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[24]Since the magnetic contribution to the isotropic volume contraction is small for NiO (Fig. 6), it is difficult to accurately determine a value for $J_2\epsilon_2$ from our lattice constant data. After several trials using various values of $J_2\epsilon_2$, we found that $J_2\epsilon_2 = 60\,°\mathrm{K}$ gave a reasonable fit to the experimental data.

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PHYSICAL REVIEW B VOLUME 3, NUMBER 3 1 FEBRUARY 1971

Current-Induced Intermediate State in Type-1 Superconductors

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We have improved our previous model of the intermediate state in type-1 current-carrying superconductors. In addition to predicting a resistance transition in reasonable agreement with experimental observations, the model gives good agreement with experimental values for the radius of the intermediate-state core as obtained by Rinderer.

In a previous paper[1] (referred to hereafter as BM) we presented a model of the intermediate state in current-carrying type-1 superconductors. The main theoretical criterion used was that, at equilibrium, the magnetic field H at all points on a normal-superconducting boundary should have the critical value H_c. Ideally the field should also be H_c throughout the whole normal region, but it is not possible to find a finite structure to satisfy this ideal condition. Supercooling is therefore involved, since the structure gives values of H in the normal regions less than H_c. We now present an improved model which produces less supercooling than the earlier model did and show that the supercooling required is acceptable.

The intermediate-state structure presented by BM for the case of critical current ($i = i_c$) is shown in Fig. 1(a), and the magnetic field distribution in each normal region is indicated in Fig. 2. It can be seen that the supercooling required is maximum near the axis of the wire. We note, however, that on the axis itself an infinite current density would be required to produce the critical field H_c and this is not only physically unrealistic but would also give rise to a temperature singularity. In practice it is reasonable to expect that the structure is such as to give a large, but finite, current density and a small finite increase in temperature near the axis. As a result, H_c would have a smaller value near the axis, and hence the supercooling requirement there is diminished.

The question now is whether or not the supercooling indicated is plausible under normal experimental conditions. We note that most resistance transition experiments are carried out near T_c and, under these conditions, Ginzburg[2] has shown that the lower supercooling limit for an ideal type-1 superconductor is given by

$$S_l = H_{sc}/H_c = \sqrt{2}\ \kappa ,$$

where H_{sc} is the lowest field at which the normal phase can persist and κ is the Ginzburg-Landau parameter. In agreement with this theory Faber[3] experimentally found values of $S_l \sim 0.16$ for tin and $S_l \sim 0.11$ for indium. Besides, recent experiments[4] with extruded plycrystalline indium wires (similar to those used in resistance-transition experiments) give $S_l \sim 0.36$. Thus the supercooling required by the structure of Fig. 1(a) is well within acceptable limits.

BM suggest that when the current increases above i_c, the superconducting cores will shrink to an equilibrium shape for each value of current so as to

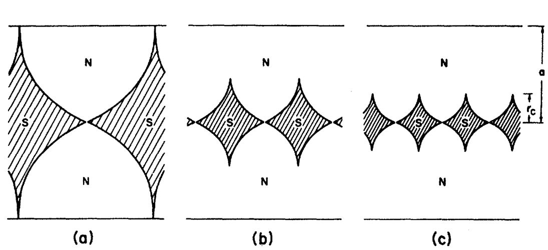

FIG. 1. Structure of the intermediate state for three values of applied current: (a) $i = i_c$, (b) $i = 1.6\ i_c$, (c) $i = 2.3\ i_c$.

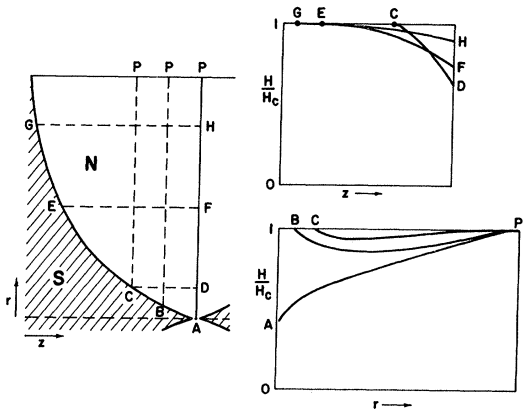

FIG. 2. Distribution of the magnetic field in a normal region. A quarter of the diametral section of a normal region is shown at left, and the field distribution along the lines AP, BP, CP and CD, EF, and GH is shown at right. r represents a radial direction and z the axial direction.

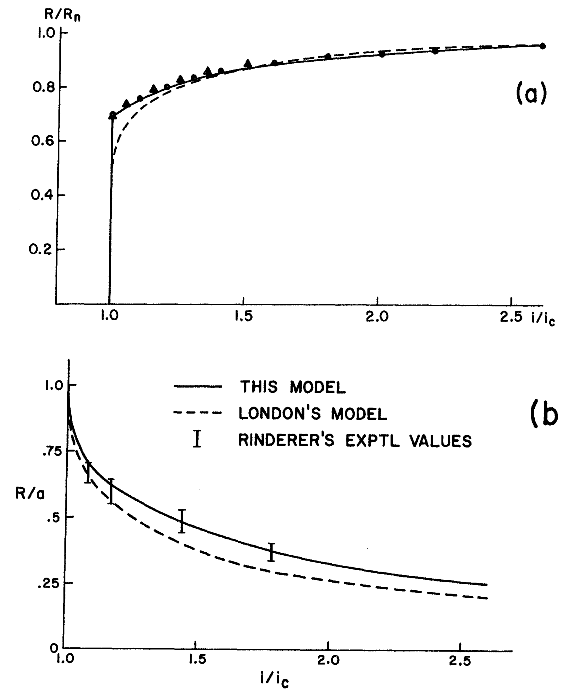

FIG. 3. (a) Return of resistance curve (R is actual resistance, R_n is fully normal resistance): solid line, this model; dashed line, Ref. 5; closed circles, experimental values for 1.65-mm-diam In wire, D. C. Baird and B. K. Mukherjee (unpublished); closed triangles, experimental values for 3-mm-diam In wire, Ref. 6. (b) Variation of the radius r_c of the intermediate state core (see Fig. 1) as a fraction of wire radius a with applied current: solid line, this model; dashed line, Ref. 5; experimental values, Ref. 7.

maintain the condition $H = H_c$ along the boundaries, and show two such structures. These structures indicate that, as the current increases, so does the axial width of the normal regions, and it follows that the supercooling required near the center of the normal regions would increase and become unacceptably large. An alternative possibility is shown in Fig. 1: As i increases over i_c, a fully normal sheath is formed surrounding an intermediate state core: within the core, the structure will be determined by the same equilibrium conditions as are valid at $i = i_c$, and we may therefore assume that the axial periodicity of the structure decreases so as to maintain the optimum $n-s$ boundary shape which allows H to equal H_c on the boundary. It follows that the supercooling called for in the normal regions is not greater than it was at $i = i_c$.

In Fig. 3(a) the resistance transition predicted by the present model is compared with the London model and with experimental values obtained for "thick" wires (diameter $\gtrsim 1$ mm). For thinner wires secondary effects occur; these have been discussed by BM and the treatment given there remains valid.

In addition to the usual consideration of resistance transitions, an independent check on the validity of any model of the intermediate state in current-carrying type-1 superconductors is provided by Rinderer's measurements of the radius of the intermediate-state core as a function of applied current.[7] Figure 3(b) shows that the values obtained by Rinderer are in good agreement with the present model, whereas they do not agree well with the values predicted by London.

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Thermal Conductivity in Pure Two-Band Superconductors in High Magnetic Fields

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The theory proposed by Maki, for the electronic thermal conductivity in the gapless region of a pure type-II superconductor, is extended to include the presence of a second superconducting band so that it can be used to describe the mixed-state thermal conductivity in clean transition-metal superconductors near H_{c2}. The general features of Maki's one-band expression remain in the two-band expression.

I. INTRODUCTION

The qualitative features of the theory proposed by Maki,[1] for the thermal conductivity in the gapless region of a pure type-II superconductor, have been seen in the recent measurements[2,3] of the thermal conductivity in the mixed state in clean niobium superconductors near the upper critical field, i.e., the $(H_{c2}-H)^{1/2}$ field dependence and the anisotropy of the conductivity with respect to the relative angle between the direction of the heat flow and the external magnetic field were seen. Quantitative agreements were achieved by treating the density of states an an adjustable parameter. Because this treatment of the density of states would imply a false behavior of the magnetic properties, many authors[4,5] have questioned the parametrization of the density of states. They believe that quantitative agreements would be achieved by the proper treatment of the anisotropy of the Fermi surface. However, it has been shown[6] that the proper treatment of the anisotropy will lead only to a small enhancement of the results based on a spherical Fermi surface (e.g., the enhancement factor for Nb is[7] 1.13).

The recent discovery of a second energy gap[8] and a second transition temperature[9] in clean niobium superconductors indicates that the usual one-