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Theory of observed cyclotron-resonance-linewidth behavior

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The quantum-limit cyclotron-resonance linewidth is studied theoretically, assuming electron scattering by acoustic phonons and ionized impurities. A minimum is predicted for the linewidth as a function of magnetic field but only for a restricted range of electron and ionized-impurity densities. Near this minimum the linewidth is proportional to the square root of the impurity density and is independent of temperature.

I. INTRODUCTION

The cyclotron-resonance absorption linewidth, and particularly its dependence on temperature and magnetic field, are very sensitive to the type of scattering mechanism affecting the carriers and hence can be used to probe these mechanisms. Many theoretical[1-8] and experimental[9-13] studies have been made of the linewidth in the quantum limit. However, there is still much which remains to be explained.

Recently the authors developed a general method for the determination of the frequency-dependent conductivity[14] and applied this to a study of the quantum-limit cyclotron resonance line shape for electrons scattered by acoustic phonons. The temperature and magnetic field dependence of the linewidth were analyzed for several temperature ranges.

However, at low enough temperatures scattering by ionized impurities is always important and must be taken into account. In this paper we study a system in which electrons are scattered by *both* acoustic phonons and ionized impurities.

An ionized impurity is assumed to give rise to a *screened* Coulomb potential where, however, the form of the screening takes into account the quantized nature of the electron motion in the magnetic field.[15] Such screening is important only if the inverse screening length is greater than the wave vector of the phonons which give the main contribution to the linewidth. The inverse screening length is proportional to the square root of the electron density n_e, and the maximum wave vector of the "important" phonons is proportional to the magnetic field B. Explicitly, we find the screening to be significant only if the electron density exceeds a critical value $n_{\rm cr}(T,B)$,

$$n_{\rm cr}(T,B)=\frac{2\epsilon\epsilon_0 k_B T}{\hbar e}B=\frac{2\epsilon\epsilon_0 m^*\omega_0 k_B T}{\hbar e^2}\;, \qquad (1.1)$$

where ϵ and m^* are the high-frequency dielectric constant and the electronic band mass, B is the magnetic field, and the resonance frequency ω_0 is given by eB/m^*. It is important that this critical density depends on temperature and magnetic field and that, in particular, screening becomes less important as the magnetic field increases.

The main conclusions of this paper may be summarized as follows: If the electron density is less than $n_{\rm cr}$, and the density n_i of ionized impurities satisfies the inequalities

$$\frac{8(k_B T)^3}{\rho_m}\left[\frac{\epsilon\epsilon_0 D m^*}{Ze^2\hbar^2 S_0}\right]^2 \ll n_i \lesssim \frac{(k_B T)^5}{40\rho_m}\left[\frac{\epsilon\epsilon_0 D}{Ze^2\hbar^2 S_0^3}\right]^2 , \qquad (1.2)$$

then (i) the linewidth γ exhibits a minimum as a function of magnetic field B (or ω_0), (ii) near this minimum γ is proportional to $n_i^{1/2}$ and is independent of T, and (iii) the magnetic field at which the minimum occurs is proportional to $n_i^{1/2}T^{-1/2}$.

In Eq. (1.2), S_0 is the sound velocity, ρ_m is the mass density, D is the deformation potential, and Ze is the charge on an impurity ion. If the impurity density lies outside the above range there is no minimum: If n_i is below the range, then γ is an increasing function of magnetic field, whereas if n_i is above the range, γ is a decreasing function. These predictions are in substantial agreement with the experiments of McCombe *et al.*[10]

The lack of a temperature dependence for the minimum linewidth has been taken to indicate that phonon scattering is unimportant[10,16] and most theoretical attempts to explain the experimental results have assumed that ionized-impurity scattering is dominant. In the present model the linewidth minimum arises because of competition between the phonon and impurity scattering: The phonon contribution to the linewidth increases with increasing magnetic field, whereas the contribution from ionized-impurity scattering decreases.

Arora *et al.*[7,8] have also considered the combined effects of phonon and impurity scattering. However, they assumed *elastic* scattering, neglected the effects of screening, and concluded in contrast to our predictions that a linewidth minimum should *always* occur.

II. FORMULATION OF THE PROBLEM

The power absorption for a circularly polarized electric field $\vec{E}(t)$ of frequency ω, perpendicular to a static magnetic field $\vec{B}$, is

$$P[\omega]=\tfrac{1}{2}E^2\mathrm{Re}[\sigma_{+-}(\omega)]$$
$$=\tfrac{1}{2}E^2\mathrm{Re}\{\sigma_{xx}(\omega)+\sigma_{yy}(\omega)+i[\sigma_{yx}(\omega)-\sigma_{yx}(\omega)]\}\ , \tag{2.1}$$

where $\sigma_{ij}(\omega)$ is the complex conductivity tensor, and the z axis is taken parallel to $\vec{B}$. In an earlier paper[1] the present authors discussed the quantum-limit cyclotron-resonance absorption in an n-type semiconductor where the only scattering mechanism was via acoustic phonons. In this paper we shall consider the more realistic system characterized by the Hamiltonian

$$H=\sum_k h(\vec{r}_k,\vec{P}_k)+\sum_{\vec{q}}\hbar\omega_{\vec{q}}b^\dagger_{\vec{q}}b_{\vec{q}}\ ,$$
$$h(\vec{r},\vec{P})=\frac{[\vec{P}+e\vec{A}(\vec{r})]^2}{2m^*}+\sum_{\vec{q}}[\gamma_{\vec{q}}(\vec{r})b_{\vec{q}}+\gamma^\dagger_{\vec{q}}(\vec{r})b^\dagger_{\vec{q}}]+\sum_\alpha V(\vec{r}-\vec{R}_\alpha)\ . \tag{2.2}$$

$b^\dagger_{\vec{q}}$ and $b_{\vec{q}}$ are the creation and annihilation operators for a phonon of wave vector $\vec{q}$ and energy $\hbar\omega_{\vec{q}}$; $\gamma_{\vec{q}}(\vec{r})$ describes the interaction of the electron and phonon and is of the form $C_{\vec{q}}\exp(i\vec{q}\cdot\vec{r})$, where $C_{\vec{q}}$ depends on the type of interaction; $V(\vec{r}-\vec{R}_\alpha)$ is the potential of the electron due to an impurity at $\vec{R}_\alpha$; $\vec{A}$ is a vector potential which gives rise to the static field $\vec{B}$.

Lodder and Fujita[2] developed a general theory of cyclotron resonance by means of a proper-connected diagram technique applied to the Kubo formula for $\sigma_{+-}(\omega)$. This theory applied to an electron-phonon system[1,3] yielded results in substantial agreement with experiments on very pure Ge.[9] Recently Suzuki and Dunn[14] developed a different and much simpler approach to the evaluation of the conductivity tensor $\sigma_{ij}(\omega)$ suitable for a strongly interacting electron-phonon system. This theory based on a resolvent super-operator technique in the weak coupling limit yielded the same equations as the diagram approach.

The extension of the theories to the present case is straightforward. If we restrict frequencies to the neighborhood of the resonance frequency $\omega\approx\omega_0$, then the power absorption is given in the quantum-limit $h\omega_0\gg k_BT$ by the following set of equations:

$$\mathrm{Re}[\sigma_{+-}(\omega)]=\frac{2e^2}{m^*V}\sum_{\vec{k}}\frac{f_{0,\vec{k}}\Gamma(k_z)}{(\omega-\omega_0)^2+\Gamma^2(k_z)}\ , \tag{2.3}$$

$$\Gamma(k_z)=\Gamma_p(k_z)+\Gamma_i(k_z)\ , \tag{2.4}$$

$$\Gamma_p(k_z)=\frac{\pi}{\hbar}\sum_{\vec{q}}|C_{\vec{q}}|^2t^2_{\vec{q}}e^{-t}_{\vec{q}}\times[(N_{\vec{q}}+1)\delta(\epsilon(k_z-q_z)-\epsilon(k_z)+\hbar\omega_{\vec{q}})+N_{\vec{q}}\delta(\epsilon(k_z-q_z)-\epsilon(k_z)-\hbar\omega_{\vec{q}})]\ , \tag{2.5}$$

$$\Gamma_i(k_z)=\frac{\pi N_i}{\hbar}\sum_{\vec{q}}|V(\vec{q})|^2t^2_{\vec{q}}e^{-t}_{\vec{q}}\delta(\epsilon(k_z-q_z)-\epsilon(k_z))\ . \tag{2.6}$$

$f_{0,\vec{k}}$ is the distribution function for the lowest Landau level; $\Gamma(k_z)$ is the total relaxation rate associated with transitions between the $n=1$ and $n=0$ Landau levels and comprises Γ_p, the phonon contribution, and Γ_i, the impurity contribution; $N_{\vec{q}}$ is the mean phonon occupation number; $\epsilon(k_z)$ is the energy associated with the motion along the magnetic field $\hbar^2k_z^2/2m^*$; $t_{\vec{q}}$ is given by

$$t_{\vec{q}}=\frac{\hbar q_\perp^2}{2m^*\omega_0}=\frac{\hbar}{2m^*\omega_0}(q_x^2+q_y^2)\ . \tag{2.7}$$

The cyclotron resonance width γ is determined in general by

$$P(\omega_{\max}+\gamma_R)=\tfrac{1}{2}P(\omega_{\max})\ ,$$
$$P(\omega_{\max}-\gamma_L)=\tfrac{1}{2}P(\omega_{\max})\ , \tag{2.8}$$
$$\gamma=\gamma_R+\gamma_L\ ,$$

where $\omega_{\max}$ corresponds to the maximum power absorption. For a symmetric line, of course, $\gamma_R=\gamma_L$.

The temperature dependence of γ arises from two sources: The electron distribution function $f_{0,\vec{k}}$, and the relaxation rate $\Gamma(k_z)$. It can therefore be very misleading to try to determine γ directly from $\Gamma(k_z)$, and it is certainly incorrect to associate, as is often done, γ with $2\Gamma(0)$. However, as a *very rough order* of *magnitude,* we may write for a nondegenerate semiconductor

$$\gamma=2\Gamma(k_z=(2m^*k_BT/\hbar^2)^{1/2})\ . \tag{2.9}$$

This formula can be used to give a guide to the temperature and magnetic field dependences of γ. In this paper we shall *not* make use of this short-cut procedure, but we evaluate γ directly by using Eqs. (2.3)—(2.8).

III. SCREENED INTERACTIONS

The effect of the electron-electron interactions can be taken into account approximately[17,18] by dividing the bare interactions by the purely electronic contribution to the static wave-vector-dependent dielectric constant $\epsilon(\vec{q})$. This electronic dielectric constant is usually evaluated in the Debye-Thomas-Fermi approximation

$$\epsilon(\vec{q})=\epsilon(1+\lambda_s^2/q^2)\ , \tag{3.1}$$

where λ_s is the classical inverse screening length and is given by

$$\lambda_s^2=\frac{n_ee^2}{\epsilon\epsilon_0k_BT}\ . \tag{3.2}$$

In the presence of a static magnetic field the motion of the electrons is quantized, and hence their screening effects are modified. Horing[15] has evaluated the dielectric constant in the random-phase approximation including the effects of the magnetic field. His result may be expressed as

$$\epsilon(\vec{q})=\epsilon(1+\lambda^2(\vec{q})/q^2)\ , \tag{3.3}$$

where the inverse screening length $\lambda(\vec{q})$ is, for a nondegenerate semiconductor, given by

$$\lambda^2(\vec{q})=\lambda_s^2\int_0^1 du\,\exp\left[-\frac{\hbar^2 q_z^2(1-u^2)}{8m^*k_BT}-\frac{\hbar q_\perp^2}{2m^*\omega_0}\left[\frac{\cosh(\hbar\omega_0/2k_BT)-\cosh(\hbar\omega_0 u/2k_BT)}{\sinh(\hbar\omega_0/2k_BT)}\right]\right]\ . \tag{3.4}$$

In the limit $\hbar\to 0$, this reduces to a classical result. The integral in Eq. (3.4) is always less than, or equal to, one, and so the presence of the magnetic field *reduces* the screening effect.

A useful approximation to Eq. (3.4), obtained by making a piecewise linear approximation to the argument of the exponential, is

$$\lambda^2(\vec{q})=\lambda_s^2\left[1-\left[\frac{[\exp(-u)-1](1+u-v)+u}{v}\right]\right]\ ,$$

$$u=\frac{\hbar^2 q_z^2}{8m^*k_BT}+\frac{\hbar q_\perp^2}{2m^*\omega_0}\left[\frac{\cosh\left[\dfrac{\hbar\omega_0}{2k_BT}\right]-1}{\sinh\left[\dfrac{\hbar\omega_0}{2k_BT}\right]}\right]\ , \tag{3.5}$$

$$v=\frac{\hbar^2(q_\perp^2+q_z^2)}{2m^*k_BT}\ .$$

Numerical tests show that this is a good approximation both for very small and very large values of $\vec{q}$. In the regions which are important for the current problem, $\hbar^2 q_z^2/2m^*\le k_BT$, $\hbar^2 q_\perp^2/2m^*\le\hbar\omega_0$, $\hbar\omega_0\gg k_BT$, the approximation exaggerates the screening effect but by less than a few percent.

Shin and co-workers[4] have, in their study of cyclotron-resonance absorption, used a much cruder approximation for the inverse screening length,

$$\lambda^2(\vec{q})\approx\lambda_s^2\frac{4m^*k_BT}{\hbar^2 q_z^2}\ . \tag{3.6}$$

A detailed analysis of expression (3.4) shows that this is valid only if

$$\frac{\hbar^2 q_z^2}{2m^*}\gg k_BT,\frac{\hbar^2 q_\perp^2}{2m^*}\ .$$

However, as we shall show, the $\vec{q}$ regions for the phonons which produce the major contributions to the linewidth are

$$\frac{\hbar^2 q_z^2}{2m^*}\le k_BT,\quad \frac{\hbar^2 q_\perp^2}{2m^*}\le\hbar\omega_0\ .$$

The approximation is therefore inappropriate. It has the unfortunate effect of greatly enhancing $\lambda^2(\vec{q})$ for small values q_z and hence of exaggerating the screening effects.

The screened ionized-impurity potential is

$$V_{\vec{q}}=-\frac{Ze^2}{\epsilon\epsilon_0 q^2 V}\frac{1}{1+\lambda^2(\vec{q})/q^2}\ , \tag{3.7}$$

and the deformation potential interaction is given by

$$C_{\vec{q}}=i\frac{D(\hbar q/2\rho_m S_0 V)^{1/2}}{1+\lambda^2(\vec{q})/q^2}\ , \tag{3.8}$$

where the deformation potential D already contains the factor ϵ^{-1}.

IV. RELAXATION RATE Γ

In order to simplify the expressions for Γ we introduce the following dimensionless variables:

$$K=\frac{\hbar k_z}{m^*S_0},\quad Q_\perp=\frac{\hbar q_\perp}{m^*S_0},\quad Q_z=\frac{\hbar q_z}{m^*S_0}\ ,$$

$$\Omega=\frac{\hbar\omega}{\frac{1}{2}m^*S_0^2},\quad \Omega_0=\frac{\hbar\omega_0}{\frac{1}{2}m^*S_0^2}\ , \tag{4.1}$$

$$\Lambda_s=\frac{\hbar\lambda_s}{m^*S_0},\quad \bar{T}=2k_BT/m^*S_0^2\ .$$

The relaxation rate Γ_p due to acoustic-phonon scattering via a deformation-potential coupling was determined previously. The result comprises contributions due to phonon emission Γ_p^+ and to phonon absorption Γ_p^-, and for positive K is

$$\Gamma_p^+(K)=\frac{A_p}{8\Omega_0^2}\Theta(K-1)\int_0^{2(K-1)}dQ_z\frac{\exp(-Q_\perp^2/\Omega_0)}{1-\exp[Q_z(Q_z-2K)/\bar{T}]}\frac{Q_\perp^4 Q_z^2(Q_z-2K)^2}{[1+\Lambda^2(Q_\perp,Q_z)/(Q_\perp^2+Q_z^2)]^2}\ , \tag{4.2}$$

$$\Gamma_p^-(K)=\frac{A_p^2}{8\Omega_0^2}\left[\int_{-\infty}^{\min\{0,2(K-1)\}}dQ_z+\int_{2(K+1)}^{\infty}dQ_z\right]\frac{\exp(-Q_\perp^2/\Omega_0)}{\exp[Q_z(Q_z-2K)/\bar{T}]-1}\frac{Q_\perp^4 Q_z^2(Q_z-2K)^2}{[1+\Lambda^2(Q_\perp,Q_z)/(Q_\perp^2+Q_z^2)]^2}\ , \tag{4.3}$$

$$Q_\perp^2=\frac{Q_z^2}{4}[(Q_z-2K)^2-4]\ , \tag{4.4}$$

$$A_p = \frac{D^2 m^{*3} S_0}{4\pi \hbar^4 \rho_m} . \tag{4.5}$$

The dimensionless inverse screening length Λ is given by

$$\Lambda^2(Q_\perp, Q_z) = \Lambda_s^2 \int_0^1 du \exp\left[-\frac{Q_z^2}{4\overline{T}}(1-u^2) - \frac{Q_\perp^2}{\Omega_0} \frac{\cosh(\Omega_0/2\overline{T}) - \cosh(\Omega_0 u/2\overline{T})}{\sinh(\Omega_0/2\overline{T})} \right] . \tag{4.6}$$

It should be noted that for $0 < K < 1$, that is, $0 < k_z < m^* s_0/\hbar$, the phonon emission process is prohibited.

The expression for the relaxation rate due to the impurity scattering is

$$\Gamma_i(K) = \frac{A_i n_i}{\Omega_0^2 |K|} \int_0^\infty dQ_\perp Q_\perp^5 \exp\left[-\frac{Q_\perp^2}{\Omega_0} \right] \left[\frac{1}{[Q_\perp^2 + \Lambda^2(Q_\perp, 0)]^2} + \frac{1}{[Q_\perp^2 + 4K^2 + \Lambda^2(Q_\perp, 2K)]^2} \right] , \tag{4.7}$$

$$A_i = \frac{1}{4\pi\epsilon_0^3 m^{*2}} \left[\frac{Ze^2}{\epsilon\epsilon_0} \right]^2 . \tag{4.8}$$

If we assume that the electron occupation $f_{0,\vec{k}}$ is simply a Boltzmann distribution, then the power absorption is proportional to

$$\mathrm{Re}[\sigma_{+-}(\omega)] = \frac{4e^2 n_e}{m^* \overline{T}^{1/2}} \int_0^\infty dK \frac{\exp(-K^2/\overline{T})\Gamma(K)}{(\omega-\omega_0)^2 + \Gamma^2(K)} , \tag{4.9}$$

where $\Gamma(K) = \Gamma_i(K) + \Gamma_p^+(K) + \Gamma_p^-(K)$.

The electron density n_e is a function of temperature in thermally generated cyclotron resonance. At low temperatures the carriers are sometimes generated by optical excitation: In such cases n_e is a function of the intensity of incident light.

An analysis of the above equations shows that the main contribution to the linewidth from the impurity scattering arises from the following ranges:

$$K \lesssim \overline{T}^{1/2}, \quad Q_z = 0, 2K, \quad Q_\perp \lesssim \Omega_0^{-1} .$$

For the phonon scattering in the high-temperature regime, $\Omega_0 \gg \overline{T} \gg \Omega_0^{1/2}$, the important ranges are

$$K \lesssim \overline{T}^{1/2}, \quad Q_z \lesssim \overline{T}^{1/2}, \quad Q_\perp \lesssim \Omega_0^{-1} .$$

With the use of these values and the fact that $\Lambda^2 < \Lambda_s^2$, we see that, in the quantum limit, screening is negligible if

$$\Lambda_s^2 \ll \Omega_0 , \tag{4.10}$$

which says that n_e must be much less than n_{cr} [Eq. (1.1)].

V. QUALITATIVE BEHAVIOR

In order to demonstrate the qualitative behavior of the linewidth as a function of magnetic field and temperature, we consider various restricted ranges.

A. No screening

If the electron density satisfies $n_e < n_{\mathrm{cr}}(T,B)$, then screening is negligible, and we can obtain approximate expressions for the relaxation rates. The ionized-impurity relaxation rate is

$$\Gamma_i(K) \approx \begin{cases} \dfrac{A_i n_i}{\Omega_0 |K|}, & K^2 \lesssim \Omega_0 \quad (5.1) \\[2ex] \dfrac{A_i n_i}{2\Omega_0 |K|}, & K^2 \gtrsim \Omega_0 . \quad (5.2) \end{cases}$$

If we further assume the high-temperature limit $\Omega_0 \gg \overline{T} \gg \Omega_0^{1/2}$, then the phonon-occupation number $N = \{\exp[Q_z(Q_z - 2K)/\overline{T}] - 1\}^{-1}$, which appears in Eqs. (4.2) and (4.3), can be approximated by $\overline{T}/Q_z(Q_z - 2K)$ and the phonon contribution to the relaxation rate is

$$\Gamma_p(K) \approx \begin{cases} 0.2 A_p \Omega_0^{3/4} \overline{T}, & K^2 \lesssim 2\sqrt{2\Omega_0} \quad (5.3) \\[2ex] \dfrac{A_p \Omega_0 \overline{T}}{|K|}, & \Omega_0 \gtrsim K^2 \gtrsim 2\sqrt{2\Omega_0} \quad (5.4) \\[2ex] \dfrac{A_p \Omega_0 \overline{T}}{2|K|}, & K^2 > \Omega_0 . \quad (5.5) \end{cases}$$

These limiting expressions for the two different scattering mechanisms are similar for the high-K and intermediate-K regions, but they differ considerably for the small-K region. The ionized-impurity relaxation rate diverges as $K \to 0$, whereas the *inelastic* nature of the phonon scattering provides a "cutoff" below $K^2 \approx 2(2\Omega_0)^{1/2}$.

In the high-temperature limit this inelastic region represents only a very small part of the K values which contribute to the linewidth $-K^2 \lesssim \overline{T}$. Hence $\Gamma_i(K)$ and $\Gamma_p(K)$ have, over most of the K region, the same K dependence and, because of this, a form of Matthiesson's rule holds,

$$\gamma = \gamma_i + \gamma_p , \tag{5.6}$$

where γ_i and γ_p are the linewidths which would occur in the presence of impurity scattering and phonon scattering, respectively. In this high-temperature region an analytic expression can be obtained for the conductivity

$$\mathrm{Re}[\sigma_{+-}(\omega)] = \frac{e^2 n_e \overline{T}}{\pi m^*} \frac{z}{\omega - \omega_0} e^{z^2} E_1(z^2) , \qquad z = \frac{A_p \Omega_0 \overline{T}^{1/2} + A_i n_i \Omega_0^{-1/2} \overline{T}^{-1/2}}{\omega - \omega_0} , \tag{5.7}$$

where $E_1(z)$ is the exponential integral.[19] The full width at half-peak height is

$$\gamma=2.561(A_p\Omega_0\bar{T}^{1/2}+A_in_i\Omega_0^{-1/2}\bar{T}^{-1/2}) . \quad (5.8)$$

This linewidth exhibits a minimum as a function of Ω_0 (and hence of magnetic field B) and the minimum

$$\gamma_{\min}=5.122(A_pA_in_i)^{1/2} \quad (5.9)$$

occurs at

$$\Omega_{0\min}=\left[\frac{A_in_i}{A_p\bar{T}}\right]^{1/2} . \quad (5.10)$$

It is important to note that the minimum linewidth is *independent* of *temperature.*

There are two approximations essential to the above result: The scattering is elastic and the phonon-occupation numbers are given by their high-temperature limits. As the temperature is lowered the elastic approximation breaks down first at $\bar{T}\approx 6\sqrt{\Omega_0}$ and the approximation to the occupation numbers breaks down at $\bar{T}\approx\sqrt{\Omega_0}$. We can therefore be somewhat more precise about the lower end of the high-temperature region,

$$\Omega_0 \gg \bar{T} \gtrsim 6\sqrt{\Omega_0} . \quad (5.11)$$

If the temperature fails to satisfy (5.11), then the phonon contribution to the linewidth is greatly reduced and no minimum occurs: The linewidth is a purely decreasing function of Ω_0 (or B). At the transition region $\bar{T}\approx 6\sqrt{\Omega_0}$, a very asymmetric and shallow minimum occurs.

If a minimum is to occur, then consistency demands that the condition (5.11) must be satisfied at $\Omega_{0\min}$. Inserting the expression for $\Omega_{0\min}$ gives a restriction on the impurity density: The linewidth will have a minimum as a function of magnetic field only if

$$\bar{T}^3\frac{A_p}{A_i} \ll n_i \lesssim \frac{\bar{T}^5A_p}{6^4A_i} . \quad (5.12)$$

When the appropriate expressions for A_p, A_i are substituted, this yields expression (1.2).

B. Strong screening

If the electron density greatly exceeds $n_{\rm cr}(T,B)$, although the expressions for the relaxation rates become more complicated, we can extract the magnetic field dependence

$$\Gamma_i \sim \Omega_0 \sim B, \quad \Gamma_p \sim \Omega_0^3 \sim B^3 . \quad (5.13)$$

The linewidth is therefore a purely *increasing* function of magnetic field for this range of electron densities.

VI. NUMERICAL RESULTS

We now turn to numerical solutions of Eqs. (4.2)–(4.9) with application to InSb. The relevant data for InSb is $m^*=0.0139m_e$,[20] $\epsilon=17.64$,[21] $S_0=3775m_s^{-1}$,[22] and $\rho_m=5780$ kg m^{-3}.[23] S_0 is calculated from the measured elastic constants to be the rms angular average of the velocity of the quasilongitudinal mode. With the use of these values, the relationships between B and T and the corresponding dimensionless quantities are

$$\Omega_0=1.47\times10^4B, \quad \bar{T}=152.5T .$$

The high-temperature region is specified by

$$96B \gg T \gtrsim 6.4\sqrt{B} , \quad (6.1)$$

and the critical electron density $n_{\rm cr}$, below which screening is unimportant, is in m^{-3},

$$n_{\rm cr}=2.5\times10^{20}BT . \quad (6.2)$$

The assignment of the deformation potential D is a problem: Published values of D span the range 7.2–45 eV.[24] We have therefore regarded D as an adjustable parameter and have chosen it to give the best fit to the linewidth data of Kaplan, McCombe and Wagner.[10] The resulting value for the deformation potential is (in eV)

$$D\approx 60 ,$$

which is just beyond the upper end of the range of measured values. This result lends weight to the argument of Tsidilkovskii and Demchuk[24] that the commonly accepted value, $D\approx 8$, is much too small.

The restricted range of impurity densities which gives rise to a linewidth minimum is then

$$1.2\times10^{15}T^3 \ll n_i \ll 2.0\times10^{16}T^5 . \quad (6.3)$$

If these conditions on the temperature and impurity density are satisfied then our "qualitative" theory predicts a minimum linewidth (in Tesla) of

$$\gamma_{\min}=1.8\times10^{-12}n_i^{1/2} , \quad (6.4)$$

which occurs at a magnetic field

$$B_{\min}=3.1\times10^{-10}\left[\frac{n_i}{T}\right]^{1/2} . \quad (6.5)$$

Figure 1 shows the relaxation rate $\Gamma(K)$, and its constit-

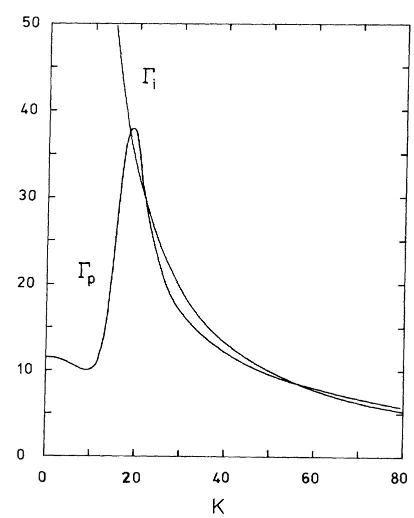

FIG. 1. Constituents $\Gamma_i(K)$ and $\Gamma_p(K)$ of the relaxation rate in units of $m^*S_0^2/2\hbar$: $B=0.5$ T, $T=4.5$ K, $n_i=3.0\times10^{19}$ m^{-3}, $n_e=4.0\times10^{19}$ m^{-3}. ($K=\hbar k_z/m^*S_0$.)

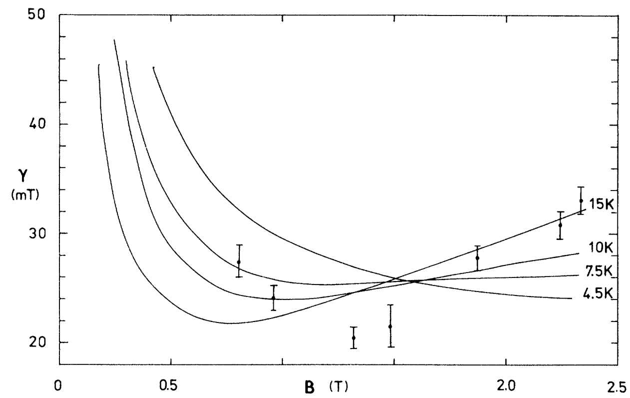

FIG. 2. Comparison of the theoretical and experimental linewidths. Solid lines show the theoretical predictions; data points indicate the experimental results of Kaplan *et al.* (Ref. 10). ($n_i = 3.0\times10^{20}$ m^{-3}, $n_e = 4.0\times10^{19}$ m^{-3}.)

uents $\Gamma_i(K)$ and $\Gamma_p(K)$ calculated for $B=0.5$ T ($\Omega_0=7350$) and $T=4.5$ K ($\overline{T}=686.25$). The characteristic K^{-1} dependence is evident for large K as is the inelastic cutoff of the phonon contribution below

$$K \approx (2\sqrt{2\Omega_0})^{1/2} \approx 16 \ .$$

These values for B and T are just at the boundary of the high-temperature region. The effects of inelasticity are just starting to be significant.

Figure 2 shows the comparison of the computed linewidths for a range of temperatures with the experimental linewidths of Kaplan *et al.*[10] measured at 4.5 K, but which are stated to be independent of temperature to within the experimental accuracy over the range 4.5–15 K. We have used $n_i = 3\times10^{20}$ m^{-3} and $n_e = 4\times10^{19}$ m^{-3}, and we have ignored the observed variation of electron density with magnetic field.

There is obviously a broad agreement between the theory and the experiment together with some discrepancies. The theory predicts minima which are shallower on the high-magnetic-field side than the experimental results, and, in particular, for $T \lesssim 6$ K the minimum disappears. The predicted temperature variation is also somewhat greater than the experimental limits. Both of these occur because the experimental conditions are only just within the high-temperature region ($T=4.5$ K is outside) at the high-magnetic-field side, and hence inelastic effects reduce the phonon contribution. However, in view of the simplicity of the model, the agreement is as good as we have any right to expect. In particular the reality in InSb is complicated by a nonparabolic energy band and, more importantly, by more complicated conduction-band wave functions than is assumed in the model. It is also complicated by (weak) piezoelectric scattering and by a Fermi energy which varies with temperature and magnetic field and is very close to the bottom of the conduction band.

VII. SUMMARY

We have shown that, in agreement with Arora *et al.*[7,8] even at low temperatures the cyclotron-resonance linewidth due to acoustic-phonon scattering should not be ignored, and that as a result of competition between the phonon and ionized-impurity scattering a minimum may occur in the linewidth as a function of magnetic field. The minimum value of the linewidth is proportional of $n_i^{1/2}$ and is only slightly dependent on temperature. Even away from the minimum only a weak temperature dependence is predicted: On the basis of Eq. (5.8), if the linewidth is set at its minimum value and the temperature is then increased or decreased by a factor of 4, keeping the magnetic field fixed, the linewidth is increased by only 25%. This is in substantial agreement with the experimental results.[10,16]

In contrast to the theory of Arora *et al.*[7,8] we have predicated that such behavior should occur only for a restricted range of impurity and carrier densities. This explains the absence of minima in other experiments.[11,12]

In the numerical studies we have concentrated on InSb but, of course, the theory applies (with somewhat less qualification) to other materials. For example, in a sample of n-Ge with $n_i = n_e = 10^{20}$ m^{-3} at $T=4.5$ K, we predict that a minimum in the linewidth should occur at a magnetic field $B \approx 8.6$ T.

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