransformation in normal space with the condition that the electric field E is the same for every valley, the electron temperature $T_e^{(j)}$ for each valley (j) is obtained. From the carrier-density normalization condition, the ratio of the population between hot and cold valleys n_h/n_c is obtained:

$$\frac{n_h}{n_c} = \left(\frac{T_{ec}}{T_{eh}}\right)^{1/2} \frac{e^{\theta_f/T} e^{-\alpha_c} + 1}{e^{\theta_f/T} e^{-\alpha_h} + 1} \frac{L(\alpha_c)}{L(\alpha_h)} \quad , \tag{8}$$

where

$$L(\alpha_i) = \tfrac{1}{2}\alpha_i e^{\alpha_i/2} K_1(\tfrac{1}{2}\alpha_i); \quad \alpha_i = \frac{\theta_f}{T_{ei}}; \quad i \equiv h, c.$$

K_1 is the modified Bessel function of order 1, where only f-type phonons of 47.4 meV (~ 540 °K) have been considered in the numerical calculations. By solving the two-equation system (1) and (2) or (1′) and (2′) for the range of validity of "energy equipartition" and "zero-point scattering," respectively, the single contribution of each valley is summed up taking into account their respective car-

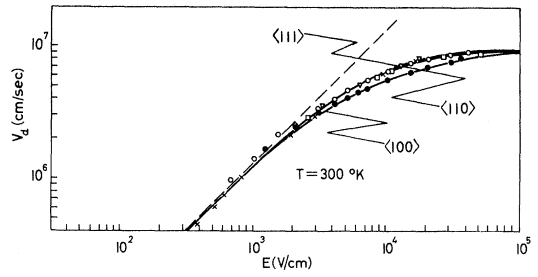

FIG. 2. Electron drift velocity v_d vs applied electric field E at 300 °K for different crystallographic directions in Si. ×, experimental values deduced from Fig. 1 of Boichenko and Vasetskii (Ref. 13) for $E \parallel \langle 111 \rangle$, reducing the current-density values to drift velocity values; □, experimental values from Fig. 2 of Sigmon and Gibbons (Ref. 14) for $E \parallel \langle 111 \rangle$. Experimental values of Canali *et al.* (Ref. 12): ○ for $E \parallel \langle 111 \rangle$, ▽ for $E \parallel \langle 110 \rangle$, ● for $E \parallel \langle 100 \rangle$.

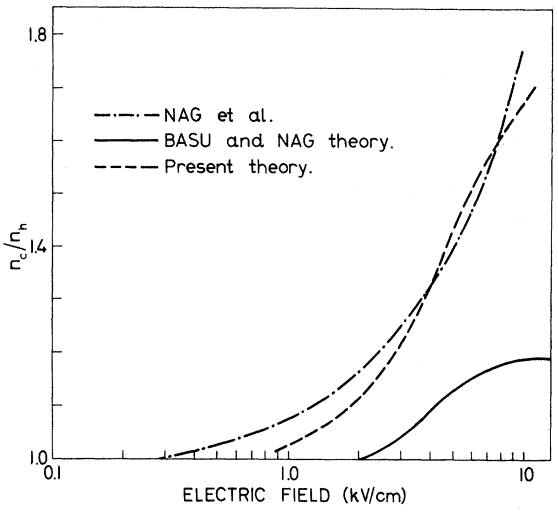

FIG. 3. Ratio of cool-to-hot valley population n_c/n_h at 300 °K vs applied electric field E parallel to the ⟨100⟩ crystallographic direction in Si. The dot-lined curve represents values obtained by Nag, Paria, and Basu (Ref. 6) from the analysis of experimental data. The full line represents the theoretical values of Basu and Nag (Ref. 1). The dashed line represents the theoretical values of Costato and Reggiani (Ref. 7).

TABLE I. Constants of n-Si used for calculations.

Quantity	Value	Units	Reference
$m_{\perp n}/m_0$	0.1905	...	26
$m_{\parallel n}/m_0$	0.9163	...	26
s	9.037×10^5	cm/sec	27
ρ	2.329	gr/cm^3	28
θ_{fLA}	540	°K	16
θ_{fTO}	680	°K	18, 19
θ_{gLA}	307	°K	16
E_1	14.5	eV	7
$(DK)_f$	3×10^8	eV/cm	19
$(DK)_g$	4.5×10^8	eV/cm	11
$2\Delta(0\,°K)$	3.69	eV	29

rier concentration in order to give the total longitudinal drift velocity v_d as a function of the applied electric field E at different lattice temperatures T.

Let us mention that the electron-phonon scattering mechanisms used by CS seem to us to have been erroneously interpreted by BN who defined the CS assumptions to be "quite different from the known scattering mechanisms in n-type Si." In fact, CS used in their simplified theoretical approach an intervalley phonon with an equivalent temperature of 343 °K obtained by weight averaging two phonon energies on their respective contribution (i.e., 56% for $\theta_g = 270$ °K, and 22% for $\theta_f = 530$ °K) following the approach of Dumke,[4] which has been confirmed by Onton[16] and Folland.[18]

Let us make the following conclusions. The theoretical approach of BN in using the Herring and Vogt[9] transformation is not fully acceptable, but their results may be considered as a useful approximation. In addition, BN used the intervalley phonon energies assumed through averages by Long[5] in the fit of Ohmic mobility data and ignored in their paper the previous results of Dumke[4] which were obtained from optical data analysis. Consequently, the physical picture of the intervalley phonon scattering mechanisms assumed by BN is open to criticism, since it has been extensively modified by theoretical and experimental findings.[16–19] Let us emphasize that Basu and Nag[23] have recently found close agreement between theory and experiment in explaining the high-field Hall mobility in n-type Si making use of the results of Dumke.

The authors are grateful to Dr. M. Asche for having clarified many fundamental aspects of the problem, to Dr. N. O. Folland for having made available his manuscript prior to publication, to Professor J. Meyer and Dr. C. Jacoboni for stimulating discussions, to Professor A. Alberigi Quaranta for experimental data support, and to Professor A. Loria for his interest in the present research.

---

†Research supported in part by the C. N. R. (National Research Council).

[1]P. K. Basu and B. R. Nag, Phys. Rev. B 1, 627 (1970).

[2]M. Costato and S. Scavo, Nuovo Cimento 52B, 236 (1968).

[3]M. Costato and S. Scavo, Nuovo Cimento 54B, 169 (1968).

[4]W. P. Dumke, Phys. Rev. 118, 938 (1960).

[5]D. Long, Phys. Rev. 120, 2024 (1960).

[6]B. R. Nag, H. Paria, and P. K. Basu, Phys. Letters 28A, 202 (1968).

[7]M. Costato and L. Reggiani, Phys. Status Solidi (to be published).

[8]Our data also improve the fit of hot mobility of NPB in comparison with the theoretical data of BN reported in Fig. 1 of Ref. 1.

[9]C. Herring and E. Vogt, Phys. Rev. 101, 944 (1956).

[10]M. Asche and O. G. Sarbei, Phys. Status Solidi 33, 9 (1969).

[11]M. Costato and L. Reggiani, Phys. Status Solidi 38, 665 (1970).

[12]C. Canali, G. Ottaviani, and A. Alberigi Quaranta J. Phys. Chem. Solids (to be published).

[13]B. L. Boichenko and W. M. Vasetskii, Fiz. Tverd. Tela 7, 2021 (1965) [Soviet Phys. Solid State 7, 1631 (1966)].

[14]P. W. Sigmon and J. F. Gibbons, Appl. Phys. Letters 15, 320 (1969).

[15]M. Asche, B. L. Boichenki, and O. G. Sarbei, in *Proceedings of the Ninth International Conference on the Physics of Semiconductors, Moscow*, 1968, edited by S. M. Ryvkin (Nauka Publishing House, Leningrad, 1968), Vol. 2, p. 793; M. Asche, W. Bondar, and O. G. Sarbei, Phys. Status Solidi 11, 255 (1965).

[16]A. Onton, Phys. Rev. Letters 22, 288 (1969).

[17]M. Lax and J. J. Hopfield, Phys. Rev. 124, 115 (1961).

[18]N. O. Folland, Phys. Rev. B 1, 1648 (1970).

[19]M. Costato, S. Fontanesi, and L. Reggiani, Nuovo Cimento Letters 1, 946 (1969).

[20]H. Fröhlich and B. V. Paranjape, Proc. Phys. Soc. (London) B69, 21 (1956).

[21]The present approach seems to be still reasonable for the case of Si in studying the carrier mobility even when the experimental concentration is two orders of magnitude lower than that necessary to validate fully the use of a displaced Maxwell-Boltzmann distribution function, as it was calculated by Stratton (Ref. 22) and reported also by Basu and Nag (Ref. 23) in a subsequent paper.

[22]R. Stratton, Proc. Roy. Soc. (London) A242, 355 (1957).

[23]P. K. Basu and B. R. Nag, J. Phys. C 2, 2396 (1969).

[24]We have found that the "zero-point scattering" approximation must be substituted in place of the "energy equipartition" approximation when the lattice temperature T is less than the quantity,

$$T = T_e^{1/2}(3\, m_\perp s^2/K_B)^{1/2}\,.$$

[25]R. A. Stradling and V. V. Zhukov, Proc. Phys. Soc. (London) 87, 263 (1966).

[26]J. C. Hensel, H. Hasegawa and N. Nakayama, Phys. Rev. 138, A225 (1965).

[27]M. Costato and S. Scavo, Ist. Naz. Fis. Nucleare Report No. INFN/FM-68-4, 1968 (unpublished).

[28]J. J. Hall, Phys. Rev. 161, 756 (1967).

[29]M. Costato and S. Scavo, Nuovo Cimento 56B, 343 (1968).